Number

What you will learn

1-1 Revisiting fractions and decimals
1-2 Approximation and estimation
1-3 Decimals and significant figures
1-4 Adding and subtracting fractions
1-5 Multiplying and dividing fractions
1-6 Converting fractions and decimals
1-7 Adding and subtracting decimals
1-8 Multiplying and dividing decimals
1-9 Indices
1-10 Scientific notation or standard form
1-11 Integers

Buying and selling

The Martinborough Fair is held twice a year in the small township of Martinborough in South Wairarapa. Over 400 stallholders sell all kinds of products to more than 25,000 visitors to each fair. Just some of the products include home preserves, clothing, jewellery, handmade furniture, gadgets for the home, handcrafts, souvenirs, and gardening tools and plants.

At any instant there could be more than 1000 mental calculations going on as people try to determine ‘Is this the best buy?’ ‘Can I afford it?’ or ‘Can I afford to sell at that price?’ The ability to make calculations mentally and to use estimation are tools of a successful stallholder and a canny bargain hunter.
New Zealand Curriculum

LEVEL 4
Number strategies and knowledge

- Use a range of multiplicative strategies when operating on whole numbers.
- Understand addition and subtraction of fractions, decimals and integers.
- Find fractions and decimals of amounts expressed as whole numbers, simple fractions and decimals.
- Apply simple linear proportions including ordering fractions.
- Know the equivalent decimal forms for everyday fractions.

LEVEL 5
Number strategies and knowledge

- Know the relative size and place value structure of decimals to three places.

- Use prime numbers, common factors, and multiples and powers (including square roots).
- Understand operations of fractions, decimals and integers.
- Know commonly used fraction and decimal conversions.
- Know and apply standard form, significant figures, rounding and decimal place value.
Do now

1. What fraction is shaded or (for part h) shown on a number line?
   a) \[
   \text{\includegraphics[width=0.3\textwidth]{circle.png}}
   \]
   b) \[
   \text{\includegraphics[width=0.3\textwidth]{rectangle.png}}
   \]
   c) \[
   \text{\includegraphics[width=0.3\textwidth]{pentagon.png}}
   \]
   d) \[
   \text{\includegraphics[width=0.3\textwidth]{triangle.png}}
   \]
   e) \[
   \text{\includegraphics[width=0.3\textwidth]{circles.png}}
   \]
   f) \[
   \text{\includegraphics[width=0.3\textwidth]{triangles.png}}
   \]
   g) \[
   \text{\includegraphics[width=0.3\textwidth]{cylinder.png}}
   \]
   h) \[
   \text{\includegraphics[width=0.3\textwidth]{bar.png}}
   \]

2. Write each fraction as a decimal.
   a) \[
   \frac{1}{2}
   \]
   b) \[
   \frac{1}{4}
   \]
   c) \[
   \frac{1}{10}
   \]
   d) \[
   \frac{1}{5}
   \]
   e) \[
   \frac{3}{4}
   \]
   f) \[
   \frac{3}{2}
   \]
   g) \[
   \frac{3}{100}
   \]
   h) \[
   \frac{7}{1000}
   \]

3. Write each decimal as a fraction.
   a) \[
   0.5
   \]
   b) \[
   0.25
   \]
   c) \[
   0.7
   \]
   d) \[
   1.5
   \]
   e) \[
   0.1
   \]
   f) \[
   0.2
   \]
   g) \[
   0.01
   \]
   h) \[
   0.003
   \]

4. Find the lowest common denominator of:
   a) \[
   12 \text{ and } 18
   \]
   b) \[
   6 \text{ and } 9
   \]
   c) \[
   25 \text{ and } 40
   \]

5. Write these fractions in simplest form.
   a) \[
   \frac{2}{4}
   \]
   b) \[
   \frac{4}{6}
   \]
   c) \[
   \frac{5}{10}
   \]
   d) \[
   \frac{4}{12}
   \]
   e) \[
   \frac{8}{24}
   \]
   f) \[
   \frac{12}{18}
   \]
   g) \[
   \frac{20}{30}
   \]
   h) \[
   \frac{27}{50}
   \]

6. Convert these measures into the unit shown in brackets.
   a) \[
   3 \text{ m} \ 27 \text{ cm} (\text{cm})
   \]
   b) \[
   150 \text{ mm} (\text{cm})
   \]
   c) \[
   5.4 \text{ kg} (\text{g})
   \]

7. Insert >, < or = to make true statements.
   a) \[
   0.6 \square 0.079
   \]
   b) \[
   \frac{2}{10} \square 0.2
   \]
   c) \[
   \frac{3}{2} \square \frac{2}{3}
   \]

8. Write the following from smallest to largest.
   a) \[
   0.8, 1.01, 0.099, 2.001
   \]
   b) \[
   \frac{3}{5}, \frac{7}{8}, \frac{1}{2}, \frac{1}{4}
   \]

PRIOR KNOWLEDGE

<table>
<thead>
<tr>
<th>Denominator</th>
<th>Improper fraction</th>
<th>Mixed number</th>
<th>Numerator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
<td>Integer</td>
<td>Multiple</td>
<td>Place value</td>
</tr>
<tr>
<td>Highest common factor (HFC)</td>
<td>Lowest common denominator (LCD)</td>
<td>Number line</td>
<td></td>
</tr>
</tbody>
</table>
The need to work with decimals and fractions is a part of everyday life. For example, suppose a stallholder is advertising peaches for $2.50 a kilogram, but a bargain hunter offers $8 for $\frac{3}{2}$ kg. Is that a fair price?

Three stallholders, Tim, Brent and Julie, each present their reasoning in different ways.

<table>
<thead>
<tr>
<th>Tim</th>
<th>Brent</th>
<th>Julie</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are 7 halves in $3\frac{1}{2}$ so $\frac{1}{2}$ kg costs $8.00 \div 7$ which is between $1.10 &amp; $1.20. 7$ lots of $1.20 = $8.40 and 7 lots of $1.10 = $7.70. I am being offered $8.00 for $3\frac{1}{2}$ kg, so it is a fair price.</td>
<td>The price should be $8.75. 3$ kg $\times$ $2$ = $6.00$ and $1.00$ shared between $3$ kg is $33$ cents, so I am being offered $2.33$ for $1$ kg. This is close to $2.50$ for a kilogram. I will accept the offer.</td>
<td>$3\frac{1}{2}$ is $3.5$. Three lots of $2.50$ is $7.50$ and $0.5$ of one lot is $1.25$, so the full price should be $8.75$. Because I am offered $8.00$ for $3\frac{1}{2}$ kg, instead of having to sell lots of small quantities, I think it is a fair price.</td>
</tr>
</tbody>
</table>

- In groups discuss the reasoning used by these three stallholders. What strategies have been used?
- Was it really necessary for Julie to work out the full price, before considering the offer? Share your answers with other groups.

**KEY IDEAS**

- A fraction has a numerator and a denominator: $\frac{\text{numerator}}{\text{denominator}}$.
- Fractions changed to equivalent fractions with a common denominator are easier to compare.
- The decimal number $2.364$ can be written in a place value house.

<table>
<thead>
<tr>
<th>Ones</th>
<th>tenths</th>
<th>hundredths</th>
<th>thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

This is $2$ whole and $3$ tenths plus $6$ hundredths plus $4$ thousandths.

$= 2 + \frac{3}{10} + \frac{6}{100} + \frac{4}{1000}$

- A fraction can be changed to a decimal by writing it as an equivalent fraction with a denominator of $10$, $100$ or $1000$: e.g. $\frac{\frac{2}{5}}{\frac{6}{10}} = 0.6$

- A decimal may be converted to a fraction by writing the digits in a place value house. For example, $4.063$ can be written as:

<table>
<thead>
<tr>
<th>Ones</th>
<th>tenths</th>
<th>hundredths</th>
<th>thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>.</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

$= 4$ plus zero tenths plus six hundredths plus three thousandths
Which is the same as $4 + \frac{0}{10} + \frac{5}{100} + \frac{3}{1000} = 4 \frac{63}{1000}$
A number with many decimal places can be rounded.

- Rounded to 1 d.p. means rounded to the nearest tenth.
- Rounded to 2 d.p. means rounded to the nearest hundredth.
- Rounded to 3 d.p. means rounded to the nearest thousandth.

When rounding decimals, look at the first digit after the required number of decimal places.

- If this digit is < 5 (less than 5), simply chop it off, along with any digits to the right of it:
  
  \( \text{e.g. } 0.3486 \text{ (1 d.p.) } \rightarrow 0.3 \)

- If this digit is \( \geq 5 \) (equal to or greater than 5), increase the previous digit by 1 before chopping off all digits to the right of the required number of decimal places:
  
  \( \text{e.g. } 0.5286 \text{ (2 d.p.) } \rightarrow 0.53 \)

### Example 1

Write \( \frac{13}{5} \) as a mixed number.

#### Solution

\[
\frac{13}{5} = 2 \frac{3}{5}
\]

**Explanation**

There are 5 fifths in one whole.

\[
1 \text{ whole } + 1 \text{ whole } + 3 \text{ fifths}
\]

OR Remember a fraction means that the numerator (number on the top) is divided by the denominator (number on the bottom).

\[
\frac{a}{b} = a \div b
\]

So \( 13 \div 5 = 2 \) and 3 left over

\[
= 2 \frac{3}{5}
\]

### Example 2

Write \( 2 \frac{3}{4} \) as an improper fraction.

#### Solution

\[
2 \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}
\]

**Explanation**

There are four quarters in one whole.

\[
1 \text{ whole } + 1 \text{ whole } + 3 \text{ quarters}
\]

\[
= 4 \text{ quarters } + 4 \text{ quarters } + 3 \text{ quarters}
\]

\[
= 11 \text{ quarters}
\]

OR \( 2 \frac{3}{4} = \frac{2 \times 4 + 3}{4} = \frac{11}{4} \)

### Example 3

Complete the equivalent fractions.

\[
a \quad \frac{2}{3} = \frac{7}{21} \\
b \quad \frac{5}{7} = \frac{7}{21}
\]
Solution Explanation

a
\( \frac{2}{3} \times 3 = \frac{6}{9} \)

By using a fraction wall we see that 3 ninths is the same as 1 third, so 2 thirds will be 6 ninths:

\( \frac{2}{3} = \frac{6}{9} \)

b
\( \frac{5}{7} \times 3 = \frac{15}{21} \)

Notice that numerator and denominator are multiplied by the same number.

EXAMPLE 4

Write the simplest equivalent fraction for \( \frac{8}{12} \).

Solution Explanation

\( \frac{8}{12} \div 4 = \frac{2}{3} \)

Use a fraction wall:

\[
\begin{array}{cccccc}
\frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\
\frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\
\frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\
\frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\
\frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\
\end{array}
\]

We can see that 8 twelfths is the same as 4 sixths and 2 thirds.

EXAMPLE 5

Write these decimals:

a from biggest to smallest: 0.028, 2.89, 0.289, 28.9

b from smallest to largest (ascending order): 0.6, 0.06, 0.61, 0.601

Solution Explanation

a 28.9, 2.89, 0.289, 0.028

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
<th>tenths</th>
<th>hundredths</th>
<th>thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
<td>.</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>.</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>.</td>
<td>2</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>.</td>
<td>.</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

28.9 is the largest, so we start with 28.9.

2.89 is the only other number with whole units, so it comes next.

0.289 has 2 tenths and the other number has zero tenths, so 0.289 is the third number and 0.028 is the smallest.
**Solution**

<table>
<thead>
<tr>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write 0.6, 0.06, 0.61, 0.601 in a place-value table. Compare ones, then tenths, then hundredths.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>.</td>
<td>0</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>.</td>
<td>6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>.</td>
<td>6</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**EXAMPLE 6**

Write each as a fraction or mixed number in its simplest form.

**a** 0.08  
**b** 6.42

<table>
<thead>
<tr>
<th>Solution</th>
<th>Explanation</th>
</tr>
</thead>
</table>
| **a** 0.08 = \( \frac{8}{100} = \frac{1}{25} = \frac{2}{25} \)  
\( \div 4 \)  
\( \frac{8}{100} = \frac{2}{25} \)  
\( \div 4 \) | To simplify, halve 8 and 100 and halve again. Or divide the numerator and denominator by 4. |
| **b** 6.42 = 6 + \( \frac{42}{100} \)  
\( \frac{42}{100} = \frac{21}{50} \)  
\( \frac{6.42}{100} = \frac{621}{100} \) | 6.42 is written as a mixed number, then the fraction part can be simplified. Halve 42 and 100. |

**EXAMPLE 7**

Write each fraction as a decimal.

**a** \( \frac{4}{5} \)  
**b** 1\( \frac{3}{4} \)  
**c** 3\( \frac{1}{5} \)

<table>
<thead>
<tr>
<th>Solution</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong> ( \frac{4}{5} = \frac{8}{10} = 0.8 )</td>
<td>( \frac{4}{5} ) is changed to an equivalent fraction. Because the denominator is 10, the fraction is written as 0.8.</td>
</tr>
</tbody>
</table>
| **b** 1\( \frac{1}{4} \) = 1.25 | 1\( \frac{1}{4} \) = one whole + one quarter  
\( \frac{1}{4} \) cannot be changed to an equivalent fraction with a denominator of 10. But \( 4 \times 25 = 100 \) so \( \frac{1}{4} = \frac{25}{100} \)  
1\( \frac{1}{4} = 1 \frac{25}{100} \) which is 1.25 |
| **c** 3\( \frac{1}{8} \) = 3 + \( \frac{1}{8} \) | \( \frac{1}{8} \) cannot be changed to an equivalent fraction with a denominator of 10 or 100. But 8 is a factor of 1000:  
\( 8 \times 25 \times 5 = 1000 \) so \( \frac{1}{8} \times \frac{25 \times 5}{25 \times 5} = \frac{125}{1000} \) |

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EXAMPLE 8

Round 0.3275 to:

<table>
<thead>
<tr>
<th>a</th>
<th>1 decimal place</th>
<th>b</th>
<th>2 decimal places</th>
<th>c</th>
<th>3 decimal places</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solution</strong></td>
<td><strong>Explanation</strong></td>
<td><strong>Solution</strong></td>
<td><strong>Explanation</strong></td>
<td><strong>Solution</strong></td>
<td><strong>Explanation</strong></td>
</tr>
</tbody>
</table>
| a 0.3275 = 0.3 (1 d.p.) | Q: Where does 0.3275 fit on a number line?  
A: Because it is required to be accurate to 1 decimal place (1 d.p.) or the closest tenth, we place it on a number line divided into tenths.  
0.3275 is between 0.3 and 0.4, but it is closer to 0.3 than it is to 0.4, so 0.3275 rounds to 0.3 (1 d.p.). | b 0.3275 = 0.33 (2 d.p.) | Q: Where does 0.3275 fit on a number line?  
A: Because it is required to be accurate to 2 decimal places (2 d.p.) or the closest hundredth, we place it on a number line divided into hundredths.  
0.3275 is between 0.32 and 0.33, but it is closer to 0.33 than it is to 0.32, so 0.3275 rounds to 0.33 (2 d.p.). | c 0.3275 = 0.328 | Q: Where does 0.3275 fit on a number line?  
A: Because it is required to be accurate to 3 decimal places (1 d.p.) or the closest thousandth, we place it on a number line divided into thousandths.  
0.3275 is halfway between 0.327 and 0.328.  
Our rounding system requires that any number falling exactly halfway between two numbers be rounded **up** to the bigger number.  
0.328 is the larger thousandth, so 0.3275 rounds to 0.328 (3 d.p.). |
EXERCISE 1a

1. **Write as mixed numbers.**
   - a \( \frac{3}{2} \)
   - b \( \frac{7}{5} \)
   - c \( \frac{9}{4} \)
   - d \( \frac{11}{7} \)
   - e \( \frac{5}{3} \)
   - f \( \frac{12}{7} \)
   - g \( \frac{43}{10} \)
   - h \( \frac{17}{4} \)
   - i \( \frac{35}{9} \)
   - j \( \frac{22}{15} \)

2. **Write as improper fractions.**
   - a \( \frac{2}{1} \)
   - b \( \frac{3}{1} \)
   - c \( \frac{2}{3} \)
   - d \( \frac{1}{15} \)
   - e \( \frac{3}{5} \)

3. **Complete the equivalent fractions.**
   - a \( \frac{1}{3} = \frac{7}{21} \)
   - b \( \frac{2}{5} = \frac{7}{10} \)
   - c \( \frac{1}{4} = \frac{3}{7} \)
   - d \( \frac{1}{4} = \frac{7}{28} \)
   - e \( \frac{2}{3} = \frac{7}{9} \)
   - f \( \frac{5}{8} = \frac{7}{10} \)
   - g \( \frac{27}{15} = \frac{7}{5} \)
   - h \( \frac{5}{8} = \frac{10}{7} \)
   - i \( \frac{45}{12} = \frac{7}{4} \)
   - j \( \frac{12}{25} = \frac{5}{7} \)

4. **Determine which fraction in each pair is the larger by first finding the lowest common denominator.**
   - a \( \frac{3}{4} \) and \( \frac{7}{8} \)
   - b \( \frac{11}{12} \) and \( \frac{7}{12} \)
   - c \( \frac{14}{21} \) and \( \frac{7}{9} \)
   - d \( \frac{5}{7} \) and \( \frac{11}{7} \)
   - e \( \frac{3}{5} \) and \( \frac{7}{10} \)
   - f \( \frac{2}{3} \) and \( \frac{7}{9} \)
   - g \( \frac{5}{7} \) and \( \frac{11}{17} \)
   - h \( \frac{3}{10} \) and \( \frac{4}{13} \)

5. **Write each fraction as simply as possible; i.e. cancel the fraction.**
   - a \( \frac{4}{8} \)
   - b \( \frac{6}{9} \)
   - c \( \frac{9}{12} \)
   - d \( \frac{10}{14} \)
   - e \( \frac{24}{30} \)
   - f \( \frac{16}{24} \)
   - g \( \frac{20}{45} \)

6. **Write each group of decimals in order from smallest to largest.**
   - a 2.645, 2.654, 2.465 and 2.564
   - b 0.456, 0.564, 0.0456 and 0.654
   - c 45.2, 45.02, 4.52 and 4.502
   - d 0.111, 0.11, 0.011 and 0.101
   - e 0.111, 0.11, 0.011 and 0.101

7. **Write each decimal as a fraction or mixed number as simply as possible.**
   - a 0.1
   - b 0.9
   - c 0.33
   - d 0.03
   - e 0.75
   - f 0.063
   - g 0.75
   - h 0.5
   - i 0.75
   - j 0.2
   - k 0.06
   - l 0.12
   - m 0.35
   - n 0.025
   - o 1.3
   - p 1.2
   - q 1.06
   - r 1.25
   - s 3.67
   - t 2.42

8. **Round these decimals to the required number of decimal places.**
   - a 0.73 (1 d.p.)
   - b 2.61 (1 d.p.)
   - c 0.49 (1 d.p.)
   - d 2.85 (1 d.p.)
   - e 0.564 (2 d.p.)
   - f 0.927 (2 d.p.)
   - g 0.675 (2 d.p.)
11 Round these decimal numbers to:
   i  one decimal place    ii  two decimal places    iii  three decimal places
   a  0.1345             b  0.2776          c  0.6689          d  0.0527
   e  0.0706             f  0.1518          g  1.4055          h  2.0068

12 Write each group of fractions in order from smallest to largest.
   a  $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{5}{6}$, $\frac{1}{9}$
   b  $\frac{3}{12}$, $\frac{7}{8}$, $\frac{15}{23}$, $\frac{3}{4}$
   c  $\frac{5}{6}$, $\frac{2}{3}$, $\frac{6}{17}$, $\frac{27}{8}$

13 Write each group of decimals and fractions in order from smallest to largest.
   a  $\frac{1}{5}$, 0.25, $\frac{1}{2}$, 0.3, 0.05
   b  $\frac{1}{4}$, $\frac{1}{3}$, 0.1, $\frac{3}{8}$, 0.75, 0.08
   c  $\frac{2}{5}$, 1.05, 1.5, 1.25, $\frac{1}{2}$
   d  0.34, $\frac{1}{2}$, 0.034, $\frac{3}{10}$, $\frac{2}{100}$

14 Write three decimals and three fractions or mixed numbers with different denominators. Arrange your numbers in order from largest to smallest.

15 For each question, is the number in column A, B or C the greatest? Justify your answer.

<table>
<thead>
<tr>
<th></th>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
<th></th>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2.789</td>
<td>0.2789</td>
<td>0.02789</td>
<td>b</td>
<td>0.0145</td>
<td>$\frac{19}{100}$</td>
<td>0.06</td>
</tr>
<tr>
<td>c</td>
<td>16.34</td>
<td>$1\frac{3}{5}$</td>
<td>0.17</td>
<td>d</td>
<td>373.278</td>
<td>372 $\frac{1}{4}$</td>
<td>370.0</td>
</tr>
<tr>
<td>e</td>
<td>6.23</td>
<td>$6\frac{1}{25}$</td>
<td>6.032</td>
<td>f</td>
<td>10</td>
<td>$\frac{101}{10}$</td>
<td>10.01</td>
</tr>
</tbody>
</table>

16 Victoria arrives home from school at 4.15 pm and goes to bed at 10.00 pm, with 6.00–6.45 pm being the family dinner time.

She uses $\frac{3}{8}$ of her free time after school watching TV, $\frac{5}{17}$ doing her homework and $\frac{5}{24}$ doing music or sports practice.

a Excluding dinner time, how much free time does Victoria have after school?
b How does Victoria spend most of that free time?
c How much time does she spend on that activity?

17 In a Lotto syndicate, the winnings of $30,000 are divided among Kate, Greg, Jack and Ella as $\frac{2}{15}$, $\frac{1}{5}$, $\frac{3}{10}$ and $\frac{7}{30}$, respectively.

a Who receives the greatest proportion of the winnings?
b How many dollars does the person with the smallest share receive?

18 When you multiply or divide the numerator and denominator of a fraction by the same number you obtain an equivalent fraction. What do you think would happen to a fraction if you added the same number to the numerator and denominator? Choose a fraction and show what happens with at least two examples.
ENRICHMENT: This is your life!

19 The aim is to construct an accurate fraction wheel to represent your average day. Typically, what fraction of the day do you spend in bed? Eating? Travelling? Playing sport?

a Write down a list of 10 activities that represent most of your time commitments.

b Now estimate the fraction of the day that each item in your list represents and draw a pie graph, using a protractor to make the slices accurately represent the time fraction. Remember that there are $360^\circ$ in a circle.
Often the exact answer to a mathematical problem is not required. We can estimate the answer if we round off the numbers to something simpler. We usually round numbers to nearest 10, 100, 1000 and so on, so that calculation estimates can be done mentally.

**KEY IDEAS**

▶ It is not always necessary to have an accurate measurement. An approximate answer is often all that is required: e.g. the amount of water needed for a shower, when you are camping.

▶ An approximate answer can indicate whether we have used a calculator correctly.

▶ An approximate measure can be used when we discuss very large measures: e.g. on 27 August 2003, Mars was 55 763 108 or approximately 56 million km from Earth, the closest it has been in 60 000 years.

▶ **Estimation** can be used to approximate an answer without measurements being taken: e.g. the perimeter of the classroom is estimated to be 30 m; the perimeter is not actually measured.

▶ Estimation provides an awareness of size of everyday objects: e.g. one pace is approximately 1 metre; the length of the room is 8 paces, so the length is estimated to be 8 metres.

▶ Easy numbers or tidy numbers make it easier for quick mental calculations to be carried out.

▶ The symbol \( \approx \) means ‘is approximately equal to’.

**EXAMPLE 9**

Round the following numbers sensibly.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>1973</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>25</td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>24 035 718</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Solution</strong></th>
<th><strong>Explanations</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong> 1973 is approximately 2000 ( (to the nearest thousand) )</td>
<td>Because 1973 is near here, it is approximately 2000.</td>
</tr>
<tr>
<td><strong>b</strong> 25 ( (to the nearest ten) )</td>
<td>25 is halfway between 20 and 30.</td>
</tr>
<tr>
<td><strong>c</strong> 24 035 718 is approximately 24 million ( (to the nearest million) ) OR 20 million ( (to the nearest ten million) ) ( i.e. 24 035 718 \approx 24 \ 000 \ 000 ) OR ( 24 035 718 \approx 20 \ 000 \ 000 )</td>
<td>When an approximate number is selected, we need to look at the calculation in which the number will be used. In a mental calculation, 20 million is easier to use than 24 million.</td>
</tr>
</tbody>
</table>
EXAMPLE 10

Approximate the following numbers.

a \(1 \frac{3}{4}\)

b 3.482

Solution | Explanations
---|---
a \(1 \frac{3}{4} = 2\)  
1 3/4 is near 2, so it is approximately 2.

b 3.482 = 3  
3.482 is shown on a number line.  
Because 3.482 is less than 3.5 and closer to 3 than 4, it is approximately 3.

OR

3.482 = 3.5  
When a more accurate approximation is required, 3.482 is closer to 3.5 than 3, so its approximation is 3.5.

EXAMPLE 11

Estimate the answers.

a \(3.2 \times 4.7\)

c \(5.31 \times 11.25 + 3.8 + 8.49\)

d \(35628 \div 1827 \times 13.2 - 115.6\)

Solution | Explanations
---|---
a \(3.2 \times 4.7 = 3 \times 5\)  
\(= 15\)  
3.2 is close to 3 and 4.7 is close to 5.  
3 \times 5 is easy to calculate.

b \(7.2 \div 3.9 = 8 \div 4\)  
\(= 2\)  
7.2 is closer to 7 than 8, but because we are dividing by 4, we approximate it to 8. Then 4 can be divided into 8 easily.  
Because 7 was increased to 8, the approximate answer of 2 will be a little larger than the true answer.

c \(5.31 \times 11.25 \div 3.8 + 8.49\)  
\(\approx 5 \times 10 \div 5 + 10\)  
\(\approx 50 \div 5 + 10\)  
\(\approx 10 + 10\)  
\(\approx 20\)  
Each number is approximated to easy numbers that are multiples of 5:  
5.31 \approx 5 and 11.25 \approx 10 and 3.8 \approx 5 and 8.49 \approx 10  
BEDMAS tells us that addition is done after multiplication.

d \(35628 \div 1827 \times 13.2 - 115.6\)  
\(\approx 40000 \div 2000 \times 10 - 100\)  
\(\approx 20 \times 10 - 100\)  
\(\approx 200 - 100\)  
\(\approx 100\)  
Sensible easy approximations are made.  
35628 \approx 40000  
1827 \approx 2000  
13.2 \approx 10  
115.6 \approx 100  
BEDMAS means subtraction is done after multiplication and division.
EXAMPLE 12

Estimate \( \frac{2}{7} \) as a decimal.

### Solution

\[
\frac{2}{7} \approx 0.3
\]

### Explanations

\( \frac{2}{7} \) is between \( \frac{1}{3} \) and \( \frac{1}{2} \) and \( \frac{1}{3} = \frac{7}{21} \) and \( \frac{1}{2} = \frac{7}{14} \).

Because \( \frac{1}{3} = 0.33 \) and \( \frac{1}{4} = 0.25 \),

\( \frac{2}{7} \) will be between 0.33 and 0.25.

---

EXERCISE 1b

1. 165 596 rounded to the nearest ten thousand is:
   - A 166 000
   - B 170 000
   - C 160 000
   - D 165 000

2. Which of the following is the closest to 3?
   - A 2\( \frac{3}{4} \)
   - B 3.1
   - C 5.9
   - D 6 - 5 + 3

3. Approximate these values to the amounts indicated.
   - a nearest 10
     i 17
     ii 28
     iii 45
     iv 94
   - b nearest hundred
     i 184
     ii 294
     iii 1576
     iv 21 350
   - c nearest thousand
     i 25 469
     ii 7777
     iii 8253
     iv 921 499
   - d nearest million
     i 4 832 179
     ii 8 040 300
     iii 911 300
     iv 30 809 210

4. Approximate these fractions or mixed numbers to the nearest whole number.
   - a \( \frac{4}{5} \)
   - b \( \frac{2}{3} \)
   - c 1\( \frac{1}{4} \)
   - d 3\( \frac{1}{5} \)
   - e 14\( \frac{7}{8} \)
   - f 2\( \frac{3}{7} \)
   - g \( \frac{11}{12} \)
   - h 4\( \frac{7}{12} \)
   - i \( \frac{1}{2} \)
   - j 3\( \frac{5}{9} \)

5. Approximate these decimal numbers to:
   - i the nearest whole number
   - ii 2 decimal places.

   For a give an example to show when each answer could be relevant.
   - a 7.435
   - b 0.584
   - c 1.275
   - d 13.333
   - e 12.125
   - f 3.504
   - g 0.073
   - h 4.875
   - i 0.005
   - j 1.064

---

Chapter 1 Number 15

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6 Estimate the answers.
   a  4.9 \times 1.1  
   b  15.4 \div 3.2  
   c  13.7 - 2.9  
   d  51.3 + 162.4  
   e  94 \times 72  
   f  14.72 \div 3.1 \times 2.7  
   g  3576 \div 878  
   h  15783 \div 3.8 \div 12.4 - 47  
   i  \left(32.6 + 17.5\right) \div 2  
   j  \left(147659 - 27352\right) \div 43  

7 Estimate these fractions or mixed numbers as decimals.
   a \frac{2}{3}  
   b \frac{3}{5}  
   c \frac{4}{7}  
   d \frac{4}{5}  
   e \frac{3}{8}  
   f \frac{6}{11}  
   g \frac{7}{12}  
   h \frac{5}{11}  
   i \frac{1}{3}  
   j \frac{8}{8}  

8 Jeremy purchased $\frac{3}{4}$ kg of peaches, $\frac{3}{8}$ kg nectarines and $3\frac{1}{2}$ kg apricots. Approximately how much fruit did he buy?

9 Hine purchased 0.5 kg beans, 1.48 kg tomatoes and 0.38 kg peas. Approximately what was the mass of vegetables in her shopping bag?

10 Cherries are sold for $14.95 a kg. Bananas are $2.68 a kg and plums are $3.20 a kg. Bill buys 0.6 kg cherries, 2.3 kg bananas and 1.7 kg of plums. Approximately how much does the fruit cost Bill?

11 Animal drench costs a farmer $230 for 5 L (5000 mL). She treats 14 steers with 17 mL each and 9 bulls with 32 mL each. Approximately how much does it cost the farmer for one round of treatment for her cattle?

12 Tyler is given $50 to buy Christmas presents for his family. He decides that he must spend half on presents for his parents. He has two brothers and one sister. Estimate how much he can spend on presents for each of his brothers and sister, assuming that he gives them presents of equal value.

13 As part of a maintenance programme Wiki Bowling Club must spray the greens for weeds.
   The diagram shows the measurements of the area to be sprayed.
   20 mL of spray is needed for every square metre (m²). Taine estimates that he will need 20 L of spray to complete the job. Is his estimate sensible? Explain.
Eating shellfish that contain high levels of toxins can result in paralytic shellfish poisoning (PSP). The maximum toxin level considered safe is 0.0008 g per 100 g of shellfish flesh. When this level is exceeded, public health authorities place a ban on collecting shellfish from affected areas. In the Bay of Islands one January, scientists recorded levels of 0.00147 g. Rounded to the nearest gram, both 0.0008 g and 0.00147 g become 0 g, yet one measure is acceptable and the other is not. Is rounding to the nearest gram sensible? What rounding would be more sensible?

Due to the experimental nature of science and engineering, not all digits in all numbers are considered important or ‘significant’. In 0.0008, the place value of 8 tells us that the measure is 8 ten-thousandths; thus the digit ‘8’ is significant. In 0.00147, when rounded to nearest ten-thousandth we have 15 ten-thousandths (0.0015), which is almost double the concentration. The number 0.0008 is written to one significant figure (1 sig. fig.) and 0.0015 is written to two significant figures (2 sig. fig.).

- In groups examine these two tables, which show different types of rounding.

<table>
<thead>
<tr>
<th>Decimal place rounding</th>
<th>Significant figures rounding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount</td>
<td>0.0008</td>
</tr>
<tr>
<td>Rounded to 1 d.p.</td>
<td>0.0</td>
</tr>
<tr>
<td>Rounded to 2 d.p.</td>
<td>0.00</td>
</tr>
<tr>
<td>Rounded to 3 d.p.</td>
<td>0.001</td>
</tr>
<tr>
<td>Rounded to 4 d.p.</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

- Using the tables above, what is similar and what is different about the two types of rounding?

The number of significant figures cannot be increased: e.g. 1.5 (2 sig. figs) cannot be ‘rounded’ to 1.50 (3 sig. figs). A coin that is 1.5 mm thick has been measured to the nearest tenth of a mm; it might be 1.48 or 1.543 rather than 1.50 mm. On the other hand, if we are told that the coin is 1.50 mm thick, we know that the thickness has been measured to the nearest hundredth of a mm; it is more than 1.49 but less than 1.51 mm.

**KEY IDEAS**

- All non-zero digits (1, 2, 3, 4, 5, 6, 7, 8, 9) are counted as significant figures.
- When the number is less than one, any zero place holders between the decimal point and the first non-zero digit DO NOT count as significant figures: e.g. 0.0356 rounds to 0.04 (1 sig. fig.); four-hundredths is the first non-zero digit, so 4 is the first significant figure.
- When the number is one or more, the first significant figure is the digit of the highest place value: e.g. 234.21 rounds to 230 (2 sig. fig.). The first two significant digits are 2 hundreds and 3 tens, which gives two hundred and thirty.
- Zeros between non-zero digits are significant: e.g. 2.0046 has 5 significant figures.
- In decimal numbers, zeros at the right are significant: e.g. 5.030 and 0.006000 each have four significant figures.
- The level of rounding for numbers should be stated: e.g. 2 907 381 = 3 000 000 (1 sig. fig.)
- ≈ means ‘is approximately equal to’.
**EXAMPLE 13**

Round to two significant figures.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>2342</td>
<td><strong>b</strong></td>
</tr>
</tbody>
</table>

**Solution**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| **a** | 2342 ≈ 2300 (2 sig. fig.) | The two highest place values are 2 thousands and 3 hundreds, so a number line would be divided into hundreds.
|   |   |   |
|   |   |   |
| **b** | 70 167.5 ≈ 70 000 (2 sig. fig.) | The two highest place values are 7 ten-thousands and zero thousands, so a number line would be divided into thousands.
|   |   |   |
|   |   |   |
| **c** | 0.006359 ≈ 0.0064 (2 sig. fig.) | The two highest place values are 6 thousandths and 3 ten-thousandths, so a number line would be divided into ten-thousandths.

**Solution Explanation**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>2342 is about here</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Because it is closer to 2300 than 2400, 2342 is rounded to 2300.</td>
<td></td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>70 167 is about here.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Because it is closer to 70 000 than 71 000, 70 167 is rounded to 70 000.</td>
<td></td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>0.006359 is about here (over halfway)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Because it is closer to 0.0064 than 0.0063, 0.006359 is rounded to 0.0064.</td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLE 14**

Estimate \(27 \times 1329.5\) by rounding each number to one significant figure.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solution</strong></td>
<td><strong>Explanation</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimation = 30 × 1000</td>
<td>27 = 30 (1 sig. fig.)</td>
<td></td>
</tr>
<tr>
<td>= 30 000</td>
<td>1329.5 = 1000 (1 sig. fig.)</td>
<td></td>
</tr>
</tbody>
</table>
EXERCISE 1C

1 Which one of the following numbers has three significant figures?
   A 5342     B 6.154     C 1.04     D 0.012

2 Count the number of significant figures in each of the following numbers.
   a 6.0     b 0.00281     c 1.015     d 3034     e 16.2

3 Round each number to two significant figures.
   a 2436     b 35 057.4     c 0.060 49     d 34.024
   e 107 892     f 0.00245     g 2.0745     h 0.7070

4 Copy and complete this table (sig. fig. means ‘significant figures’).

<table>
<thead>
<tr>
<th>Number</th>
<th>4783</th>
<th>8921</th>
<th>7549</th>
<th>86321</th>
<th>60 407</th>
<th>20 005</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 sig. fig.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 sig. fig.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 sig. fig.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 Copy and complete this table.

<table>
<thead>
<tr>
<th>Number</th>
<th>24.863</th>
<th>97.142</th>
<th>125.06</th>
<th>48.209</th>
<th>370.726</th>
<th>6070.802</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 sig. fig.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 sig. fig.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 sig. fig.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6 Copy and complete this table.

<table>
<thead>
<tr>
<th>Number</th>
<th>0.1234</th>
<th>0.6178</th>
<th>0.80649</th>
<th>0.08097</th>
<th>0.03409</th>
<th>0.0009906</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 sig. fig.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 sig. fig.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 sig. fig.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7 Estimate the answers by rounding each number to one significant figure.
   a 567 + 3126     b 795 – 35.6     c 97.8 \times 42.2
   d 965.98 + 321.2 – 763.2     e 4.23 – 1.92 \times 1.827
   f 17.43 – 2.047 \times 8.165
   g 0.0704 + 0.0482     h 0.023 \times 0.98
   i 0.027 \div 0.0032
   j 41.034^2     k 0.078 \times 0.9803^2
   l 1.8494^2 + 0.972 \times 7.032

8 Estimate the answers by rounding each number to one significant figure.
   a 49.2 \times 9.23
   b 9.79 \times 2.807^2
   c 67.87 + 26.13
   d 32.65 – 19.087
   b 9.95 + 13.1 \times 2.71
   c 3.02 \times 3.24^2 + 1.97
Seven children visit the zoo. Three wear orange T-shirts, 2 wear blue T-shirts and the rest wear green T-shirts.

In pairs, discuss how Sam may have made his decisions.

Another group of children were identified by the colour of their hats.

\[
\begin{align*}
\frac{2}{7} & \text{ wear red hats,} \\
\frac{5}{14} & \text{ wear green hats,} \\
\frac{1}{7} & \text{ wear yellow hats} \\
\text{and } \frac{3}{14} & \text{ wear blue hats.}
\end{align*}
\]

Is Tom correct?

Ella says there are 28 children in the group. Is Ella correct? How did she work out that there were 28 children in the group?

Discuss your ideas with another group, and then share your explanations with the rest of the class.

**KEY IDEAS**

- Fractions with the same denominator can be added or subtracted.
- Fractions with different denominators must be made into equivalent fractions with the same denominator, before being added or subtracted.
- Answers to fraction problems should always be expressed in their simplest form.
EXAMPLE 15

Find the answers.

\( \frac{5}{12} + \frac{1}{12} \)  \( \frac{8}{11} - \frac{5}{11} \)  \( \frac{4}{7} + \frac{3}{14} \)

<table>
<thead>
<tr>
<th>Solution</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{12} + \frac{1}{12} = \frac{5 + 1}{12} = \frac{6}{12} = \frac{1}{2} )</td>
<td>Because both fractions are twelfths, the numerators can be added.</td>
</tr>
<tr>
<td>( \frac{8}{11} - \frac{5}{11} = \frac{8 - 5}{11} = \frac{3}{11} )</td>
<td>Because both fractions are elevenths, the numerators can be subtracted.</td>
</tr>
<tr>
<td>( \frac{4}{7} + \frac{3}{14} = \frac{8 + 3}{14} = \frac{11}{14} )</td>
<td>Both fractions need a common denominator before they can be added or subtracted, so ( \frac{3}{7} ) is changed to an equivalent fraction.</td>
</tr>
</tbody>
</table>

The fractions can now be added.

\( \frac{4}{7} = \frac{8}{14} \)

\( \frac{3}{14} \)
EXAMPLE 16

Find the answers.

\[ a \quad 2 \frac{2}{5} - 1 \frac{1}{5} \]

**Solution**

\[ a \quad 2 \frac{2}{5} - 1 \frac{1}{5} = \frac{12}{5} - \frac{6}{5} = \frac{6}{5} \]

Write as improper fractions.

Write fractions with the same denominator.

LCD is 15.

\[ \times 5 \]

\[ \frac{8}{3} = \frac{40}{15} \]

\[ \times 3 \]

\[ \frac{6}{5} = \frac{18}{15} \]

\[ \frac{22}{15} = 1 \frac{7}{15} \]

Write as a mixed number.

\[ b \quad 3 \frac{1}{5} + 2 \frac{1}{10} \]

**Solution**

\[ b \quad 3 \frac{1}{5} + 2 \frac{1}{10} = (3 + 2) + \left( \frac{1}{5} + \frac{1}{10} \right) \]

\[ = 5 + \frac{2}{10} + \frac{1}{10} \]

\[ = 5 \frac{3}{10} \]

For addition problems, instead of writing as improper fractions, you could first add the integers, then write the fractions with the same denominator and add them.

EXAMPLE 17

Georgia eats \( \frac{1}{4} \) of an orange, Tom eats \( \frac{2}{5} \) of another orange and Scott eats the leftovers of both oranges. What quantity of orange did Scott eat?

**Solution**

Georgie leaves:

\( 1 - \frac{1}{4} = \frac{3}{4} \)

This is the fraction left by Georgia.

Tom leaves:

\( 1 - \frac{3}{5} = \frac{2}{5} \)

This is the fraction left by Tom.

Scott eats:

\( \frac{3}{5} + \frac{2}{10} + \frac{1}{10} \)

\[ = \frac{6}{10} + \frac{1}{10} \]

\[ = \frac{12}{20} \]

\[ = \frac{7}{20} \]

Scott eats the fractions left by Georgia and Tom. Because the fractions have different denominators, we need to change them to equivalent fractions with the same denominator. 20 is the lowest number that is divisible by 4 and 5; i.e. 20 is the **lowest common denominator (LCD)**.

\[ \times 5 \]

\[ \frac{3}{4} = \frac{15}{20} \]

\[ \times 4 \]

\[ \frac{3}{5} = \frac{12}{20} \]

Write as a mixed number.
1 A circle is divided into 8 equal parts. Three parts and half of another part are shaded:

a What fraction of the circle is shaded?
b What fraction of this quadrant is shaded? (A quadrant is a quarter of a circle).
c What is the difference between your answers to a and b?

Example 1.5 Find the answers.

a \(\frac{2}{5} + \frac{1}{5}\)  
b \(\frac{4}{5} - \frac{2}{5}\)  
c \(\frac{21}{25} + \frac{18}{25}\)  
d \(\frac{14}{15} - \frac{3}{19}\)

e \(\frac{5}{7} + \frac{3}{14}\)  
f \(\frac{3}{8} - \frac{5}{16}\)  
g \(\frac{11}{12} - \frac{1}{6}\)  
h \(\frac{3}{4} - \frac{3}{10}\)

i \(\frac{4}{5} + 1\frac{1}{5}\)  
j \(2\frac{3}{4} - 4\frac{5}{8}\)  
k \(1\frac{2}{3} + 2\frac{2}{3}\)  
l \(2\frac{1}{2} - 1\frac{3}{4}\)

Example 1.6 Find the answers.

a \(\frac{2}{3} + \frac{1}{4}\)  
b \(\frac{1}{2} - \frac{1}{5}\)  
c \(\frac{3}{7} + \frac{12}{3}\)  
d \(\frac{2}{4} - \frac{5}{6}\)

e \(\frac{3}{5} - \frac{4}{9}\)  
f \(\frac{3}{2} + \frac{5}{12}\)  
g \(2\frac{5}{9} - 1\frac{1}{2}\)  
h \(6\frac{1}{2} + 2\frac{4}{7}\)

Example 1.7 Tui has organised a pot luck party for her friends. She asks \(\frac{2}{5}\) to bring pizza, \(\frac{3}{10}\) to bring drinks and \(\frac{1}{6}\) to bring chips and dips. The rest will bring ice-cream. What fraction brings ice-cream to the party?

6 Rimu High School held a mini gala to raise funds for sunshades. Each form class held a stall. Three-eighths of all stalls sold food, \(\frac{1}{4}\) held guessing competitions, and the remainder tested students’ skills, such as hitting the bullseye with a dart. What fraction of the stalls tested skills?

7 Anne’s fruit cake recipe required a large amount of dried fruit. Half the amount was sultanas, \(\frac{1}{8}\) was currants, \(\frac{1}{5}\) was cherries and \(\frac{1}{8}\) was raisins. The rest of the dried fruit was lemon peel. What fraction was lemon peel?

8 Write at least three different sums using addition and/or subtraction which give an answer of \(\frac{2}{3}\).

9 Pauline decided to varnish her table and allowed 5 hours for the job. It actually took her \(3\frac{1}{2}\) hours in the afternoon and \(\frac{1}{2}\) hour in the evening to complete it. By how much did she overestimate her time?
10 Nadia plans to make four curtains that are each $2\frac{1}{3}$ m long.

a What length of material does she need to purchase?

b If the chosen material has a pattern that repeats every $\frac{1}{2}$ metre, what length will she need to purchase? How much material will be wasted?

11 To make a fruit punch Reuben combined $1\frac{1}{2}$ litres of lemonade, $2\frac{1}{3}$ litres of orange juice and $\frac{3}{4}$ of a litre of pineapple juice. How much soda water does he need to add to make 6 litres of fruit punch?

ENRICHMENT: TV fractions

12 A TV station has to ensure that programs and advertisements will exactly fit into blocks of time that last for 30 minutes, 60 minutes and so on. To ensure this happens, advertisements are designed to last a specific fraction of a minute. Using a wall clock, or the stopwatch feature on your watch or mobile phone, time the advertisements during one evening.

a What different fractions of a minute are used?

b What fraction of a minute is the most popular?
Sue was explaining multiplication of fractions to her friends.

On the overhead projector she placed two grids, partly shaded to make the blue rectangle and the yellow rectangle below.

She slid the yellow rectangle on top of the blue rectangle and formed a third shaded rectangle. (Blue and yellow produce green.) She wrote the fractions under the rectangles.

Explain how Sue obtained the fraction \( \frac{12}{20} \).

Jack explained division of fractions using the following thinking.

- Six pies are divided into thirds.

Think: How many thirds are there in 6 pies? i.e. \( 6 ÷ \frac{1}{3} = ? \)

The answer is 18.

How did Jack obtain the answer 18?

He then used the following thinking.

- How many thirds in one-third? i.e. \( \frac{1}{3} ÷ \frac{1}{3} = ? \) Answer: \( ? = 1 \)
- How many thirds in two-thirds? i.e. \( \frac{2}{3} ÷ \frac{1}{3} = ? \) Answer: \( ? = 2 \)
- How many thirds in three-thirds? i.e. \( \frac{3}{3} ÷ \frac{1}{3} = ? \) Answer: \( ? = 3 \)
- How many thirds in four-thirds? i.e. \( \frac{4}{3} ÷ \frac{1}{3} = ? \) Answer: \( ? = 4 \)

Investigate how Jack obtained these answers.
Harvey posed the question: If I share four and two-thirds apples, how many people can have two-thirds of an apple each?

On the board he wrote:

\[
4 \frac{2}{3} + \frac{2}{3} = ? \\
\frac{14}{3} + \frac{2}{3} = ? \\
= 7
\]

\[\therefore 7 \text{ people can have a two-third share of an apple.}\]

Investigate Harvey’s strategy. When is Harvey’s strategy possibly the better one to use?

**KEY IDEAS**

**Multiplying fractions**

- To multiply fractions, multiply the numerators and then multiply the denominators: \(\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}\)
- Mixed numbers are changed to improper fractions before multiplying.
- To find a fraction of something, ‘of’ can be replaced by a multiplication sign: e.g. \(\frac{3}{7} \times \frac{2}{3}\)
  (That is, ‘of’ tells you to multiply.)

**Multiplying fractions**

- \(4 \div 3\) means ‘How many lots of 3 are there in 4?’ Similarly \(\frac{3}{4} \div \frac{1}{5}\) means: ‘How many lots of \(\frac{1}{5}\) are there in \(\frac{3}{4}\)?’
- To divide by a fraction you can multiply by the reciprocal of the fraction. (The reciprocal is the fraction turned upside down; i.e. the numerator and denominator are swapped.)

The reciprocal of \(\frac{a}{b}\) is \(\frac{b}{a}\), so \(\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}\)

**EXAMPLE 18**

Simplify:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Solution</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (\frac{3}{5}) of 40</td>
<td>(\frac{3}{5} \times 40 = 24)</td>
<td>‘Of’ means multiply. OR (\frac{3}{5}) of 40 = 8 (\frac{3}{5}) of 40 = 8 \times 8 = 24</td>
</tr>
<tr>
<td>b (\frac{2}{7} \times \frac{3}{5})</td>
<td>(\frac{2}{7} \times \frac{3}{5} = \frac{6}{35})</td>
<td>Numerators are multiplied. Denominators are multiplied. Because there are no common factors of 6 and 35, the fraction cannot be simplified.</td>
</tr>
</tbody>
</table>
EXAMPLE 19

Find the answers.

a  $4 \div \frac{2}{5}$ 

Solution

\[ \text{How many lots of } \frac{2}{5} \text{ are there in } 4? \]

\[ \begin{align*}
  4 \div \frac{2}{5} &= 4 \times \frac{5}{2} \\
  &= \frac{20}{2} \\
  &= 10
\end{align*} \]

There are 10 lots.

b  $\frac{4}{5} \div \frac{2}{3}$

Solution

\[ \begin{align*}
  \frac{4}{5} \div \frac{2}{3} &= \frac{4}{5} \times \frac{3}{2} \\
  &= \frac{6}{5} \\
  &= \frac{3}{3}
\end{align*} \]

Write an equivalent multiplication statement.
Multiply fractions.
Simplify fraction.

Solution

\[ \begin{align*}
  2 \frac{1}{2} \div \frac{2}{3} &= \frac{5}{2} \div \frac{2}{3} \\
  &= \frac{5}{2} \times \frac{3}{2} \\
  &= \frac{15}{4} = 3 \frac{3}{4}
\end{align*} \]

Change the mixed number to an improper fraction.
Write the division by the fraction as a multiplication by its reciprocal.
Do the multiplication and change the improper fraction to a mixed number.

Solution Explanation

EXERCISE 1e

1 Which of the following is an example of incorrect cancelling?

A  $\frac{3}{4} \times \frac{4}{5}$
B  $\frac{3}{5} \times \frac{4}{5}$
C  $\frac{4}{5} \times \frac{3}{5}$
D  $\frac{4}{5} \times \frac{5}{1} $

EXAMPLE 1e

2 Find these amounts.

a  $\frac{2}{3}$ of 60 g
b  $\frac{7}{10}$ of 480 kg
c  $\frac{3}{4}$ of 1.2 km
d  $\frac{2}{3}$ of 144 m
e  $\frac{5}{6}$ of 4 $\frac{1}{2}$ days
f  $\frac{2}{9}$ of 72 hours
g  $\frac{3}{4}$ of $2.20$
h  $\frac{3}{10}$ of $480 000$
i  $\frac{5}{7}$ of $8400$
Find the answers.

3. Find the answers.

\[ \begin{align*}
\text{a} & \quad \frac{2}{3} \times 24 \\
\text{b} & \quad \frac{1}{2} \times 60 \\
\text{c} & \quad 81 \times \frac{2}{3} \\
\text{d} & \quad 125 \times \frac{4}{5} \\
\text{e} & \quad \frac{1}{3} \times \frac{4}{7} \\
\text{f} & \quad \frac{3}{8} \times \frac{5}{6} \\
\text{g} & \quad \frac{2}{15} \times \frac{5}{8} \\
\text{h} & \quad \frac{6}{21} \times \frac{5}{9} \\
\text{i} & \quad 8 \times \frac{2}{3} \\
\text{j} & \quad 6 \times \frac{3}{4} \\
\text{k} & \quad \frac{17}{8} \times 16 \\
\text{l} & \quad 2 \frac{1}{2} \times 8 \\
\text{m} & \quad 1 \frac{1}{2} \times 2 \frac{1}{3} \\
\text{n} & \quad 2 \frac{2}{3} \times 2 \frac{1}{2} \\
\text{o} & \quad \frac{10}{21} \times 1 \frac{2}{5} \\
\text{p} & \quad \frac{1}{2} \times 1 \frac{1}{2}
\end{align*} \]

4. Find the answers.

\[ \begin{align*}
\text{a} & \quad 6 \div \frac{1}{4} \\
\text{b} & \quad 15 \div \frac{5}{3} \\
\text{c} & \quad \frac{4}{5} \div 8 \\
\text{d} & \quad 1 \frac{8}{9} \div 6 \\
\text{e} & \quad \frac{2}{5} \div \frac{3}{7} \\
\text{f} & \quad \frac{3}{8} \div \frac{2}{7} \\
\text{g} & \quad \frac{9}{8} \div \frac{7}{5} \\
\text{h} & \quad \frac{6}{7} \div \frac{4}{5} \\
\text{i} & \quad 1 \frac{7}{8} \div 8 \\
\text{j} & \quad 2 \frac{1}{4} \div 1 \frac{1}{7} \\
\text{k} & \quad 2 \frac{2}{3} \div 3 \frac{1}{4} \\
\text{l} & \quad 2 \frac{3}{4} \div 5 \\
\text{m} & \quad 3 \frac{4}{9} \div 1 \frac{2}{7} \\
\text{n} & \quad 5 \frac{1}{3} \div 3 \frac{2}{5}
\end{align*} \]

5. Simplify:

\[ \begin{align*}
\text{a} & \quad \frac{17}{18} \div \frac{4}{9} \\
\text{b} & \quad \frac{4}{7} \div \frac{8}{9} \\
\text{c} & \quad 4 \div \frac{7}{13} \\
\text{d} & \quad 6 \div \frac{3}{4} \\
\text{e} & \quad \frac{8}{9} \div 12 \\
\text{f} & \quad \frac{50}{35} \div 25 \\
\text{g} & \quad \frac{3}{8} \div \frac{2}{13} \\
\text{h} & \quad \frac{1}{4} \div \frac{3}{7} \\
\text{i} & \quad 3 \frac{3}{4} \div 1 \frac{2}{7} \\
\text{j} & \quad 3 \frac{1}{4} \div 2 \frac{2}{7} \\
\text{k} & \quad 2 \frac{3}{4} \div 3 \frac{1}{4} \\
\text{l} & \quad 1 \frac{7}{8} \div 2 \frac{1}{3}
\end{align*} \]

6. Write three sums involving fractions and at least two of the operations +, −, ×, ÷ that give an answer of \(\frac{3}{4}\).

7. A swimming club has 350 members. If \(\frac{3}{5}\) of the members are girls, how many boys are members?

8. There are 720 students at Kauri Secondary College. \(\frac{3}{20}\) of the students are in Year 12. How many Year 12 students are there?

9. There are 128 students in Year 8. What fraction of the students are in Year 8?

10. If the area of a triangle is calculated by the rule \(\frac{1}{2} \times \text{base} \times \text{height}\), find the total area of the shape at the right.

11. Jacqueline found that one of her landscaping jobs required \(3 \frac{1}{2} \text{ m}^3\) (cubic metres) of topsoil. She estimates that the next job will require \(2 \frac{1}{3}\) times as much. How much topsoil will she need to order for both jobs?

12. A security guard at a shopping mall knows that it takes \(\frac{1}{3}\) of an hour to walk around the mall when on patrol. How many times can he walk around the mall in \(\frac{4}{5}\) of an hour?

13. In a 2 \(\frac{1}{2}\) hour maths exam, \(\frac{1}{5}\) of the time is allocated as reading time. How long is the reading time?

14. A driveway is to be constructed with \(15 \frac{1}{2} \text{ m}^3\) of crushed rock. If a small truck can carry \(2 \frac{1}{3} \text{ m}^3\) of crushed rock, how many truckloads will be needed?
14 Feroza is paid $5 an hour for working at her local cafe. If she works $6 \frac{3}{4}$ hours how much will she be paid?

15 Eight-ninths of the students in Year 9 participated in the school swimming carnival. If there are 171 pupils in Year 9, how many participated?

16 Regan worked for $7 \frac{1}{2}$ hours in a sandwich shop. Three-fifths of her time was spent cleaning up and the rest serving customers. How much of her time did she spend serving customers?

**ENRICHMENT: Choosing a strategy**

17 Choose a strategy to perform these calculations. Remember to follow BEDMAS.

\[
\begin{align*}
\text{a} & \quad \frac{5}{9} + \frac{2}{3} \times \frac{15}{22} \\
\text{b} & \quad \frac{5}{5} + \frac{2}{3} \times \frac{15}{22} \\
\text{c} & \quad \frac{3}{5} \div \frac{3}{4} \times \frac{1}{9} \\
\text{d} & \quad 1 \frac{7}{8} + 2 \frac{1}{4} \times 4 \frac{1}{2} \\
\text{e} & \quad 3 \frac{3}{5} \div 2 \frac{1}{3} \times 3 \frac{3}{5} \\
\text{f} & \quad 2 \frac{2}{3} \times 3 \frac{1}{7} + 1 \frac{1}{7} \\
\text{g} & \quad 1 \frac{1}{2} + 3 \frac{3}{4} \times \frac{8}{9} \\
\text{h} & \quad 1 \frac{3}{15} - \frac{3}{4} \times \frac{2}{27} \\
\text{i} & \quad 2 \frac{7}{14} + 1 \frac{3}{4} - \frac{3}{7}
\end{align*}
\]
A class is asked by their teacher, ‘What does $\frac{2}{5}$ mean?’

Ella says that when one whole is shared into 5 parts, she would have 2 of the parts.

Jack says he would have 0.4 of a whole as shown on the double number line.

Kym draws a rectangle divided into tenths and shades four divisions.

Mere uses a decimat divided into 100 small squares and shades 40 of them.

In groups discuss who you think is correct. Explain.

Mere later explains that if she uses a decimat and shades half the number of squares, she has shaded 5 out of 10, or 50 out of 100 or 500 out of 1000, so she has the fractions:

$\frac{1}{2} = \frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$

She writes the fractions in the place value houses and she now has decimal numbers.

Other common fractions are: $\frac{1}{4} = \frac{25}{100} = \frac{250}{1000} = 0.25 = 0.250$

and $\frac{1}{8} = \frac{125}{1000} = 0.125$

These students have changed the fraction to an equivalent fraction with the denominator 10, 100 or 1000.

It is not always easy to write a fraction with a denominator of 10, 100 or 1000, so we need to look at a strategy to divide the numerator by the denominator.
KEY IDEAS

- A fraction is changed to a decimal by dividing the numerator by the denominator.
- A decimal may be changed to a fraction by writing the place value of the digits:
  \[0.173 = \frac{173}{1000}\]

EXAMPLE 20

a  Write \(\frac{1}{5}\) as:
  i  an equivalent fraction with 10, 100, and 1000 as the denominator
  ii  a decimal

b  Write as a fraction in its simplest form.
  i  0.13
  ii  0.276

Solution | Explanation
--- | ---
\[\text{a i } \frac{1}{5} = \frac{2}{10} = \frac{20}{100} = \frac{200}{1000}\] | Using a decimal, one-fifth can be divided into two, which gives two-tenths.

\[\text{When decimals are divided into hundredths and into thousandths, the shaded area contains 20 hundredths or 200 thousandths.}\]

\[\text{ii } 0.2\] | \(\frac{2}{10}\) in place value house

\[\text{b i } 0.13 = \frac{13}{100}\] | Because there is one tenth and 3 hundredths, we can write the fraction \(\frac{13}{100}\)

\[\text{ii } 0.276 = \frac{276}{1000} = \frac{138}{500} = \frac{69}{250}\] | Because there are 276 thousandths, we can write the fraction \(\frac{276}{1000}\). This can be simplified by halving the numerator and the denominator.
EXAMPLE 21

Without using a calculator or a decimat, change to a decimal.

\[ \begin{align*}
\text{a} & \quad \frac{1}{5} \\
\text{b} & \quad \frac{3}{4}
\end{align*} \]

<table>
<thead>
<tr>
<th>Solution</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{5} \times 2 )</td>
<td>The denominator (5) divides exactly into 10, so rewrite ( \frac{1}{5} ) out of 100.</td>
</tr>
<tr>
<td>( \frac{1}{5} \times 2 = \frac{2}{10} = 0.2 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{4} \times 25 )</td>
<td>The denominator (4) does not divide exactly into 10, so try 100.</td>
</tr>
<tr>
<td>( \frac{3}{4} \times 25 = \frac{75}{100} = \frac{3}{4} \times 25 = 0.75 )</td>
<td>4 divides exactly into 100 so rewrite ( \frac{3}{4} ) out of 100.</td>
</tr>
</tbody>
</table>

EXERCISE 1f

1. Write as:
   i. an equivalent fraction with 10, 100, 1000 as a denominator
   ii. a decimal

   \[ \begin{align*}
   \text{a} & \quad \frac{1}{10} = \boxed{\frac{1}{10}} = \boxed{\frac{\Delta}{1000}} \\
   \text{b} & \quad \frac{2}{5} = \boxed{\frac{2}{5}} = \boxed{\frac{\Delta}{1000}} \\
   \text{c} & \quad \frac{1}{4} = \boxed{\frac{1}{4}} = \boxed{\frac{\Delta}{1000}} \\
   \text{d} & \quad \frac{3}{4} = \boxed{\frac{3}{4}} = \boxed{\frac{\Delta}{1000}} \\
   \text{e} & \quad \frac{3}{8} = \boxed{\frac{3}{8}} = \boxed{\frac{\Delta}{1000}} \\
   \text{f} & \quad \frac{3}{20} = \boxed{\frac{3}{20}} = \boxed{\frac{\Delta}{1000}} \\
   \text{g} & \quad \frac{9}{25} = \boxed{\frac{9}{25}} = \boxed{\frac{\Delta}{1000}} \\
   \text{h} & \quad \frac{9}{50} = \boxed{\frac{9}{50}} = \boxed{\frac{\Delta}{1000}}
   \end{align*} \]

2. Write as a fraction in its simplest form.

   \[ \begin{align*}
   \text{a} & \quad 0.3 \\
   \text{b} & \quad 0.15 \\
   \text{c} & \quad 0.75 \\
   \text{d} & \quad 0.5 \\
   \text{e} & \quad 0.32 \\
   \text{f} & \quad 0.98 \\
   \text{g} & \quad 0.273 \\
   \text{h} & \quad 0.767 \\
   \text{i} & \quad 0.483 \\
   \text{j} & \quad 0.325
   \end{align*} \]

3. Change the fraction to a decimal.

   \[ \begin{align*}
   \text{a} & \quad \frac{7}{10} \\
   \text{b} & \quad \frac{3}{5} \\
   \text{c} & \quad \frac{1}{2} \\
   \text{d} & \quad \frac{1}{8} \\
   \text{e} & \quad \frac{3}{2} \\
   \text{f} & \quad 2 \frac{3}{10}
   \end{align*} \]
4 Without using a calculator, use the best strategy to change the fraction to a decimal.
   a \( \frac{2}{3} \)  
   b \( \frac{3}{25} \)  
   c \( \frac{7}{20} \)  
   d \( \frac{5}{40} \)

5 Which of the following is greater than 0.76?
   A \( \frac{76}{100} \)  
   B 0.706  
   C \( \frac{39}{50} \)  
   D \( \frac{18}{25} \)

6 How many spaces can you fill in from memory? Students are expected to know these fractions and decimals and their conversions.

Copy and complete the table.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>1/3</td>
<td>0.25</td>
<td>1/5</td>
<td>1/6</td>
<td>0.125</td>
<td>1/10</td>
<td>1/20</td>
</tr>
<tr>
<td>i</td>
<td>j</td>
<td>k</td>
<td>l</td>
<td>m</td>
<td>n</td>
<td>o</td>
<td>p</td>
</tr>
<tr>
<td>0.02</td>
<td>2/3</td>
<td>3/8</td>
<td>3/4</td>
<td>0.625</td>
<td>2/5</td>
<td>2/3</td>
<td>b</td>
</tr>
</tbody>
</table>

After you have completed the chart, test your partner.

ENRICHMENT: A snug fit

7 Some very old mathematics books state that the circumference of a circle is three times the diameter. This means that they considered \( \pi = 3 \).

Your parents would have used \( \pi = 3 \frac{1}{7} \) or 3.14 or even 3.14159 as an approximation.

Using your calculator and the symbol for \( \pi \), you can use a very accurate value.

   a How do these different values of \( \pi \) affect the calculation of the circumference and the area of a circle?
   b Calculate the circumference and the area of a circle of radius 100 cm using each of these values for \( \pi \).
Adding and subtracting decimals

Which of the following strategies do you think is the best for calculating $3.6 + 5.9$?

<table>
<thead>
<tr>
<th>Anna’s method</th>
<th>Jack’s method</th>
<th>Tim’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td>+5</td>
<td>Four tenths need to be added to 3.6 to make a whole number, and 5.9 = 0.4 + 5 + 0.5</td>
<td>5.9 is 5 whole and 9 tenths. 3.6 is 3 whole and 6 tenths. When I add the whole numbers I get 8. This leaves me 9 tenths plus 6 tenths. I need another tenth to add to 9 tenths to make another whole. I now have 5 tenths left. Answer: $8 + 1 + 0.5 = 9.5$ OR $5 + 3 + 0.9 + 0.1 + 0.5 = 9.5$</td>
</tr>
<tr>
<td>3.6</td>
<td>= 3.6 + 0.4 + 5 + 0.5</td>
<td>= 4 + 5 + 0.5</td>
</tr>
<tr>
<td>8.6</td>
<td>= 9 + 0.5</td>
<td>= 9.5</td>
</tr>
<tr>
<td>9.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I can split 5.9 into 5 whole and 9 tenths. I add 3.6 and 5 to get 8.6. Another 9 tenths gives 9.5.

Can you think of another strategy you could use? Is one strategy always better than another?

Discuss these methods with your partner and share your ideas with the class.

KEY IDEAS

- Number line addition or subtraction allows decimal numbers to be broken into convenient component parts.
- Decimal numbers can be made into whole numbers. For example, in Jack’s method, 4 tenths is subtracted from 5.9 and added to 3.6, to make it the whole number 4. We then add 4 to the 5.5 that is left.

EXAMPLE 22

Find the answer.

<table>
<thead>
<tr>
<th>a 4.8 + 2.5</th>
<th>b 4.3 + 5.9</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Solution</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a $4.8 + 2.5 = 4.8 + 2 + 0.5$</td>
<td>+2</td>
</tr>
<tr>
<td>= 6.8 + 0.5</td>
<td>+0.5</td>
</tr>
<tr>
<td>= 7.3</td>
<td>4.8 6.8 7.3</td>
</tr>
<tr>
<td>b $4.3 + 5.9 = 5.9 + 0.1 + 4.2$</td>
<td>5.9 requires one more tenth to make the whole number 6. To compensate, 4.3 is reduced by one tenth to 4.2. Addition is easy: 6 + 4.2</td>
</tr>
<tr>
<td>= 6 + 4.2</td>
<td>4.8</td>
</tr>
<tr>
<td>= 10.2</td>
<td></td>
</tr>
</tbody>
</table>
### EXAMPLE 23

Find the answers.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>4.8 − 2.5</td>
<td><strong>b</strong></td>
</tr>
</tbody>
</table>

#### Solution

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| **a** | 4.8 − 2.5 = 4.8 − 2 − 0.5  
   = 2.8 − 0.5  
   = 2.3 |

#### Explanation

This strategy is called rounding and compensating.  
4.8 is rounded to 5 by adding 2 tenths. To compensate, 2 tenths is also added to 6.2. So it is now easy to subtract 5 from 6.4.

### EXAMPLE 24

Find the answers.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>0.34 + 0.9</td>
</tr>
</tbody>
</table>

#### Solution

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
</table>
| **a** | 0.34 + 0.9 = 0.3 + 0.04 + 0.9  
   = 0.3 + 0.9 + 0.04  
   = 1.24 + 0.04 |

#### Explanation

The numbers can be split into their place values:  
0.3 = 3 tenths plus 4 hundredths  
and 0.9 = 9 tenths  
0.3 + 0.9 = 1.2  
1.2 + 4 hundredths = 1.24

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>b</strong></td>
<td>6.564 + 7.628</td>
</tr>
</tbody>
</table>

#### Solution

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| **b** | 6.564 + 7.628 = 6 + 7 + 0.564 + 0.628  
   = 13 + 0.564 + 0.628  
   = 13 + 0.5 + 0.06 + 0.004  
   + 0.8 + 0.02 + 0.008  
   + 0.5 + 0.6 + 0.06 + 0.02 + 0.004 + 0.008 |

#### Explanation

The whole numbers can be added before the decimal fractions are split into their place values.  
Numbers are added in this order:  
- whole numbers  
- tenths  
- hundredths  
- thousandths

### Chapter 1 Number

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**EXAMPLE 25**

Find the answers.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td><strong>b</strong></td>
</tr>
<tr>
<td>$2.34 - 1.89$</td>
<td>$1.55 - 0.673$</td>
</tr>
</tbody>
</table>

**Solution**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td><strong>b</strong></td>
</tr>
<tr>
<td>$2.34 - 1.89 = 2.34 + 0.11 - (1.89 + 0.11)$</td>
<td>$1.55 - 0.673 = 1.55 + 0.007 - (0.673 + 0.007)$</td>
</tr>
<tr>
<td>$= 2.45 - 2$</td>
<td>$= 1.557 - 0.68$</td>
</tr>
<tr>
<td>$= 0.45$</td>
<td>$= 1.557 + 0.32 - (0.68 + 0.32)$</td>
</tr>
<tr>
<td></td>
<td>$= 1.877 - 1$</td>
</tr>
<tr>
<td></td>
<td>$= 0.877$</td>
</tr>
</tbody>
</table>

**Explanation**

This example uses the strategy of rounding and compensating. To make 1.89 a whole number, 0.11 is added. To compensate, 0.11 is also added to 2.34 to make it 2.45. It is now easy to take 2 away from 2.45.

OR Adapt the written method for subtracting whole numbers. Write the second number below the first and align the numbers on the decimal point:

$2.34 - 1.89 - 0.45$

Rounding and compensating are applied twice.

OR Adapt the written method for subtracting whole numbers. Align the numbers on the decimal point. Write a zero after 1.55 so that it has the same number of decimal places as 0.673:

$1.550 - 0.673 = 0.877$

**EXERCISE 1g**

1 Use the strategy you think is best to find each answer.

- **a** $4.2 + 3.7$
- **b** $3.8 + 2.2$
- **c** $6.4 + 0.9$
- **d** $3.2 + 1.7$
- **e** $1.6 + 4.4$
- **f** $0.6 + 0.7$
- **g** $1.9 + 1.9$
- **h** $3.6 + 1.8$

2 Find the answers.

- **a** $1.2 - 0.8$
- **b** $0.9 - 0.2$
- **c** $2.3 - 1.8$
- **d** $6.7 - 4.8$
- **e** $3.2 - 1.6$
- **f** $8.1 - 2.7$
- **g** $1.1 - 0.3$
- **h** $4.4 - 1.5$

3 Sonny says 0.56 + 3.2 is 3.76, Max says the answer is 3.58 and Jim says 0.88 is correct. Use two strategies to find the correct answer. Who is correct? Explain the errors two of the students made.

4 Which of the following is the best rounding strategy for calculating $5.67 - 2.91$?

- **A** $5.76 - 3$
- **B** $5.71 - 2.95$
- **C** $5.63 - 2.87$
- **D** $4.76 - 2$

5 What is the missing digit in the calculation below?

```
12.34 -
  9 0 7
---  
  2 6 7
```
EXAMPLE 25a  
Use the strategy you think is best to find each answer.

| a | 5.23 + 0.6 | b | 3.57 + 0.6 | c | 5.72 + 0.4 | d | 9.61 + 2.7 |
| e | 0.26 + 0.43 | f | 1.52 + 0.74 | g | 1.03 + 5.69 | h | 5.84 + 2.19 |
| i | 0.88 + 4.79 | j | 42.87 + 2.24 | k | 5.67 + 2.09 | l | 7.34 + 1.07 |
| m | 0.99 + 0.93 | n | 22.08 + 2.57 | o | 36.56 + 14.87 | p | 0.45 + 1.08 |
| q | −1.25 + 0.33 | r | −0.46 + 0.32 | s | −0.84 + 0.25 | t | −2.64 + 1.57 |

EXAMPLE 25b  
Sonny, Max and Jim again have different answers for this problem. A builder requires two lengths of timber of 1.057 m and 0.86 m to make steps. Sonny says the builder will need a total length of 9.657 m, Max says she will need 1.917 m and Jim says she will need 1.143 m. Who is correct this time? Explain your answer.

EXAMPLE 25c  
Rachael needs a bookshelf. She has a space on her wall 1.675 m long, but the shelf is 0.95 m long. How much space is left? Our friends Sonny, Max and Jim discuss the problem and tell her she will have 1.18 m left. She disagrees and says there will be 0.725 m left.

Explain the error the boys have made and why Rachael is correct.

EXAMPLE 25d  
Use strategies of your choice to find the answers.

| a | 9.38 − 7.1 | b | 3.84 − 2.6 | c | 0.36 − 0.3 | d | 4.21 − 3.9 |
| e | 7.42 − 5.23 | f | 2.63 − 1.42 | g | 1.96 − 0.75 | h | 2.07 − 1.06 |
| i | 2.93 − 1.66 | j | 7.52 − 5.65 | k | 3.46 − 2.64 | l | 6.42 − 3.71 |
| m | 3.65 − 1.82 | n | 2.73 − 1.48 | o | 8.56 − 7.89 | p | 5.37 − 2.19 |
| q | 1.26 − 0.57 | r | 5.42 − 3.65 | s | 6.27 − 4.59 | t | 3.03 − 1.17 |

EXAMPLE 25e  
Use strategies of your choice to find the answers.

| a | 8.913 − 8.501 | b | 2.154 − 1.021 | c | 6.305 − 5.002 |
| d | 6.853 − 6.762 | e | 3.674 − 2.559 | f | 2.751 − 1.338 |
| g | 4.582 − 4.489 | h | 1.025 − 0.537 | i | 2.164 − 1.189 |
| j | 3.651 − 1.897 | k | 2.421 − 1.543 | l | 5.027 − 4.348 |
| m | 1.234 − 1.2 | n | 0.564 − 0.13 | o | 1.642 − 0.759 |

EXAMPLE 25f  
Use strategies of your choice to find the answers.

| a | 0.045 + 2.67 + 42.3 | b | 7.89 + 34.9 + 26.67 |
| c | 5.64 + 2.789 + 12.9 | d | 17.4 + 34.65 + 90 |
| e | 3.26 + 2.49 − 0.02 | f | 17.04 − 1.74 + 12.2 |
| g | 3.98 − 2.76 + 2 − 1.5 | h | 5.771 − (2.2 + 0.095) + 0.6 |
| i | 15.678 − 3.6 − 2.07 − 1.008 | j | (2.80 + 0.15) − (1.75 + 1.20) |
| k | 2.2 − 0.4 + 0.3 | l | 3.5 + 0.4 |
| m | 1.8 − 0.2 | n | 8.2 − 6.4 |
| o | 8.9 − 9.8 | p | 9.78 − 7 |
13 Write at least three sums using addition which give an answer of 5.423.

14 Write at least three sums using subtraction which give an answer of 1.538.

15 Emma bought shampoo, flea rinse and a collar for her dog. The items cost $4.75, $2.57 and $9.00 respectively. She gave the veterinarian a $20 note.
   a What was the total cost of the items?
   b What change (to the nearest 10 cents) did she receive?

16 For a party, Rae bought two cakes for $17.49 and $13.75, and two for $20 each. How much (to the nearest 5 cents) did Rae pay?

17 An automotive service centre charges $456 for a major car service. If it cost the centre $179.56 for the parts and lubricants, what did the centre charge for labour and profit?

18 A giraffe was measured at the end of each month to check its growth rate.

<table>
<thead>
<tr>
<th>Month</th>
<th>January</th>
<th>February</th>
<th>March</th>
<th>April</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth</td>
<td>0.237 m</td>
<td>0.246 m</td>
<td>0.32 m</td>
<td>0.345 m</td>
</tr>
</tbody>
</table>

   a What was the total growth over the 4-month period?
   b If the giraffe was 3.67 m tall at the beginning of January, how tall was it at the end of February?

ENRICHMENT: Decimal nonogram

19 Make an accurate copy of these three number lines, with divisions on the number lines 2 cm apart. Position a ruler so that it connects a number on the top line and a number on the bottom line. The sum of those numbers is shown on the middle line.

```
<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0.8</td>
<td>1.2</td>
<td>1.6</td>
<td>2.0</td>
<td>2.4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
<td>1.2</td>
</tr>
</tbody>
</table>
```

Explain, with examples, how the number lines could be used to subtract decimals.
David is building a ramp for his skateboard. He works out that he will need 3.6 m of bracing timber because he wants 4 pieces 0.9 m long. What strategy could David have used to do this calculation?

Our three friends, Sonny, Max and Jim offer their advice.

<table>
<thead>
<tr>
<th>Sonny’s advice</th>
<th>Max’s advice</th>
<th>Jim’s advice</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can round and compensate: 9 tenths plus 1 tenth makes one whole: 0.9 + 0.1 = 1.0 4 lots of 1 m = 4 m and 4 lots of 0.1 m = 0.4 m (0.1 + 0.1 + 0.1 + 0.1 = 0.4) That is: 4 × 1 = 4 and 4 × 0.1 = 0.4 Therefore: amount of timber = 4 – 0.4 = 3.6 m</td>
<td>9 tenths times 4 is 36 tenths. When I write this number in the place value house like this:</td>
<td>I need one more tenth to make a whole. Three tenths from the purple bar make the other three bars into whole bars, which is 3 whole, and I have 6 tenths left over.</td>
</tr>
</tbody>
</table>

\[ \therefore 0.1 \times 4 = 3.6 \]

Which strategy do you prefer? Discuss your reasons with members of your group.

### Key Ideas

#### Multiplying Decimals

- **To multiply** by a decimal:
  - determine how many decimal places there are in the numbers being multiplied
  - perform normal multiplication (using an algorithm, if appropriate)
  - write your answer with the total number of decimal places in the question.

- **Multiplying any number by a decimal less than one gives a smaller number**:
  - e.g. \( 6 \times 0.3 = 1.8 \quad 0.6 \times 0.3 = 0.18 \)

#### Dividing Decimals

- **To divide** by a decimal, first adjust both numbers to make the divisor into a whole number:
  - e.g. for \( 35.79 \div 0.2 \), multiply both numbers by 10 and then calculate \( 357.9 \div 2 \).

- **Dividing a number by a decimal less than one gives a bigger number**:
  - e.g. \( 3 \div 0.5 = 6 \quad 1.2 \div 0.04 = 30 \)
EXAMPLE 26

Find the answers.

\[ a \quad 4.2 \times 7 \]
\[ b \quad 4.23 \times 0.5 \]
\[ c \quad 6.32 \times 0.001 \]

<table>
<thead>
<tr>
<th>Solution</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong> 42 \times 7</td>
<td>Perform normal multiplication without the decimal point.</td>
</tr>
<tr>
<td>29.4</td>
<td>There is one decimal place in the numbers being multiplied, i.e. 4.2 and 7</td>
</tr>
<tr>
<td>4.2 \times 7 = 29.4</td>
<td>Write the answer, positioning the decimal point to give one decimal place.</td>
</tr>
<tr>
<td><strong>b</strong> 423 \times 5</td>
<td>Because 4.23 is multiplied by a decimal less than one (0.5), the answer will be less than 4.23.</td>
</tr>
<tr>
<td>2\cdot1\cdot15</td>
<td>Perform normal multiplication without the decimal points.</td>
</tr>
<tr>
<td>4.23 \times 0.5 = 2.115</td>
<td>There are three decimal places in the numbers being multiplied.</td>
</tr>
<tr>
<td><strong>c</strong> 632 \times 1 = 632</td>
<td>Write the answer, positioning the decimal point to give 3 decimal places.</td>
</tr>
<tr>
<td>6.32 \times 0.001 = 0.00632</td>
<td>The problem asks us to find one-thousandth of 6.32. So the answer must be very much smaller than the starting number.</td>
</tr>
<tr>
<td></td>
<td>There are five decimal places in the numbers being multiplied.</td>
</tr>
<tr>
<td></td>
<td>Write the answer, positioning the decimal point to give 5 decimal places.</td>
</tr>
</tbody>
</table>

EXAMPLE 27

Find the answers.

\[ a \quad 4.32 \div 4 \]
\[ b \quad 3.24 \div 0.6 \]
\[ c \quad 5.62 \div 1000 \]

<table>
<thead>
<tr>
<th>Solution</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong> 4.32 \div 4</td>
<td>Multiply 4.32 by 100 to make it a whole number. To compensate, divide by 100.</td>
</tr>
<tr>
<td>= 4.23 \times 100 \div 100 \div 4</td>
<td>Divide 432 by 4.</td>
</tr>
<tr>
<td>= 432 \div 4 \div 100</td>
<td>Then divide 108 by 100.</td>
</tr>
<tr>
<td>= 108 \div 100</td>
<td>= 1.08</td>
</tr>
<tr>
<td><strong>b</strong> 3.24 \div 0.6</td>
<td>Because 0.6 is less than one, the answer will be greater than 3.24.</td>
</tr>
<tr>
<td>= 324 \div 60</td>
<td>Multiply both 3.24 and 0.6 by 100.</td>
</tr>
<tr>
<td>= 324</td>
<td>60 \times 5 + 24 = 324</td>
</tr>
<tr>
<td>= 54</td>
<td></td>
</tr>
<tr>
<td>= 10</td>
<td></td>
</tr>
<tr>
<td>= 5.4</td>
<td></td>
</tr>
<tr>
<td><strong>c</strong> 5.62 \div 1000</td>
<td>The answer will be a very small number.</td>
</tr>
<tr>
<td>= 0.00562</td>
<td>To divide by 1000, move the decimal point 3 places to the left, since 1000 has 3 zeros.</td>
</tr>
</tbody>
</table>
EXERCISE 1h

1 Which one of the following is not equal to 2.3 \times 30?
   A 23 \times 3  \quad B 4.6 \times 15  \quad C 11.5 \times 6  \quad D 2.3 \times 60 \div 3

2 Evaluate:
   a 3.2 \times 30  \quad b 4.1 \times 40  \quad c 6.3 \times 20  \quad d 1.2 \times 50

EXAMPLE 26a
3 Find the answers.
   a 1.5 \times 7  \quad b 4.6 \times 3  \quad c 6.5 \times 8  \quad d 0.3 \times 9
   e 4 \times 6.7  \quad f 3 \times 5.7  \quad g 12 \times 2.8  \quad h 6 \times 1.2

EXAMPLE 26b
4 Find the answers.
   a 0.7 \times 0.6  \quad b 2.6 \times 0.8  \quad c 5.7 \times 0.6  \quad d 2.8 \times 0.5
   e 1.52 \times 0.4  \quad f 2.64 \times 0.8  \quad g 8.2 \times 0.6  \quad h 9.4 \times 0.7
   i 3.46 \times 0.3  \quad j 6.34 \times 0.4  \quad k 7.3 \times 0.7  \quad l 12.63 \times 0.9

EXAMPLE 26c
5 Find the answers.
   a 3.2 \times 0.01  \quad b 6.34 \times 0.01  \quad c 0.23 \times 0.01  \quad d 0.6403 \times 0.01
   e 0.567 \times 0.01  \quad f 0.567 \times 0.001  \quad g 6.8 \times 0.001  \quad h 254.3 \times 0.001
   i 8.9754 \times 0.001

EXAMPLE 27a
6 Find the answers.
   a 2.56 \div 4  \quad b 5.13 \div 3  \quad c 4.08 \div 4  \quad d 2.17 \div 7
   e 4.59 \div 9  \quad f 2.88 \div 3  \quad g 14.04 \div 4  \quad h 15.47 \div 7
   i 3.58 \div 4  \quad j 2.82 \div 4  \quad k 4.16 \div 5  \quad l 15.32 \div 8

EXAMPLE 27b
7 Find the answers.
   a 0.2 \div 0.5  \quad b 2 \div 0.5  \quad c 0.2 \div 5  \quad d 0.07 \div 0.2
   e 0.18 \div 0.3  \quad f 0.06 \div 0.2  \quad g 5.12 \div 0.4  \quad h 12.33 \div 0.3
   i 42.49 \div 0.7  \quad j 9.12 \div 0.8  \quad k 76.88 \div 0.8  \quad l 0.005 \div 0.5
   m 1.236 \div 0.6  \quad n 2.02 \div 0.4  \quad o 5.06 \div 0.5  \quad p 3.07 \div 0.4
   q 2.4 \div 0.03  \quad r 36 \div 0.06  \quad s 3.6 \div 0.06  \quad t 3.06 \div 0.03

EXAMPLE 27c
8 a Find the answers.
   i 3.73 \div 1000  \quad ii 3.58 \div 1000  \quad iii 6.45 \div 1000  \quad iv 45.231 \div 1000
   v 6.49 \div 100  \quad vi 8.43 \div 100  \quad vii 42.33 \div 10  \quad viii 85.02 \div 10
   ix 0.0035 \div 1000  \quad x 1.005 \div 1000  \quad xi 0.0006 \div 100  \quad xii 0.0087 \div 100

b Examine your answers to these questions and then write a rule that you could use when dividing by 10, 100 or 1000. Discuss your rule with your partner.

9 a Use a place value house to work out the answers.
   i 3.57 \div 0.001  \quad ii 5.48 \div 0.001  \quad iii 0.246 \div 0.001
   iv 0.645 \div 0.001  \quad v 0.0573 \div 0.001  \quad vi 0.00763 \div 0.001
   vii 0.645 \div 0.01  \quad viii 0.0573 \div 0.01  \quad ix 0.00763 \div 0.01
   x 0.645 \div 0.1  \quad xi 0.0573 \div 0.1  \quad xii 0.00763 \div 0.1

b Examine your answers to these questions and then write a rule that you could use when dividing by 0.1, 0.01 or 0.001. Discuss your rule with your partner.

10 A hose 125 m long is cut into 100 pieces. How long is each piece of hose?
11 The scoring at diving events involves averaging the judges’ scores and then multiplying by a number that represents the degree of difficulty of that dive. If the judges’ average score was 7.2 and the dive’s degree of difficulty was 3, what did the diver score?

12 Megan fills the tank of her car with 38.5 litres of petrol. If petrol costs $2.20 per litre how much did she pay?

13 How many pizza bases weighing 175 g can be made from 2.45 kg of pizza base dough?

14 a The heights in metres of five players of a netball team are 1.6, 1.7, 1.6, 1.8 and 1.9. Find their mean (average) height.
   b If the average height of five netballers in a second team is 1.75 m, give at least two sets of possible values for the heights.

15 Marie has estimated that the cost of the labour and materials for painting is $15.50 per square metre. How much should she quote to paint a hall with a total wall surface 15.5 m by 4 m?

16 The volume of soil removed to prepare the site for a house slab was 5.75 m by 20 m by 0.6 m deep. How much soil was removed?

17 A washing-machine repairer charges a call-out fee of $65 plus $32.50 per hour, plus parts.
   a If a job involves a call of 42 minutes and parts that total $14.56, what is the bill?
   b If the cost of the job was $254.50, write at least two different sums showing the length of the call and the cost of parts.

18 A cement mixer delivers 0.1 m³ of cement in one load. How many loads will be needed to make a floor that measures 2.5 m by 3.1 m by 10 m thick?

ENRICHMENT: All that glitters!

19 Just 1 gram of gold can be rolled into a very thin sheet covering 50 000 square centimetres.
   a How many grams of gold would be needed to cover your body with gold leaf? The area of your body can be calculated approximately by the formula $2 \times h \times t$, where $h$ is your height and $t$ is your thigh circumference in centimetres.
   b How much would a gold leaf skin cost? Use the internet to obtain gold prices.
Indices are also called ‘powers’ or exponents.

How many cells are there in each of these arrays? Without actually counting the cells, how can you work out the number of cells?

Draw the next two terms in the pattern.

In pairs, explain how to find the number of cells in any array of the pattern. Share your answers with the rest of the class.

Now investigate how many blocks you need to build a pattern of cubes whose faces are the arrays above. Explain how to find the number of blocks in any term of the pattern.

Indices can be used to describe the squares and cubes above. Can you draw a pattern in which the $n$th term has $n^4$ cubes?

**KEY IDEAS**

- $3^4 = 3 \times 3 \times 3 \times 3 = 81$
- **Exponent** is another name for index number or power.
- To multiply numbers with the same base, add the indices:
  - e.g. $3^4 \times 3^2 = 3^{4+2} = 3^6$
  - $a^m \times a^n = a^{m+n}$
To divide numbers with the same base, subtract the indices:
e.g. $3^5 ÷ 3^2 = 3^{5-2} = 3^3$

$$a^m ÷ a^n = a^{m-n} \text{ OR } \frac{a^m}{a^n} = a^{m-n}$$

Square root is the opposite operation to finding the square of a number:
e.g. $\sqrt{16} = \sqrt{4 \times 4} = 4$

Cube root is the opposite operation to finding the cube of a number:
e.g. $\sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$

### EXAMPLE 28

Evaluate:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$2^3$</td>
<td>b</td>
</tr>
</tbody>
</table>

**Solution**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$2^1 = 8$</td>
<td>Because $2^1$ is 2 cubed, it means 2 must be multiplied by itself 3 times: $2 \times 2 \times 2$</td>
</tr>
<tr>
<td>b</td>
<td>$4^5 = 1024$</td>
<td>Four to the power of 5 means $4 \times 4 \times 4 \times 4 \times 4$</td>
</tr>
</tbody>
</table>
| c | $\sqrt{144} = 12$ | The ‘opposite operation’ means we need to find the number that, when it is multiplied by itself, gives 144: $\square \times \square = 144$
From our multiplication tables we know $12 \times 12 = 144$, so $\square$ must be 12. |

### EXAMPLE 29

Calculate, leaving the answers in index form.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$2^3 \times 2^2$</td>
<td>b</td>
</tr>
</tbody>
</table>

**Solution**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| a | $2^1 \times 2^2 = 2^3$ | $2^1 = 2 \times 2 \times 2$
and $2^2 = 2 \times 2 \times 2 \times 2$
so $2^1 \times 2^2 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
OR $2^1 \times 2^2 = 2^{(1+2)}$ |
| b | $4^5 ÷ 4^2 = 4^3$ | $4^5 ÷ 4^2 = \frac{4^5 \times 4 \times 4 \times 4 \times 4}{4^2 \times 4^2}$
$= 4^2$
OR $4^5 ÷ 4^2 = 4^{(5-2)}$ |
EXERCISE 1

1 Evaluate:
   a $3^3$
   b $3^4$
   c $4^2$
   d $4^3$
   e $5^4$
   f $6^2$
   g $7^3$
   h $8^3$
   i $10^2$
   j $10^3$
   k $\sqrt{81}$
   l $\sqrt{49}$
   m $\sqrt{241}$
   n $\sqrt{400}$

2 Calculate each of the following, leaving the answers in index form.
   a $3^2 \times 3^5$
   b $5^3 \times 5^4$
   c $13^3 \times 13^4$
   d $8^6 \times 8^2$
   e $23^7 \times 23^5$
   f $102^4 \times 102^5$
   g $3^6 \div 3^3$
   h $9^6 \div 9^2$
   i $7^{12} \div 7^8$
   j $6^8 \div 6^3$
   k $26^{45} \div 26^{39}$
   l $100^{10} \div 100^7$
   m $4^3 \div 4^5$
   n $6^2 \div 6^5$
   o $11^8 \div 11^{11}$

3 Simplify each of the following, leaving your answers in index form.
   a $2^3 \times 2^6$
   b $2^6 \times 2^4$
   c $4^5 \times 4^5$
   d $7^3 \times 7^3$
   e $7^8 \div 7^4$
   f $10^8 \times 10^2$
   g $156^5 \div 156^3$
   h $9.9^2 \times 9.9^3$

4 Simplify where possible, leaving your answers in index form.
   a $3^4 \times 3^2 \times 3^2$
   b $3.6^3 \times 3.6^2 \times 3.6^4$
   c $2^9 \times 2^5 \times 3^3$
   d $2^{12} \div (2^6 \times 2^2)$
   e $9^8 \div (9^3 \times 9^1)$
   f $3^4 \times 3^3 \div 3^6$

5 Is the statement $4^3 \times 4^2 = 16^7$ true or false?

6 True or false? $\frac{3^2 \times 3^4}{3^2} = 3^6$

7 Find the square root of:
   a 25
   b 49
   c 64
   d 144
   e 169
   f 225
   g 625
   h 102
   i 10 000

8 Find:
   a $\sqrt{27}$
   b $\sqrt{625}$
   c $\sqrt{243}$
   d $\sqrt{81}$
   e $\sqrt{216}$
   f $\sqrt{1024}$
ENRICHMENT: Busy bacteria

9 Some bacteria can divide into two cells every 20 minutes to produce a new generation. A ‘family tree’ for one bacterial cell has been started for you. Use your knowledge of indices to assist you.

(Assumptions: No cell dies and every cell divides into two new cells.)

a Investigate the number of generations needed to produce a population of 32 768 bacteria.
b How long does it take?
c How much longer will it take to produce a population of more than 1 000 000 bacteria?

<table>
<thead>
<tr>
<th>Generation</th>
<th>Number in generation</th>
<th>Time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
When dealing with very large numbers or very small numbers it is easier to record them in a type of ‘shorthand’ method using powers of 10. Scientific notation, also called standard form, is used to represent very large numbers or very small numbers.

Scientists who calculate the number of bacteria in a food sample need to record many millions of bacteria, so they use scientific notation: e.g. if a pie was found to contain 127 392 000 000 bacteria.

Scientists who measure the amount of chemical in a sample are measuring extremely small amounts, and they too use scientific notation: e.g. if a chlorine test showed that there was 0.00000000000023 g of chlorine in 100 mL of water.

Sally says, ‘I think it has something to do with using powers because $10^5 = 10^1$, $100 = 10^2$, $1000 = 10^3$.’

Harry says, ‘That is okay for large numbers, but what about the small ones?’

Petra draws up the following table and fills in some of the cells. She adds arrows with comments about some of the things she notices.

Copy and complete the table for Petra.

<table>
<thead>
<tr>
<th></th>
<th>10^6</th>
<th>10^5</th>
<th>10^4</th>
<th>10^3</th>
<th>10^2</th>
<th>10^1</th>
<th>10^0</th>
<th>10^-1</th>
<th>10^-2</th>
<th>10^-3</th>
<th>10^-4</th>
<th>10^-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 000</td>
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<td></td>
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<tr>
<td>10 000</td>
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<tr>
<td>1 000</td>
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<td>100</td>
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<td>0.1</td>
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<tr>
<td>0.01</td>
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<tr>
<td>0.001</td>
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<tr>
<td>0.0001</td>
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<td></td>
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<tr>
<td>0.00001</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

This number must be 1000 000.

If I follow the same pattern this will be $10^6$. The powers are one less each time, so it must be $10^5$.

See how the pattern goes. It is one less each time.
**KEY IDEAS**

To write a number in standard form, we write it as a number between 1 and 10, multiplied by a power of 10:

- e.g. $127392000000 = 1.27392 \times 10^{11}$
- $0.0000000000023 = 2.3 \times 10^{-13}$

**EXAMPLE 30**

Write these ordinary decimal numbers in standard form.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>$4500000$</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>$0.0000004$</td>
</tr>
</tbody>
</table>

**Solution**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>$4500000 = 4.5 \times 10^6$</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>$0.0000004 = 4.0 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

**Explanation**

- **a**  
  Standard form means that the number must be between 1 and 10 and multiplied by a power of 10.  
  The number between 1 and 10 will be 4.5, so $4500000 = 4.5 \times 10^6$.  
  1 000 000 can also be written as $10^6$.

- **b**  
  The number between 1 and 10 will be 4.0, so $0.0000004 = 4.0 \times 0.0000001$ and $0.0000001 = \frac{1}{10^7} \Rightarrow \frac{1}{10^7} = 10^{-7}$  
  Since the ordinary decimal is less than 1 the power will be negative.

**EXAMPLE 31**

Write these standard form numbers as ordinary numerals.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>$3.1 \times 10^{12}$</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>$2.153 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

**Solution**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>$3.1 \times 10^{12} = 310000000000$</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>$2.153 \times 10^{-7} = 0.0000002153$</td>
</tr>
</tbody>
</table>

**Explanation**

- **a**  
  $10^{12} = 100000000000$  
  To multiply by $10^{12}$ move the decimal point 12 places to the right.

- **b**  
  $10^{-7} = 0.0000001$  
  To multiply by $10^{-7}$ move the decimal point 7 places to the left.
EXERCISE 1j

1. Write in scientific notation:
   a. 3 000 000
   b. 15 000 000 000
   c. 1200 000 000 000
   d. 21 000
   e. 4 500 000
   f. 64 000
   g. 5 million
   h. 34 thousands
   i. 500 hundreds

2. Write in scientific notation:
   a. 0.0004
   b. 0.000 003
   c. 0.000 000 024
   d. 0.000 000 000 73
   e. 0.000 03
   f. 0.000 000 000 125
   g. 0.000 034 5
   h. 0.008 76
   i. 0.000 000 008 09

3. Write in standard form:
   a. 6000
   b. 720 000
   c. 324.5
   d. 7869.03
   e. 8459.12
   f. 0.2
   g. 0.000 328
   h. 0.009 87
   i. 0.000 01
   j. 460 100 000
   k. 17 467
   l. 128

4. 25 \times 10^4 \text{ expressed in standard form is:}
   A 2.5 \times 10^3
   B 2.5 \times 10^5
   C 250 \times 10^3
   D 0.25 \times 10^4

5. 0.42 \times 10^9 \text{ expressed in standard form is:}
   A 4.2 \times 10^2
   B 4.2 \times 10^4
   C 42 \times 10
   D 42 \times 100

6. Write in standard form.
   a. 52 300 \times 10^4
   b. 6200 \times 10^9
   c. 450 \times 10^3
   d. 80 \times 10^5

7. Express (write) each of the following as an ordinary decimal numeral.
   a. 2.4 \times 10^6
   b. 3.6 \times 10^6
   c. 4.7 \times 10^{10}
   d. 5.86 \times 10^7
   e. 4.23 \times 10^8
   f. 3.78 \times 10^{12}
   g. 1.26 \times 10^9
   h. 3.03 \times 10^2
   i. 9.783 \times 10^5

8. Express (write) each of the following as a basic numeral.
   a. 3.6 \times 10^{-8}
   b. 4.6 \times 10^{-6}
   c. 2 \times 10^{-12}
   d. 2.74 \times 10^{-5}
   e. 3.678 \times 10^{-1}
   f. 7.89 \times 10^{-7}
   g. 4 \times 10^{-5}
   h. 3 \times 10^{-1}
   i. 9 \times 10^{-7}

9. Convert to a basic numeral:
   a. 9.43 \times 10^1
   b. 8.561 \times 10^2
   c. 4.59 \times 10^2
   d. 1.27 \times 10^3
   e. 7 \times 10^3
   f. 3 \times 10^9
   g. 6 \times 10^{-2}
   h. 7.213 \times 10^{-2}
   i. 9.57 \times 10^{-4}
   j. 41 \times 10^0
   k. 600 \times 10^3
   l. 450.6 \times 10^6
   m. 8900 \times 10^{-7}
   n. 40 100 \times 10^8
   o. 91 \times 10^{-6}
   p. 2819 \times 10^{-2}

10. Express these approximate numbers using scientific notation.
    a. The mass of the Earth is 6 000 000 000 000 000 000 000 000 kg.
    b. The diameter of the Earth is 40 000 000 m.
    c. The diameter of an atom is 0.000 000 000 1 m.
    d. The radius of the Earth’s orbit around the Sun is 150 000 000 km.
    e. The universal constant of gravitation is 0.000 000 000 066 7 Nm^2/kg^2.
    f. The half-life of polonium-214 is 0.000 15 seconds.
    g. Uranium-238 has a half-life of 4 500 000 000 years.
11 Express as a basic numeral.
   a Pluto is approximately $5.9 \times 10^9$ km from Earth.
   b A population of bacteria contained $6 \times 10^{15}$ organisms.
   c The Moon is approximately $3.84 \times 10^5$ km from Earth.
   d A 50-cent coin is approximately $1.8 \times 10^{-3}$ m thick.
   e The diameter of the nucleus of an atom is approximately $1 \times 10^{-14}$ m.
   f The population of a city is $5.6 \times 10^5$.

ENRICHMENT: Olympic costs

12 When Sydney was planning for the 2000 Olympic Games, the Olympic Organising Committee made the following predictions.
   - The cost of staging the games would be A$1.7 billion ($1.7 \times 10^9$), excluding infrastructure.
   - The cost of constructing or upgrading infrastructure would be A$807 million.

In fact, an extra A$140 million was spent on staging the games.

Use standard form to express the following costs (in Australian dollars A$).
   a The actual total cost of staging the 2000 Olympic Games
   b The total cost of staging the games and constructing or upgrading the infrastructure

13 Use the internet to investigate some very large and very small numbers. For example, you could look at the distances between planets, the mass of atoms, or the size of bacteria and viruses.
In our number system we use both positive and negative numbers. Temperature, for example, uses positive numbers for temperatures above zero and negative numbers for temperatures below zero.

Positive and negative whole numbers, along with zero, are called integers and include $\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots$

Like positive numbers, negative numbers can be added, subtracted, multiplied or divided.

**KEY IDEAS**

- **On a number line, negative numbers are to the left of zero.**

- **Addition and subtraction:**
  
  $a + (+b) = a + b \quad a - (-b) = a + b \quad a + (-b) = a - b \quad a - (+b) = a - b$

- **Multiplication:**
  
  - positive number $\times$ positive number = positive number
  - positive number $\times$ negative number = negative number
  - negative number $\times$ positive number = negative number
  - negative number $\times$ negative number = positive number

- **Division:**
  
  - positive number $\div$ positive number = positive number
  - positive number $\div$ negative number = negative number
  - negative number $\div$ positive number = negative number
  - negative number $\div$ negative number = positive number

- **When multiplying or dividing:**
  
  - if the signs are the *same* the answer is positive
  - if the signs are *different* the answer is negative

- **The order of BEDMAS applies for integers.**

**EXAMPLE 32**

Insert < (is less than) or > (is greater than) between:

- $a \quad -2 \quad$ and $\quad 3$
- $b \quad -1 \quad$ and $\quad -4$

<table>
<thead>
<tr>
<th>Solution</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \quad -2 \quad$ and $\quad 3$</td>
<td>$-2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$</td>
</tr>
<tr>
<td>$b \quad -1 \quad$ and $\quad -4$</td>
<td>$-4 \quad -3 \quad -2 \quad -1 \quad 0$</td>
</tr>
</tbody>
</table>

Chapter 1 Number 51

Uncorrected 3rd sample pages • Cambridge University Press © Brookie et al., 2015 • ISBN 978-1-107-59551-4 • Ph 03 8671 1400
Evaluate:
\[ \begin{align*}
\text{a} & \quad 2 + 3 = 2 - 3 = -1 \\
\text{b} & \quad -1 - 3 = -1 + 3 = 2 \\
\text{c} & \quad 2 \times 3 = (2 \times 3) = -6 \\
\text{d} & \quad 4 \div 2 = (4 \div 2) = 2
\end{align*} \]

**Solution Explanation**

\[ \begin{align*}
\text{a} & \quad 2 + 3 = 2 - 3 = -1 \\
\text{b} & \quad -1 - 3 = -1 + 3 = 2 \\
\text{c} & \quad 2 \times 3 = (2 \times 3) = -6 \\
\text{d} & \quad 4 \div 2 = (4 \div 2) = 2
\end{align*} \]

**EXERCISE 1k**

1. Insert < (is less than) or > (is greater than) between:
   \[ \begin{align*}
   \text{a} & \quad 2 \quad \text{and} \quad 5 \\
   \text{b} & \quad 4 \quad \text{and} \quad 1 \\
   \text{c} & \quad -1 \quad \text{and} \quad 0 \\
   \text{d} & \quad -4 \quad \text{and} \quad -2 \\
   \text{e} & \quad -7 \quad \text{and} \quad 1 \\
   \text{f} & \quad -4 \quad \text{and} \quad -10 \\
   \text{g} & \quad 2 \quad \text{and} \quad -3 \\
   \text{h} & \quad -10 \quad \text{and} \quad -12 \\
   \text{i} & \quad -16 \quad \text{and} \quad -4
   \end{align*} \]

2. Write down a number that is:
   \[ \begin{align*}
   \text{a} & \quad 4 \quad \text{less than} \quad 3 \\
   \text{b} & \quad 2 \quad \text{more than} \quad -2 \\
   \text{c} & \quad 6 \quad \text{more than} \quad -11 \\
   \text{d} & \quad 3 \quad \text{less than} \quad -4
   \end{align*} \]

3. Write the next two numbers in each pattern.
   \[ \begin{align*}
   \text{a} & \quad 4, 2, 0, \ldots, \ldots \\
   \text{b} & \quad -10, -7, -4, \ldots, \ldots \\
   \text{c} & \quad -1, -8, -15, \ldots, \ldots \\
   \text{d} & \quad 40, 10, -20, \ldots, \ldots
   \end{align*} \]

4. Evaluate:
   \[ \begin{align*}
   \text{a} & \quad +5 - 7 \\
   \text{b} & \quad -1 + 3 \\
   \text{c} & \quad -2 + 3 \\
   \text{d} & \quad -1 + 4 \\
   \text{e} & \quad +5 - 2 \\
   \text{f} & \quad +2 - 4 \\
   \text{g} & \quad -2 - 3 \\
   \text{h} & \quad -4 - 5
   \end{align*} \]

5. 14 is obtained from -2 by:
   \[ \begin{align*}
   \text{A} & \quad \text{subtracting} \ 12 \\
   \text{B} & \quad \text{subtracting} \ 16 \\
   \text{C} & \quad \text{multiplying} \ 7 \\
   \text{D} & \quad \text{adding} \ (-15)
   \end{align*} \]

6. Evaluate:
   \[ \begin{align*}
   \text{a} & \quad +2 \times 3 \\
   \text{b} & \quad +2 \times 4 \\
   \text{c} & \quad -3 \times 4 \\
   \text{d} & \quad -5 \times 2 \\
   \text{e} & \quad -10 + 2 \\
   \text{f} & \quad +12 + 6 \\
   \text{g} & \quad +20 + 10 \\
   \text{h} & \quad -18 + 6
   \end{align*} \]
7 Which one of the following is less than $-12$?
   A $5 \times 2$
   B $6 \times 3$
   C $-24 \div 3$
   D $-24 \times 0.5$

8 Use BEDMAS to evaluate:
   \( a \ (1 - 3) \times 2 \)
   \( b \ (-4 - 2) \div 2 \)
   \( c \ (8 - 1) \times (20 + 19) \)
   \( d \ 4 + (-2) \times 3 + 1 \)
   \( e \ -1 - 4 \div (-2) - 1 \)
   \( f \ 12 \times (-1 + 2) + 7 \times 2 \)

9 Insert brackets to make the following true.
   a $1 + 2 \times 3 = -3$
   b $4 \div 2 \times 2 + 1 = -2$
   c $-1 \times 2 - 4 \div (-2) = -1$
   d $6 - 2 \times 3 + 1 = 10$

10 If $a = 2$, $b = -3$ and $c = -4$ evaluate each of the following.
   a $a + b$
   b $a + c$
   c $a + b + c$
   d $a - b$
   e $b - c$
   f $c - a + b$
   g $a - b + c$
   h $c - b - a$
   i $ab$
   j $abc$
   k $3a + bc$
   l $2(ab) - b$

11 In a scientific experiment the temperature inside a chamber increased and decreased over a period of time. It started at $10^\circ \text{C}$ and decreased by $18^\circ \text{C}$ in the first 5 minutes. Over the next 20 minutes the temperature then:
   - increased by $10^\circ \text{C}$
   - decreased by $7^\circ \text{C}$
   - increased by $4^\circ \text{C}$
   - decreased by $8^\circ \text{C}$
   What was the final temperature?

12 In a multiple-choice test with 30 questions, the following rules apply.
   - 5 marks for a correct answer
   - $-1$ marks for an unanswered question
   - $-3$ marks for an incorrect answer
   a Find the total score for the following test results.
     i 20 correct answers, 4 unanswered questions and 6 incorrect answers
     ii 18 correct answers, 3 unanswered questions and 9 incorrect answers
   b Is it possible to obtain the following scores? If so give an example.
     i 46
     ii 58
     iii $-6$

13 ENRICHMENT: Exponents

13 When working with exponents, take care with negative signs. For example $(-4)^2 = -4 \times -4 = 16$ but $-4^2 = -(4 \times 4) = -16$. Find:
   a $(-2)^2$
   b $-(-2)^2$
   c $-2 \times 2^3$
   d $4 \times (-3)^2$
   e $(-2)^3 - (-2)^2$
   f $-4 \times (-5)^2$
   g $(-1)^4$
   h $(-1)^5$
   i $1 - (1)^4$
   j $-(-1)^3$
   k $1 \times (-1)^6$
   l $-1 \times (-1)^5$
   m $(-1)^m$ where $m$ is an even number
   n $(-1)^m$ where $m$ is an odd number
1 Catering at Eden Park

A catering company needs to decide how many pies, pasties, sausage rolls etc. will be required for the cricket final at Eden Park.

Estimation

From this photo of a portion of a crowd at the cricket, determine the approximate fractions of men, women and children.

Calculating crowd statistics

Using the fractions you calculated, estimate the approximate number of men, women and children at a game at Eden Park attended by:

a 20 000 people  b 35 000 people  c 60 000 people

Applying assumptions

On average:
- each man consumes 2 pies, 2 litres of cola and 1.5 ice-creams
- each woman consumes 1 pie, 0.7 litres of cola and 0.9 ice-creams
- each child consumes 1.3 pies, $\frac{11}{3}$ litres of cola and $2\frac{1}{4}$ ice-creams.

Determine the number of pies, the amount of cola and the number of ice-creams that must be ordered.

Using the data

a Assuming a profit of 5 cents is made on each pie, 15 cents on each litre of cola, and 12 cents on each ice-cream, what is the overall expected profit?
b In reality, the types of food and the quantities of each that are consumed vary for different sports. Even between test matches, the food and beverage choices vary. Choose your three favourite foods and determine the profit margins for each. Then calculate the profit again.

Write a report

List your chosen foods, and beverages, the number of sales and the profit margins. Give reasons for your choices. How could your profit be increased?

2 Using technology to investigate recurring decimals

Decimals can be classified as terminating decimals or recurring decimals.

For example, \( \frac{1}{4} = 0.25 \), which is an example of a terminating decimal as it ‘finishes’ after the 5. In contrast, \( \frac{1}{3} = 0.3333\ldots \) is an example of a simple recurring decimal, as the 3 keeps repeating. Some recurring decimals involve more than just one digit repeating.

Calculating the decimal equivalents of fractions

A spreadsheet like the one shown here can be used to calculate the decimal equivalents of fractions from \( \frac{1}{1} \) to \( \frac{1}{11} \).

a Open your spreadsheet.

i Enter 1 in A2 and in B1.

ii Enter the formula shown in A3 and Fill Down to enter the formula in cells A4 to A12.

iii Enter the formula shown in B2 and Fill Down to cells B3 to B12.

You have now created the decimal equivalents of the fractions \( \frac{1}{1}, \frac{1}{2}, \ldots, \frac{1}{10}, \frac{1}{11} \).

Refine the investigation

Now highlight cells B2 to B12 and increase the number of decimal places to 13. (Go to Format, Cells, Number, type in 13, then press OK.)

Reporting your findings

Describe the decimal patterns for \( \frac{1}{1}, \frac{1}{2}, \ldots, \frac{1}{10}, \frac{1}{11} \), noting which are terminating decimals and which are recurring decimals.
Chapter summary

Fractions

▶ Fraction parts
\( \frac{3}{4} \) numerator
\( \frac{5}{4} \) denominator

▶ Whole numbers and mixed numbers can be written as improper fractions:
\[ 3 \frac{1}{4} = \frac{3 \times 4}{4} + \frac{1}{4} = \frac{12 + 1}{4} = \frac{13}{4} \]

▶ Equivalent fractions: Multiply or divide the numerator and denominator by the same number.
\( \frac{2}{3} = \frac{4}{6} = \frac{8}{12} \)

▶ Simplifying a fraction: Cancel by the highest common factor.

▶ To add or subtract fractions:
  - Change any mixed numbers to improper fractions.
  - Convert each fraction into its equivalent fraction with the lowest common denominator (LCD).
  - Add or subtract the numerators and simplify the answer by cancelling.
  - Write the answer as a mixed number if larger than 1.

▶ To multiply fractions:
  - Change mixed numbers to improper fractions and cancel where possible.
  - Multiply the numerators and multiply the denominators.

▶ To divide fractions:
  - Change mixed numbers to improper fractions.
  - Write as equivalent fractions with a common denominator. Divide the numerators.
  - OR To divide by a fraction, multiply by the reciprocal.

Changing fractions and decimals

▶ Changing decimals to fractions: Write with digits of the decimal as the numerator and powers of 10 as the denominator; e.g. \( 1.023 = 1 \frac{23}{1000} \)

▶ Changing fractions to decimals: Write as an equivalent fraction with the power of 10 as the denominator. The numerator becomes the decimal.
  OR Divide the numerator by the denominator.
Decimals

▶ Adding and subtracting decimals

Strategies: Rounding and compensating
- Round up to create one whole number, subtracting the same amount to compensate:
  e.g. $3.8 + 5.7 = (3.8 + 0.2) + (5.7 - 0.2) = 4 + 5.5$
- When subtracting, add the same amount to both decimals, creating a number that is easy to subtract:
  e.g. $6.2 - 3.9 = (6.2 + 0.1) - (3.9 + 0.1) = 6.3 - 4$
- Use a number line.

▶ Multiplying decimals
- Split decimal into components and use an array.
- To multiply by 10, move the decimal point one place to the right. (This is because each digit moves left to a higher place value.)

▶ Dividing decimals
- Division and multiplication are inverse operations: e.g. $0.8 \div 0.2 = 4$ and $4 \times 0.2 = 0.8$
- To divide by 10, move the decimal point one place to the left. (This is because each digit moves right to a lower place value.)

▶ Rounding decimals

Decimal numbers are often rounded to the nearest whole number (or tenth, hundredth etc.), so that the number retains meaning but is easier to use.
- ‘Round to 2 d.p.’ means round to the nearest hundredth.
- ‘Round to 2 sig. fig.’ means round to the first 2 non-zero digits:
  e.g. $23.589 = 24000$ (2 sig. fig.)
- Zeros between non-zero digits are significant: e.g. 2.0046 has 5 significant figures.
- In decimal numbers, zeros at the right are significant: e.g. 5.030 and 0.006000 each have 4 significant figures.

Indices

▶ $a^n = a \times a \times a \times \ldots \times a$ (index, exponent, index or power)
- $3^4 = 3 \times 3 \times 3 \times 3 = 81$
- $a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n}$

Standard form, scientific notation

▶ Written as a number between 1 and 10 multiplied by a power of 10: e.g. $6.25 \times 10^3$
▶ For numbers less than 1 the power will be negative: e.g. $0.00285 = 2.85 \times 10^{-3}$
1. Copy and complete the equivalent fractions:
   \[ \frac{1}{2} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} \]

2. Write as a decimal: \( \frac{562}{1000} \)

3. Write as a fraction: 0.3

4. Find: \( \frac{3}{4} + \frac{4}{7} \)

5. Find: \( 1\frac{3}{5} - \frac{4}{5} \)

6. Find: \( \frac{3}{4} \times \frac{4}{9} \)

7. Find: \( \frac{5}{11} - \frac{2}{11} \)

8. Find: \( 0.35 \times 0.2 \)

9. Write as a decimal (without using a calculator): \( \frac{2}{3} \)

10. Round to the nearest thousand: 256 789

11. Estimate: \( 56 \times 32 = 125 \)

12. Copy and complete the equivalent fractions.
   \[ a \quad \frac{3}{4} = \frac{7}{12} \quad b \quad \frac{48}{24} = \frac{24}{7} \]
   \[ c \quad \frac{8}{5} = \frac{16}{10} = \frac{7}{5} \]

13. Evaluate:
   \[ a \quad \frac{3}{5} - \frac{1}{4} \quad b \quad \frac{3}{5} + \frac{2}{3} \]
   \[ c \quad \frac{7}{2} - \frac{3}{4} - \frac{1}{8} \quad d \quad \frac{3}{5} - \frac{1}{3} \]
   \[ e \quad 5 \times 3 \quad f \quad \frac{5}{8} \times \frac{4}{13} \]
   \[ g \quad \frac{4}{7} \times \frac{4}{5} \quad h \quad \frac{4}{9} \times \frac{12}{35} \]
   \[ i \quad 4 \frac{2}{3} + 1 \frac{1}{2} \quad j \quad \frac{11}{5} + \frac{14}{15} \times \frac{35}{33} \]
   \[ k \quad \frac{24}{3} - \left( \frac{1}{2} + \frac{2}{3} \right) \quad l \quad \left( \frac{3}{4} + \frac{2}{3} \right) + 7 \frac{3}{5} \]

14. Calculate:
   \[ a \quad 3.84 + 2.63 \quad b \quad 12.09 + 3.6 \]
   \[ c \quad 10.85 + 32.7 \quad d \quad 6.59 - 3.24 \]
   \[ e \quad 7.4 - 3.75 \quad f \quad 98.37 - 20.4 + 13.2 \]

15. Evaluate:
   \[ a \quad 5.3 \times 100 \quad b \quad 0.29 \times 1000 \]
   \[ c \quad 4.8 \times 9 \quad d \quad 9.7 \times 0.15 \]
   \[ e \quad 156 \div 100 \quad f \quad 52.5 \div 1000 \]
   \[ g \quad 156.84 \div 3 \quad h \quad 152.84 \div 0.004 \]

16. Copy and complete this table.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ( \frac{3}{4} )</td>
<td></td>
</tr>
<tr>
<td>b ( 0.667 )</td>
<td></td>
</tr>
<tr>
<td>c ( 2\frac{3}{5} )</td>
<td></td>
</tr>
</tbody>
</table>

17. Melanie bought six packets of photocopy paper for $41.10. What is the cost of one packet?

18. Alan bought 12 standard rose bushes at $15.65 each. What is the total cost of the roses?

19. For each of the following:
   i. estimate the answer
   ii. explain whether your estimate is reasonable

   \[ a \quad 4\frac{2}{5} + 1\frac{3}{4} \quad b \quad 0.026 \times 35.3 \]
   \[ c \quad 5.6 \times 10^4 + 1.3 \times 10^2 \quad d \quad 4 \times 6 \times \sqrt{12 \times 80} \times 0 \times 1 \]
   \[ e \quad 5.78 \times 10^3 \times 2.37 \times 10^{-6} \]
20 a On 20 December 2007 the population of New Zealand was 4 254 054. The area of New Zealand is 262 992 square kilometres. How many people were there for every square kilometre?

b Use the internet to find the latest census figures for New Zealand. Has the area per person increased or decreased? If it has, find by how much.

21 Tom was late for maths and missed the first $\frac{3}{8}$ of the 80-minute lesson. He then spent another $\frac{3}{10}$ of the lesson at his locker looking for his books.

a How many minutes did Tom:

i miss by being late to class?

ii spend at his locker?

iii actually spend in the maths class?

b For what fraction of the maths lesson was he present?