

## Solutions: A10

The probability is 0.441, so to find  $n$ , solve  $n \times 0.3 \times 0.7^{n-1} = 0.441$ , or equivalently  $n \times 0.7^{n-1} = 1.47$  (dividing through by 0.3).

Method 1: use trial and error. Try  $n$  equal to 1, 2, 3, 4, 5 in turn to see which gives 1.47.

Here is a table showing this approach:

$n$	$n \times 0.7^{n-1}$
1	1
2	1.4
3	1.47 (as required)
4	1.372
5	1.2005

Method 2: Use a CAS calculator or computer software with a 'solve' command. This gives  $n = 2.6161$  or  $n = 3$ , but as  $n$  must be an integer,  $n = 3$  is the answer.

### Question 305

a. There are 20 items on Shelf B. Only one is a CD, so:

$$\Pr(\text{a CD} \mid \text{it comes from shelf B}) = \frac{1}{20}.$$

b.  $\Pr(\text{a CD})$

$$\begin{aligned} &= \frac{1}{3} \times \frac{4}{20} + \frac{1}{3} \times \frac{1}{20} + \frac{1}{3} \times \frac{7}{10} \\ &= \frac{1}{3} \left( \frac{4}{20} + \frac{1}{20} + \frac{7}{10} \right) = \frac{19}{60}. \end{aligned}$$

### Question 306 E

The first two alternatives are not transition matrices. Alternative C is not correct since the column probabilities do not sum to 1. In the remaining alternatives, label the first and second column 'last holiday in Australia' and 'last holiday overseas' respectively and the first and second rows as 'next holiday in Australia' and 'next holiday overseas'

respectively. Then you can check that the entries in the matrix in alternative E work. In alternative D, the entry in the second row and second column reads: 'the probability that their next holiday is overseas given that their last holiday was overseas is 0.80'. This is the opposite of what is given.

### Question 307 306

From the information given, you could set up a transition matrix, or draw a tree diagram, or simply note that there are just two possibilities: TT|C (tea on Tuesday and on Wednesday given coffee on Monday) or CT|C (coffee on Tuesday then tea on Wednesday given coffee on Monday). The probability is  $0.7 \times 0.4 + 0.3 \times 0.7 = 0.49$ .

### Question 308 C

If  $S_2 = TS_1$ , then  $S_2 = T^{-1}S_1$ . You could find the inverse matrix and perform the calculation by hand or use a CAS calculator or computer software. Define  $T$  and  $S_1$  as given and simply enter  $T^{-1}S_1$ .

$$\text{The result is } S_2 = \begin{bmatrix} 1100 \\ 900 \end{bmatrix}.$$

### Question 309 C

An appropriate transition matrix is  $\begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix}$ , where the first column denotes 'win today' etc. Use a CAS to raise this matrix to consecutive values of a power  $n$  until the elements of the matrix are the same, correct to two decimal places. You will find that the seventh and eighth powers each give  $\begin{bmatrix} 0.67 & 0.67 \\ 0.33 & 0.33 \end{bmatrix}$ . So the long term probability of winning is 0.67.

## Solutions: A10

### Question ~~310~~ 307

You could draw a tree diagram or simply note that there are just three possibilities: CCD|C (Cino on next two Fridays, Dendy on the third Friday given Cino this Friday), CDC|C or DCC|C.

The probability is

$$0.4 \times 0.4 \times 0.6 + 0.4 \times 0.6 \times 0.5 + 0.6 \times 0.5 \times 0.4 = 0.336.$$

### Question ~~311~~ 308

a. i.  $\Pr(3 \text{ heads}) = p^3.$

ii. There are 3 possibilities, HHT or HTH or THH, each with probability  $p^2(1-p)$ . So  $\Pr(2 \text{ heads and } 1 \text{ tail}) = 3p^2(1-p).$

b. Solve  $p^3 = 3p^2(1-p).$

$$p^3 - 3p^2(1-p) = 0$$

$$p^2(p - 3(1-p)) = 0$$

$$p^2(p - 3 + 3p) = 0$$

$$p^2(4p - 3) = 0$$

So  $p = 0$  or  $p = \frac{3}{4}.$

(Don't assume  $p$  cannot be zero; this could be a very biased coin!)

### Question ~~312 D~~ 309 D

If there are  $n$  tosses, the probability of obtaining a head on each trial is  $0.5^n$ . This probability is to be less than 0.0005.

Method 1: Use trial and error. Try  $n$  equal to 8, 9, 10, 11, 12 in turn to find when the answer drops below 0.0005. Here is a table showing this approach:

$n$	$0.5^n$
8	0.00391
9	0.00195
10	0.00098
<b>11</b>	<b>0.00049</b>
12	0.00024

So the minimum number of tosses is 11.

Method 2: Use a CAS calculator or computer software with a 'solve' command: solve  $0.5^n = 0.0005$ , giving  $n = 10.9658$ . But as  $n$  must be the smallest integer to give less than 0.0005,  $n = 11$  is the answer.

### Question ~~313 D~~ 310 D

If  $n$  games are played and John loses all of them, then the probability is  $0.7^n$ . But the probability is given as 0.0576. So solve  $0.7^n = 0.0576$ .

Method 1: Check each alternative, rounding to four decimal places where necessary.

A:  $0.7^1 = 0.7$

B:  $0.7^2 = 0.49$

C:  $0.7^5 = 0.1681$

D:  $0.7^8 = 0.0576$  (just what is required)

E:  $0.7^{12} = 0.0138$

Method 2: Use a CAS calculator or computer software with a 'solve' command: solve  $0.7^n = 0.0576$ , giving  $n = 8.0023$ . In this case, choose  $n = 8$  as the probability given is not exact.