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CHAPTER 3

Linear graphs and models

- What is a linear graph?
- How do we determine the slope of a straight-line graph?
- How do find the equation of a straight line from its graph?
- How do we sketch a straight-line graph from its equation?
- How do we use straight-line graphs to model practical situations?

3.1 Drawing straight line graphs

Plotting straight line graphs

Relations defined by equations such as

$$y = 1 + 2x \quad y = 3x - 2 \quad y = 10 - 5x \quad y = 6x$$

are called **linear** relations because they generate **straight line graphs**.

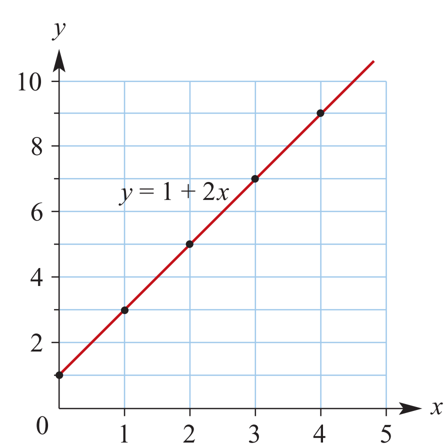
For example, consider the relation $y = 1 + 2x$. To plot a graph, we first need to form a table of values.

x	0	1	2	3	4
y	1	3	5	7	9

We can then plot the values from the table on a set of axes, as shown opposite.

The points appear to lie on a straight line.

A ruler can then be used to draw in this straight line to give the graph of $y = 1 + 2x$.



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Example 1 **Constructing a graph from a table of values**

Plot the graph of $y = 8 - 2x$ by forming a table of values of y using $x = 0, 1, 2, 3, 4$.

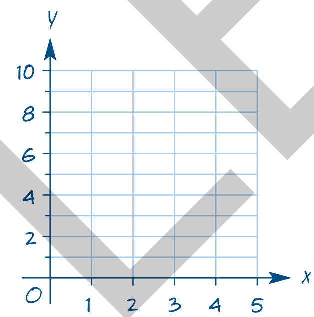
Solution

1 Set up table of values.

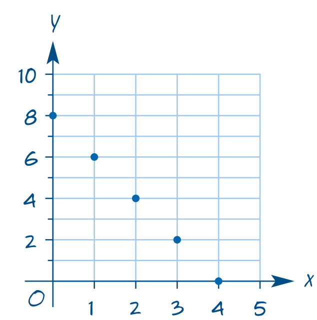
When $x = 0$, $y = 8 - 2 \times 0 = 8$.
When $x = 1$, $y = 8 - 2 \times 1 = 6$, and so on.

x	0	1	2	3	4
y	8	6	4	2	0

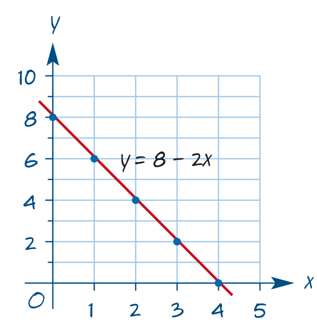
2 Draw, label and scale a set of axes to cover all values.



3 Plot the values in the table on the graph by marking with a dot (•).
The first point is (0, 8).
The second point is (1, 6), and so on.



4 The points appear to lie on a straight line.
Use a ruler to draw in the straight line.
Label the line $y = 8 - 2x$.



A graphics calculator can also be used to draw straight-line graphs, although it can take some fiddling around with scaling to get just the graph you want. One bonus of using a graphics calculator is that, in drawing the graph, it automatically generates a table of values for you.

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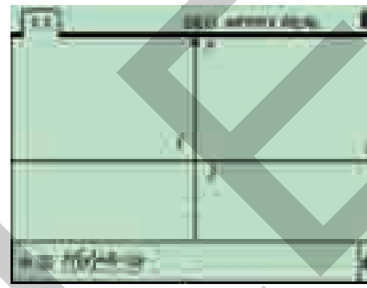
How to draw a straight-line graph and show a table of values using the TI-Nspire CAS

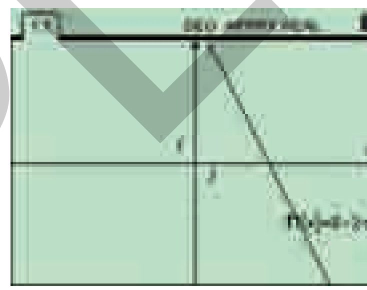
Use a graphics calculator to draw the graph $y = 8 - 2x$ and show a table of values.

Steps

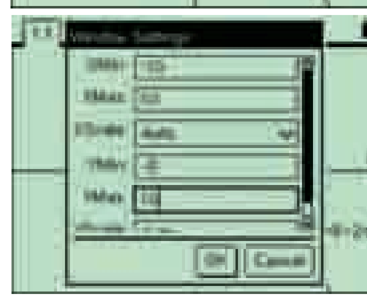
1 Start a new document (by pressing ctrl + N) and select **2: Add Graphs & Geometry**.



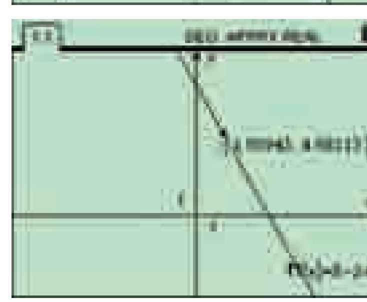
2 Type in the equation as shown. Note that $f1(x)$ represents the y term. Press enter to obtain the graph below.
Hint: ctrl + G will hide the entry line to give more screen view.



3 Change the window setting to see the key features of the graph. Press menu **/4:Window/1:Window Settings** and edit as shown. Use the tab key to move between the entry lines. Press enter when you have finished editing the settings. The re-scaled graph is shown below.



4 To show values on the graph, press: menu **/5:Trace/1:Graph Trace** and then use ◀ ▶ arrows to move along the graph.



5 To show a table of values, press ctrl + T.
Use the ▲ ▼ arrows to scroll through the values in the table.

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How to draw a straight-line graph and show a table of values using the ClassPad

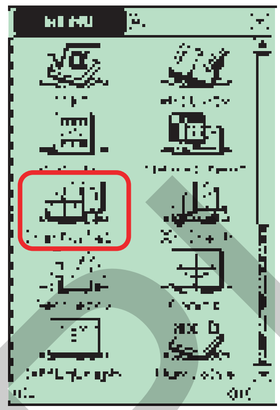
Use a graphics calculator to draw the graph of $y = 8 - 2x$ and show a table of values.

Steps

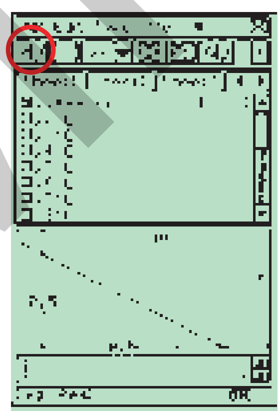
1. Open the built-in **Graphs and Tables** application.
2. Tap the icon and complete the **View Window** as shown to get a graph more like the one plotted by hand.

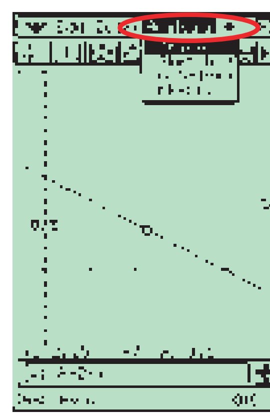
   Notes:

   1. Making ***x* min** and ***y* min** $= -0.5$, rather than zero, enables you to see the axes.
   2. The **dot** value gives the trace increment for the graph and is set automatically.



3. Enter the equation into the graph editor window by typing **8 − 2*x*** and then pressing EXE.
   Tap the icon to plot the graph.
4. Tapping resize (Resize) from the toolbar increases the size of the graph window. Selecting **Analysis** from the menu and then **Trace** will place a cursor on the graph and the equation will be displayed in a window at the bottom of the screen. The coordinates of the point (2.25, 3.5) are also shown.





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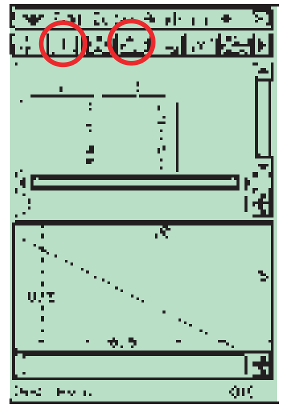
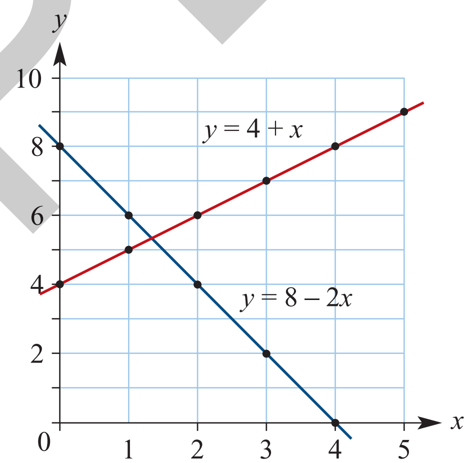
5 Tapping the icon from the toolbar will display a table of values.
Tapping the icon from the toolbar opens the **Table Input** dialog box. The values displayed in the table can be adjusted by changing the values in this window.



Exercise 3A

1 Two straight-line graphs, $y = 4 + x$ and $y = 8 - 2x$, are plotted as shown opposite.



a Reading from the graph of $y = 4 + x$, determine the missing coordinates: (0, ?), (2, ?), (?, 7), (?, 9).

b Reading from the graph of $y = 8 - 2x$, determine the missing coordinates: (0, ?), (1, ?), (?, 4), (?, 2).

2 Plot the graph of the linear equations below by first forming a table of values of y using $x = 0, 1, 2, 3, 4$.

a $y = 1 + 2x$ **b** $y = 2 + x$ **c** $y = 10 - x$ **d** $y = 9 - 2x$

3 For each of these linear equations, use a graphics calculator to do the following:

i Plot a graph for the window given.
ii Generate a table of values.

a $y = 4 + x$
$-10 \le x \le 10$
$-10 \le y \le 10$

b $y = 2 + 3x$
$-0.5 \le x \le 5$
$-0.5 \le y \le 20$

c $y = 10 + 5x$
$-0.5 \le x \le 5$
$-0.5 \le y \le 40$

d $y = 5x$
$-5 \le x \le 5$
$-25 \le y \le 25$

e $y = -5x$
$-5 \le x \le 5$
$-25 \le y \le 25$

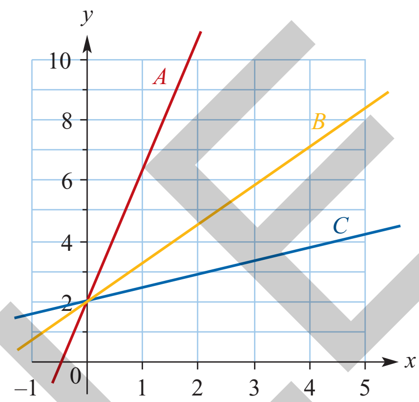
f $y = 100 - 5x$
$-0.5 \le x \le 25$
$-25 \le y \le 125$

3.2 Determining the slope of a straight line

Positive and negative slopes

One of the things that make one straight-line graph look different from another is its steepness or **slope**. Another name for slope is **gradient**.

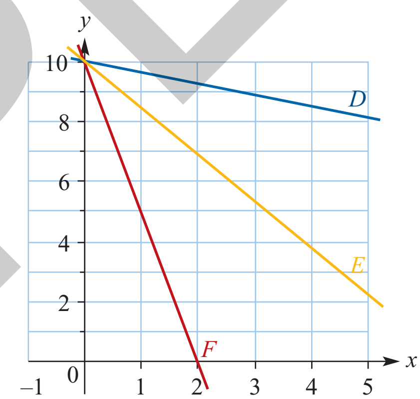
For example, the three straight lines on the graph opposite all cut the y-axis at $y = 2$, but they have quite different slopes.



Line A has the steepest slope while Line C has the gentlest slope. Line B has a slope somewhere in between.

In all cases, the lines have **positive** slopes; that is, they rise from left to right.

Similarly, the three straight lines on the graph opposite all cut the y-axis at $y = 10$, but they have quite different slopes.



In this case, Line D has the gentlest slope while Line F has the steepest slope. Line E has a slope somewhere in between.

In all cases, the lines have **negative** slopes; that is, they fall from left to right.

Giving slope a value: the slope

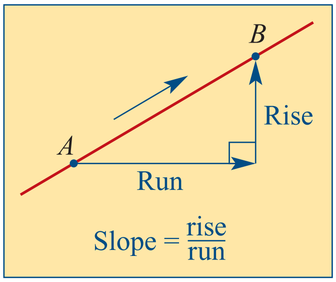
When talking about the slope of a line (Line C, for example), we want to be able to do more than say that it has a gentle positive slope. We would like to be able to give the slope a number that reflects this fact. We do this as follows.

First, two points A and B on the line are chosen.

As we go from A to B (left to right), we move:

- a distance vertically, called the **rise**
- a distance horizontally, called the **run**.

The **slope** of the line is found by dividing the rise by the run.



Example 2 **Finding the slope of a line from a graph: positive slope**

Find the slope of the line through the points (1, 4) and (4, 8).

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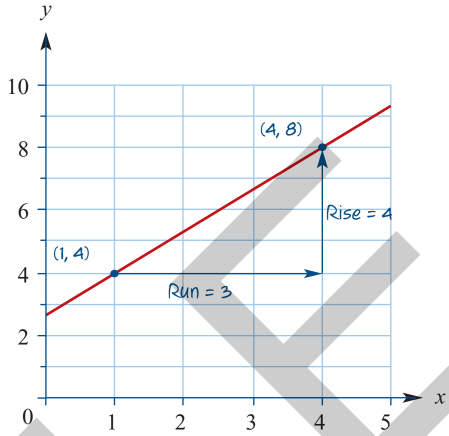
Solution

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Rise} = 8 - 4 = 4$$

$$\text{Run} = 4 - 1 = 3$$

$$\therefore \text{Slope} = \frac{4}{3} = 1.33 \text{ (correct to 2 d.p.)}$$



Example 3 **Finding the slope of a line from a graph: negative slope**

Find the slope of the line through the points (0, 10) and (4, 2).

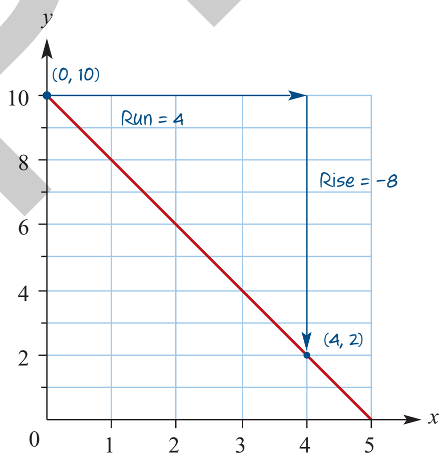
Solution

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Rise} = 2 - 10 = -8$$

$$\text{Run} = 4 - 0 = 4$$

$$\therefore \text{Slope} = \frac{-8}{4} = -2$$



Note: In this example, we have a *negative* 'rise' or a 'fall'.

A formula for finding the slope of a line

While the 'rise/run' method for finding the slope of a line will always work, some people prefer to use a formula for calculating the slope. The formula is derived as follows.

Label the coordinates of point $A: (x_1, y_1)$.

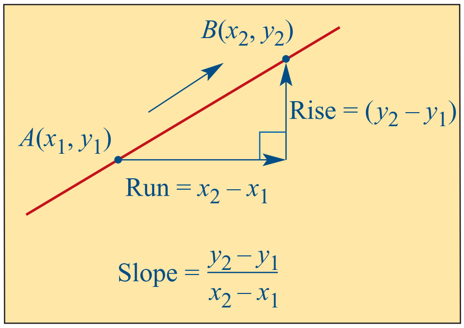
Label the coordinates of point $B: (x_2, y_2)$.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Rise} = y_2 - y_1$$

$$\text{Run} = x_2 - x_1$$

$$\therefore \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



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Example 4 **Finding the slope of a line using the formula**

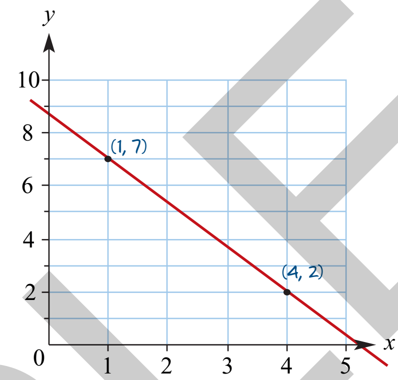
Find the slope of the line through the points (1, 7) and (4, 2) using the formula.

Solution

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

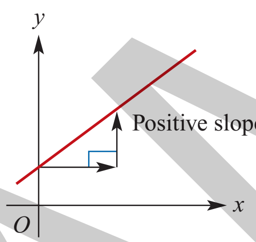
Let $(x_1, y_1) = (1, 7)$ and $(x_2, y_2) = (4, 2)$.

$$\therefore \text{Slope} = \frac{2 - 7}{4 - 1} = -1.67 \text{ (to 2 d.p.)}$$

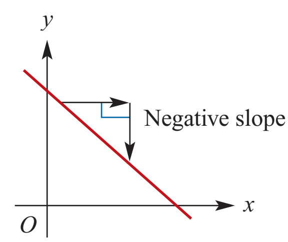


Summary

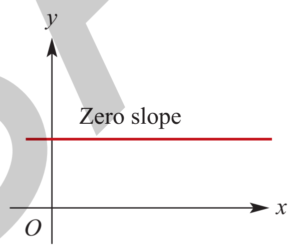
A straight-line graph that **rises** from left to right is said to have a **positive slope** (positive rise).



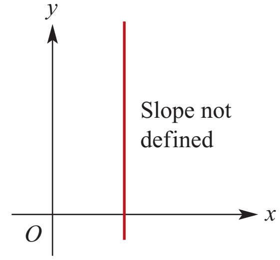
A straight-line graph that **falls** from left to right is said to have a **negative slope** (negative rise).



A straight-line graph that is **horizontal** has **zero slope** ('rise' = 0).

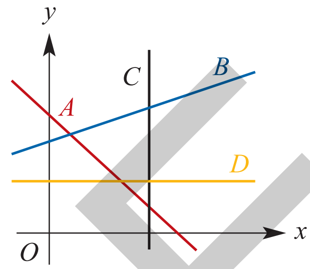


The **slope is not defined** for a straight-line graph that is **vertical**.

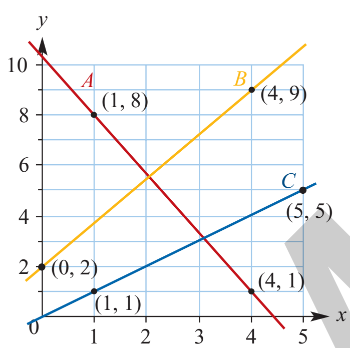


Exercise 3B

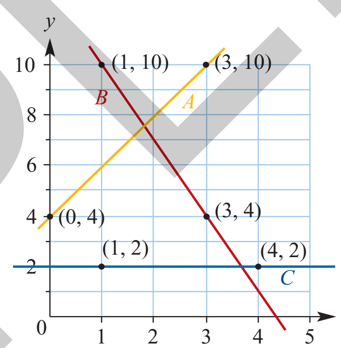
1 For each of the straight-line graphs A, B, C and D, identify the slope as: positive, negative, zero, or not defined.



2 Find the slope of each of the lines (A, B, C) shown on the graph below.

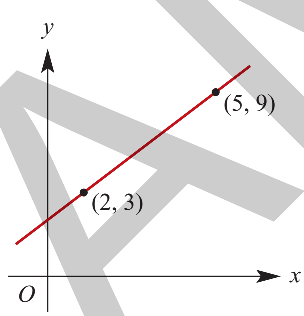


3 Find the slope of each of the lines (A, B, C) shown on the graph below.

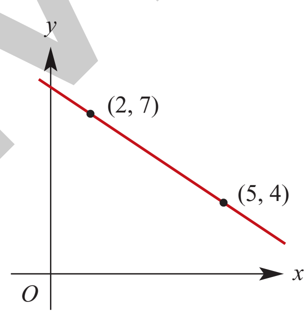


4 Find the slope of each of the lines shown.

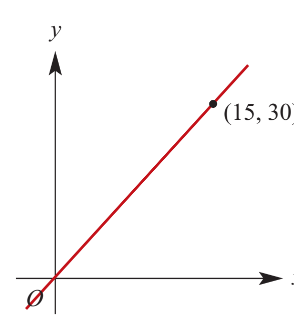
a



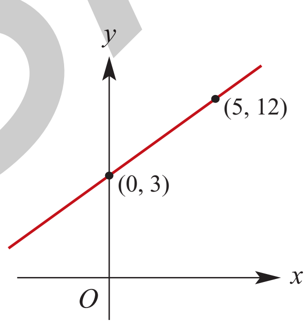
b



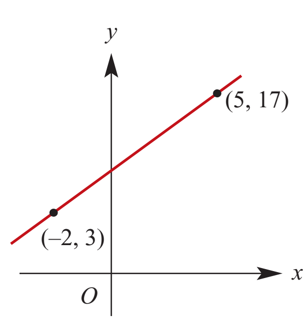
c



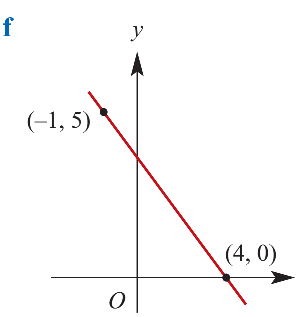
d



e



f



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3.3 The intercept–slope form of the equation of a straight line

Determining the intercept and slope of a straight-line graph from its equation

When we write the equation of a straight line in the form

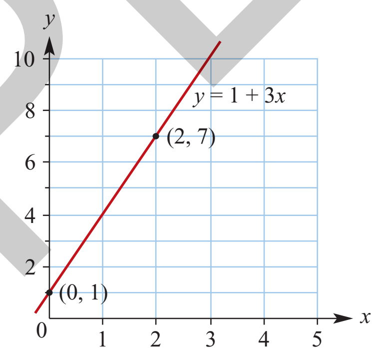
$$y = a + bx$$

we are using what is called the **intercept–slope** form of the equation of a straight line. This is because:

- a = the ***y*-intercept** of the graph
- b = the **slope** of the graph.

The intercept–slope form of the equation of a straight line is useful in modelling relationships in many practical situations. It is also the form used in **bivariate** (two-variable) statistics.

An example of the equation of a straight line written in intercept–slope form is $y = 1 + 3x$. Its graph is shown opposite.



From the graph we see that the

$$y\text{-intercept} = 1$$

$$\text{slope} = \frac{7-1}{2-0} = \frac{6}{2} = 3$$

That is,

- the y-intercept corresponds to the first (**constant**) term in the equation (intercept = 1)
- the slope is given by the **coefficient of *x*** in the equation (slope = 3).

> If the equation of a straight line is in the intercept–slope form
>
> $$y = a + bx$$
>
> then: a = the y-intercept of the graph
> b = the slope of the graph

Example 5 **Finding the intercept and slope of a line from its equation**

Write down the y-intercept and slope of each of the straight line graphs defined by the following equations.

a $y = -6 + 9x$ **b** $y = 10 - 5x$ **c** $y = -2x$ **d** $y - 4x = 5$

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Solution

For each equation:

1. Write the equation.
   If it is not in intercept–slope form, rearrange it.
2. Write down the y-intercept and slope. When the equation is in intercept–slope form:
   $a = y$-intercept (constant term)
   $b =$ slope (coefficient of x).

a $y = -6 + 9x$
$\therefore$ y-intercept $= -6$, slope $= 9$

b $y = 10 - 5x$
$\therefore$ y-intercept $= 10$, slope $= -5$

c $y = -2x$
$\therefore$ y-intercept $= 0$, slope $= -2$

d $y - 4x = 5$
Transpose to intercept–slope form.
$y = 5 + 4x$
$\therefore$ y-intercept $= 5$, slope $= 4$

Sketching straight-line graphs

Because only two points are needed to draw a straight line, all we need to do is find two points on the graph and then draw a line passing through these two points. When the equation of a straight line is written in intercept–slope form, one point on the graph is immediately available: the y-intercept. A second point can then be quickly calculated by substituting a suitable value of x into the equation.

When we draw a graph in this manner, we call it a **sketch graph**.

Example 6 **Sketching a straight-line graph from its equation**

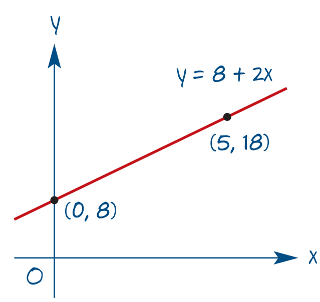
Sketch the graph of $y = 8 + 2x$.

Solution

1. Write the equation of the line.
2. As the equation is in intercept–slope form, the y-intercept is given by the constant term. Write it down.
3. Find a second point on the graph. Choose an x-value (not 0) that makes the calculation easy: $x = 5$ would be suitable.
4. To sketch the graph:
   - Draw a set of labelled axes.
   - Mark in the two points with their coordinates.
   - Draw a straight line through the points.
   - Label the line with its equation.

$y = 8 + 2x$
y-intercept $= 8$

When $x = 5$, $y = 8 + 2(5) = 18$
$\therefore$ (5, 18) is a point on the line.



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Exercise 3C

1 Write down the y-intercepts and slopes of the straight lines with the following equations.

a $y = 5 + 2x$ **b** $y = 6 - 3x$ **c** $y = 15 - 5x$ **d** $y = 5 - 2x$
e $y = 10 - 3x$ **f** $y = -5 - 2x$ **g** $y = 3x$ **h** $y - 3x = 6$
i $2x - y = 5$ **j** $y = 5x - 10$ **k** $x + y = 10$ **l** $y - 2x = 0$

2 Write down the equation of a line that has:

a y-intercept $= 2$, slope $= 5$
b y-intercept $= 5$, slope $= 10$
c y-intercept $= -2$, slope $= 4$
d y-intercept $= 12$, slope $= -3$
e y-intercept $= -2$, slope $= -5$
f y-intercept $= 1.8$, slope $= -0.4$
g y-intercept $= 2.9$, slope $= -2$
h y-intercept $= -1.5$, slope $= -0.5$

3 Sketch the graphs of the straight lines with the following equations, clearly showing the y-intercepts.

a $y = 5 + 2x$ **b** $y = 5 + 5x$ **c** $y = 20 - 2x$
d $y = -10 + 10x$ **e** $y = 4x$ **f** $y = 16 - 2x$

3.4 Determining the equation of a straight line from its graph: the intercept–slope method

We have learnt how to construct a straight-line graph from its equation. How do we determine the equation from a graph? If the graph shows the y-intercept, it is a relatively straightforward procedure.

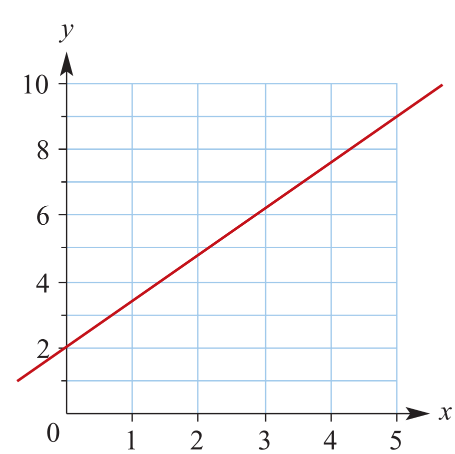
The intercept-slope method for finding the equation of a line

To find the equation of a straight line in intercept–slope form ($y = a + bx$) from its graph:

1. Read off the value of the y-intercept. This gives the value of a.
2. Use two points on the graph to find the slope. This gives the value of b.
3. Substitute these two values into the standard equation $y = a + bx$.

Example 7 **Finding the equation of a line: intercept–slope method**

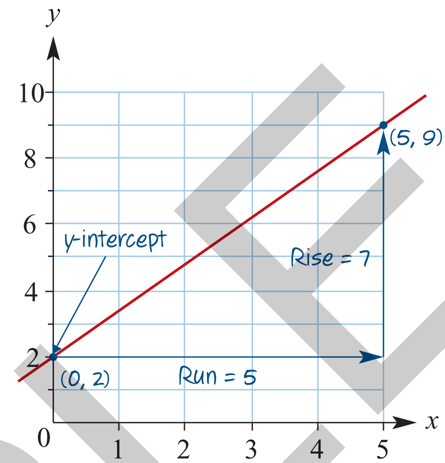
Determine the equation of the straight-line graph shown opposite.



Solution

1 Write the general equation of a line in intercept–slope form.

$y = a + bx$



2 Read off the y-intercept from the graph.

y-intercept $= 2 \quad \therefore a = 2$

3 Find the slope using two well-defined points on the line; for example, (0, 2) and (5, 9).

$\text{Slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{7}{5} = 1.4 \quad \therefore b = 1.4$

4 Substitute the values of a and b into the equation.

$\therefore y = 2 + 1.4x$

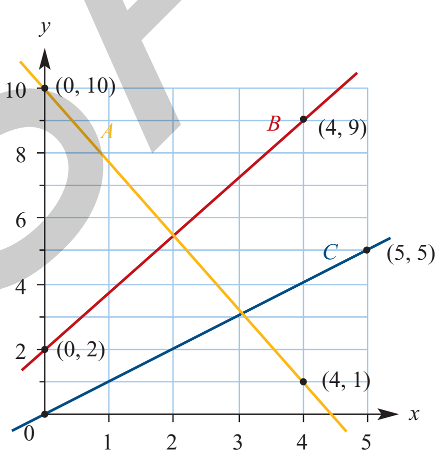
5 Write your answer.

The equation of the line is $y = 2 + 1.4x$.

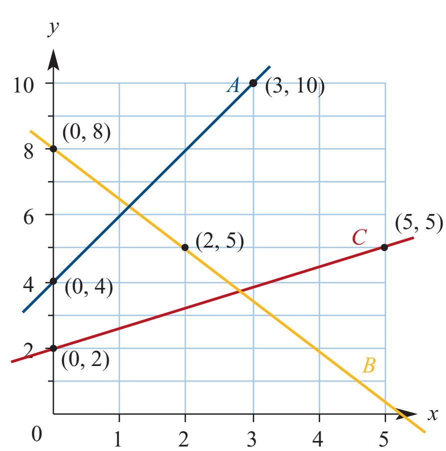
Skillsheet TEACHER

Exercise 3D

1 Find the equation of each of the lines (A, B, C) shown on the graph below.



2 Find the equations of each of the lines (A, B, C) shown on the graph below.



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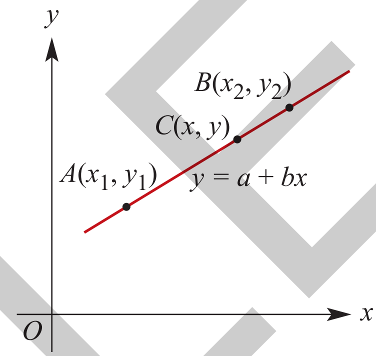
3.5 Determining the equation of a straight line from its graph: the two-point method

Unfortunately, not all straight-line graphs show the y-intercept. When this happens we have to use the two-point method for finding the equation of the line.

Suppose we have three points on the line $y = a + bx$. Two of the points, A and B, are known points. The third point, C, is a general point somewhere on the line.

- Label the coordinates of known point A: (x_1, y_1).
- Label the coordinates of known point B: (x_2, y_2).
- Label the coordinates of general point C: (x, y).



The slope is the same all along the line, so we can write:

$$\text{slope } AC = \text{slope } AB$$

$$\therefore \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$\text{or } y - y_1 = \text{slope} \times (x - x_1) \quad \text{where} \quad \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

This is known as the **two-point form** of the equation of a straight line (you need the two points to find the slope). The advantage of this formula is that it can be used to find the equation of a line given *any* two points on the line. It is not necessary for one of the points to be the y-intercept.

While the two-point form of the equation of a straight line looks complicated, x_1, x_2, y_1 and y_2 are just numbers, so the expression simplifies quickly.

The two-point method for finding the equation of a line

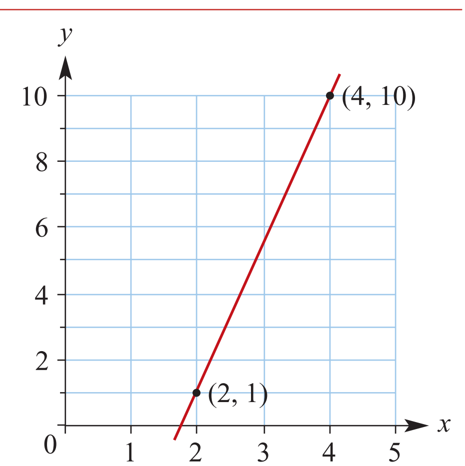
The points $A(x_1, y_1)$ and $B(x_2, y_2)$ lie on a straight line.

The two-point form of the equation of this line is given by

$$y - y_1 = \text{slope} \times (x - x_1) \quad \text{where} \quad \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 8 **Finding the equation of a line: two-point method**

Find the equation of the line that passes through the points (2, 1) and (4, 10).



Solution

1 Write down the two-point formula.

$$y - y_1 = \text{slope} \times (x - x_1) \quad \text{where} \quad \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

2 Write down the values of x_1, x_2, y_1 and y_2.

Note: It does not matter whether the point (2, 1) is called (x_1, y_1) or (x_2, y_2).

$x_1 = 2,\ y_1 = 1;\ x_2 = 4,\ y_2 = 10$

3 Substitute the values of x_1, y_1, x_2 and y_2 into the formula.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 1}{4 - 2} = 4.5$$

$$\therefore y - y_1 = 4.5(x - x_1)$$

4 Simplify to make y the subject.

$$y - 1 = 4.5(x - 2)$$
$$y - 1 = 4.5x - 9$$
$$y = 4.5x - 8$$

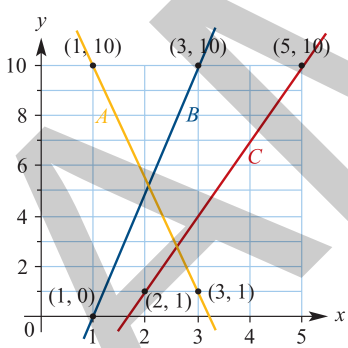
5 Write your answer.

The equation of the line is $y = 4.5x - 8$

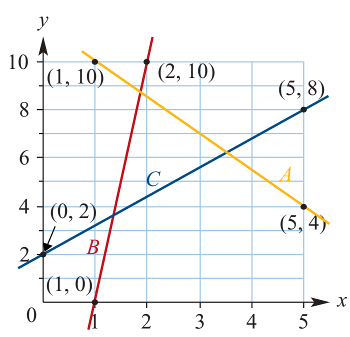
Skillsheet TEACHER

Exercise 3E

1 Find the equation of each of the lines (A, B, C) shown on the graph below.



2 Find the equations of each of the lines (A, B, C) shown on the graph below.



3.6 Finding the equation of a straight line from its graph: the graphics calculator method

While the intercept–slope method of finding the equation of a line from its graph is relatively quick and easy to apply, the two-point method can be tedious to apply. An alternative to using either of these methods is to use the line-fitting facility of your graphics calculator. If you go on to study Further Mathematics, you will use this facility extensively. It is known as **linear regression**.

The advantage of the graphics calculator method is that it works all the time, *provided the coordinates of the points are entered in the correct order*. The disadvantage of using linear regression is that it will give you the wrong results if you do not enter the coordinates of the points in the correct order. So take care.

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How to find the equation of a line from two points using the TI-Nspire CAS

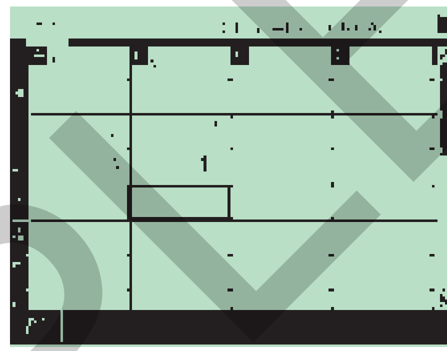
Find the equation of the line that passes through the two points (2, 1) and (4, 10).

Steps

1 Write the coordinates of the two points. Call one point A, the other B.

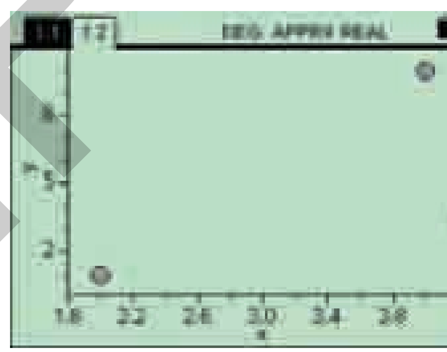
The line passes through the points $A(2, 1)$ and $B(4, 10)$.

2 Start a new document (by pressing ctrl + N) and select **3:Add Lists & Spreadsheet**. Enter the coordinate values into lists named $\boldsymbol{x}$ and $\boldsymbol{y}$.



3 Plot the two points on a scatterplot. Press ⌂ and select **5:Data & Statistics**.

Note: A random display of dots will appear – this is to indicate list data are available for plotting. It is not a statistical plot.



To construct a scatterplot

a Move the cursor to the textbox area below the horizontal (or x-) axis. Press ☝ when prompted and select the variable $\boldsymbol{x}$. Press enter to paste the variable $\boldsymbol{x}$ onto that axis.

b Move the cursor towards the centre of the vertical (or y-) axis until a textbox appears. Press ☝ when prompted and select the variable $\boldsymbol{y}$. Press enter to paste the variable $\boldsymbol{y}$ onto that axis and generate the required scatterplot.

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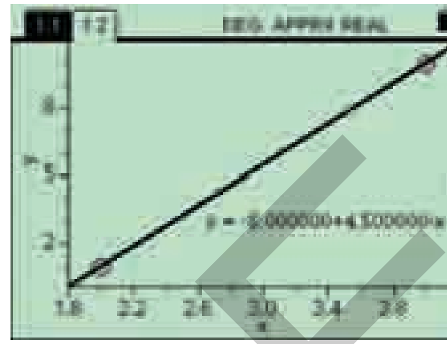
4 Use the **Regression** command to draw a line through the two points and determine its equation.
Press (menu)**/4:Analyze/6:Regression/2: Show Linear (a + bx)** to complete the task.
Correct to 1 decimal place, the equation of the line is $y = -8.0 + 4.5x$.



5 Write your answer.

The equation of the line is $Y = -8 + 4.5x$.

How to find the equation of a line from two points using the ClassPad

Find the equation of the line that passes through the two points (2, 4) and (4, 10).

Steps

1 Open the **Statistics** application and enter the coordinate values into lists named ***x*** and ***y***, as shown.

2 Plot the two points on a scatterplot.

a Tap [icon] from the toolbar to open the **Set StatGraphs** dialog box.

b Complete the dialog box as follows:

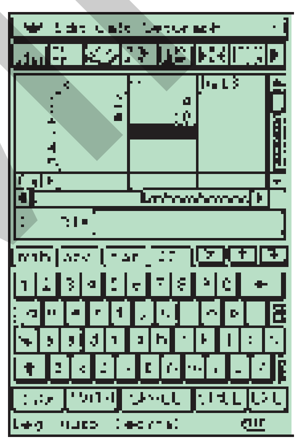
For **Type:** select **Scatter** (▾)

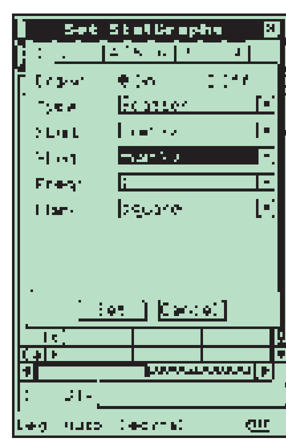
For **XList: select main \ *x*** (▾)

For **YList:** select **main \ *y*** (▾)

Leave **Freq:** as **1**

Tap [SET] to confirm your selections.





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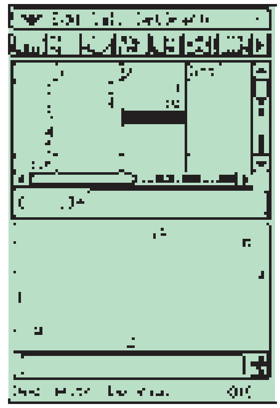
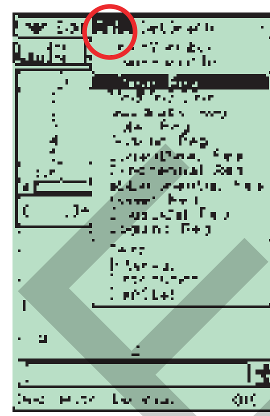
3 Plot the graph

Tapping from the toolbar at the top of the screen automatically plots a scaled graph in the lower-half of the screen.





4 To find the equation of the line $y = ax + b$ that passes through the two points, select **Calc** from the menu bar, and then **Linear Reg**. This opens the **Set Calculation** dialog box below.

5 Complete the **Set Calculation** dialog box as shown.

For XList: select **main** \ x (▾)

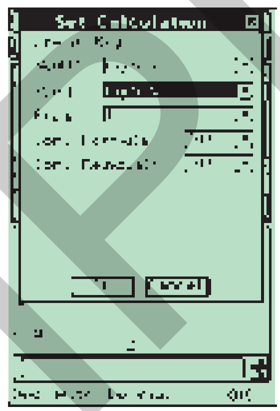
For YList: select **main** \ y (▾)

Leave **Freq**: as **1**

Copy Formula: **Off**

Copy Residual: **Off**

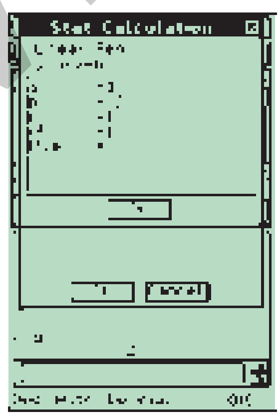
Tap **OK** to confirm your selections.



6 The results are given in a **Stat Calculation** dialog box.

The equation of the line is $y = 3x - 2$.

Note: Tapping **OK** will automatically display the graph window with the line drawn through the two points. This confirms that the line passes through the two points.



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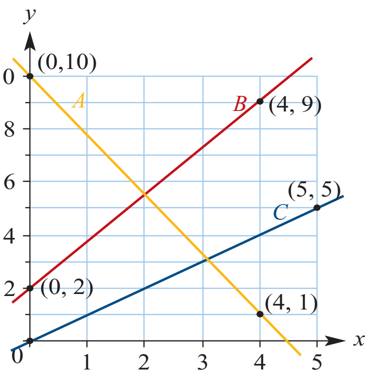
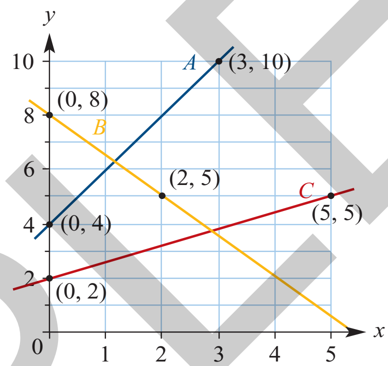
Exercise 3F

Note: This exercise repeats Exercises 3D and 3E, but this time using a graphics calculator.

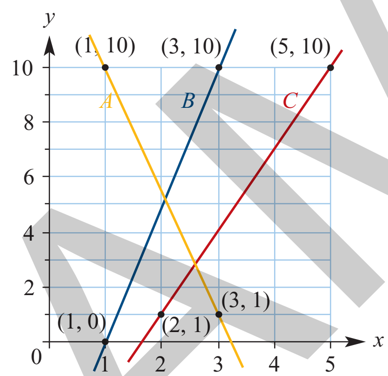
1 Use a graphics calculator to find the equation of each of the lines (A, B, C) shown on the graph below.



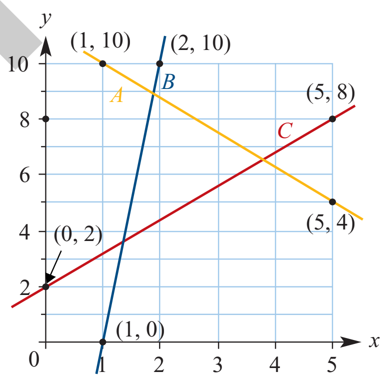
2 Use a graphics calculator to find the equation of each of the lines (A, B, C) shown on the graph below.



3 Use a graphics calculator to find the equation of each of the lines (A, B, C) shown on the graph below.



4 Use a graphics calculator to find the equation of each of the lines (A, B, C) shown on the graph below.



3.7 Linear modelling

Linear modelling refers to the many situations in real life where the relationship of two variables can be modelled (described mathematically) by a linear equation; that is, one whose graph is a straight line. For example, the intercept–slope form of a straight-line graph can sometimes be used to model plant growth.

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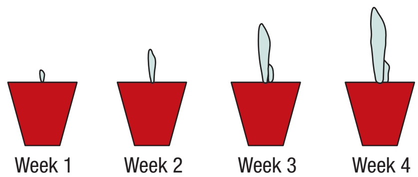
Modelling plant growth with a linear equation

Some pot plants grow remarkably quickly, reaching a height of 40 cm in just a few weeks.



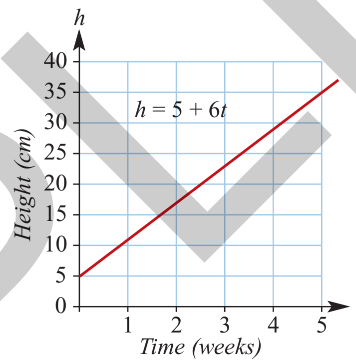
In some instances we can model this growth with a linear equation. Suppose that, when first measured, the height of this pot plant was 5 cm, and that plants of this type grow 6 cm per week. Using this information, we can write down a linear equation to model the plant's growth.

Letting h be the height (in cm) and t be the time (in weeks), we can write

$$h = 5 + 6t \quad \text{for } t \geq 0$$

Note: We have put in the condition $t \geq 0$ because time cannot be negative.

The graph of this linear model has been drawn opposite.



Two important features of this linear model should be noted:

- The ***h*-intercept** gives the initial height of the plant; that is, when $t = 0$.
  The plant was 5 cm tall at the start.
- The **slope** gives the growth rate of the plant. The plant grows at a rate of 6 cm per week; that is, each week the plant increases its height by 6 cm.

Using the key ideas of intercept and slope, we are now in a position to investigate other relationships that can be modelled by straight line graphs.

Interpreting straight line graphical models

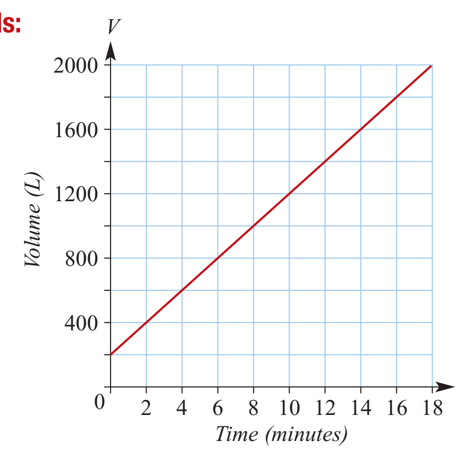
Example 9 **Interpreting graphical models: positive slope**

Water is pumped into a partially full tank. The graph gives the volume of water V (in litres) after t minutes.

a How much water is in the tank at the start ($t = 0$)?

b How much water is in the tank after 10 minutes ($t = 10$)?

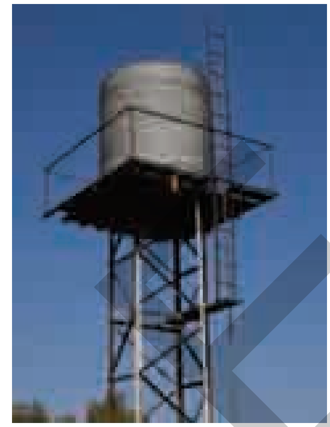
c The tank holds 2000 L. How long does it take to fill?



d Find the equation of the line in terms of V and t.

e Use the equation to calculate the volume of water in the tank after 15 minutes.

f At what rate is the water pumped into the tank; that is, how many litres are pumped into the tank each minute?



Solution

a Read from the graph (when $t = 0$, $V = 200$).

b Read from the graph (when $t = 10$, $V = 1200$).

c Read from the graph (when $V = 2000$, $t = 18$)

d The equation of the line is $V = a + bt$.

- a is the V-intercept. Read from the graph.
- b is the slope. Calculate using two points on the graph, say (0, 200) and (18, 2000).

Note: You can use your calculator to find the equation of the line if you wish.

e Substitute $t = 15$ into the equation. Evaluate.

f The rate at which water is pumped into the tank is given by the slope of the graph, 100.

a 200 L

b 1200 L

c 18 minutes

d $V = a + bt$

$a = 200$

$b = \text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{2000 - 200}{18 - 0}$

$= 100$

$\therefore V = 200 + 100t \quad (t \geq 0)$

e $V = 200 + 100(15) = 1700$ L

f 100 L/min

Example 10 Interpreting graphical models: negative slope

The value of new cars depreciates with time. The graph shows how the value V (in dollars) of a new car depreciates with time t (in years).

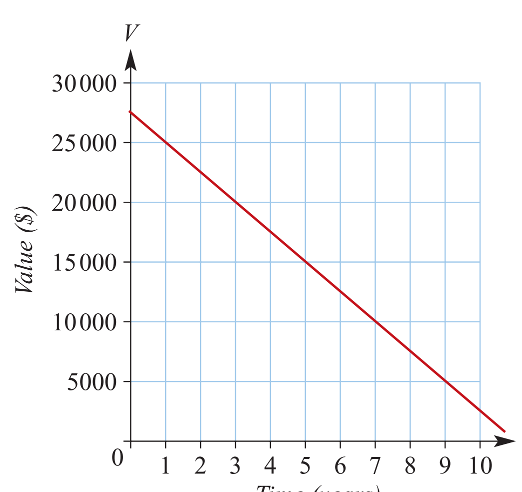
a What was the value of the car when it was new?

b What was the value of the car when it was 5 years old?

c Find the equation of the line in terms of V and t.

d When does the equation predict the car will have no (zero) value?

e At what rate does the value of the car depreciate with time; that is, by how much does its value decrease each year?



Solution

a Read from the graph (when $t = 0$, $V = 27\,500$).

a $27 500

b Read from the graph (when $t = 5$, $V = 15\,000$).

b $15 000

c The equation of the line is $V = a + bt$.

- a is the V-intercept. Read from the graph.
- b is the slope. Calculate using two points on the graph, say (1, 25 000) and (9, 5000).

Note: You can use your calculator to find the equation of the line if you wish.

c $V = a + bt$

$a = 27\,500$

$b = \text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{25\,000 - 5000}{1 - 9} = -2500$

$\therefore V = 27\,500 - 2500t \quad (t \geq 0)$

d The car will have zero value when $V = 0$. Substitute into the equation and solve for t.

d $0 = 27\,500 - 2500t$

$2500t = 27\,500$

$\therefore t = \dfrac{27\,500}{2500} = 11$

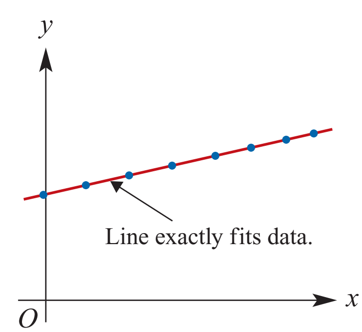
The car will have no value after 11 years.

e The slope of the line is –2500, so the car depreciates in value by $2500 per year.

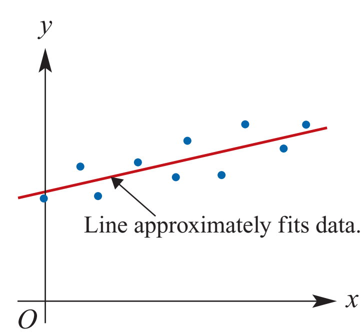
e $2500 per year

Line of best fit

So far all the relationships we have dealt with can be **exactly modelled** by a straight line. This is because all the data points lie exactly on a straight line. However, in practice, this is often not the case. Rather, while the data values tend to follow a linear trend, they do not lie exactly on a straight line. In such cases, the relationship between the variables can be only **approximately modelled** by a straight line. This situation happens most frequently when analysing statistical data. You will learn more about this in Chapter 4.



What we have to do when a straight line does not exactly fit a set of data is to draw in a **line of best fit**. There are many ways of doing this. The simplest is to draw a line that seems to balance out the points around the line. This is called fitting a line 'by eye'.



Finding the line of best fit is not the concern of this chapter. What we are concerned with here is, given the graph of a line of best fit: what is its equation, what do its slope and intercept tell us, and how we can use it to make predictions?

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Example 11 Fitting a straight line to data: line of best fit

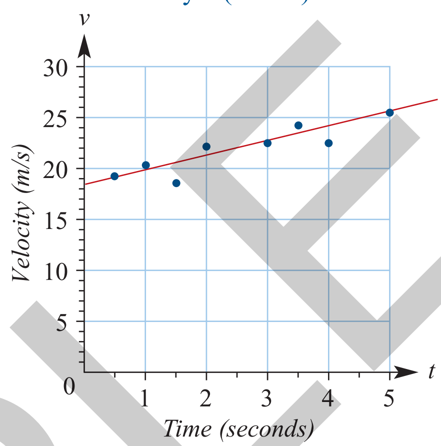
A straight line has been fitted to a set of data that recorded the velocity v (in m/s) of an accelerating car at time t seconds.

a What does the line predict the car's speed to be when $t = 0$?

b The slope of the line can be used to find the car's average acceleration (in m/s^2). What is the value of the slope?

c Write down the equation of the line in terms of v and t.

d If the car keeps accelerating at the same rate, when will its velocity reach 30 m/s?



Solution

a Read from the graph (when $t = 0$, $v = 18.5$).

a $v = 62$ m/s

b Calculate the slope by using two points on the graph, say (0, 18.5) and (5, 25.5).

b Slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{25.5 - 18.5}{5 - 0} = 1.4$

$\therefore$ Average acceleration $= 1.4$ m/s^2

c The equation of the line is $v = a + bt$.
- a is the v-intercept. Read from the graph.
- b is the slope, already calculated.

c
$$v = a + bt$$
$$a = 18.5$$
$$b = 1.4$$
$$\therefore v = 18.5 + 1.4t$$

d Substitute $v = 30$ into the equation and solve for t.

d
$$30 = 18.5 + 1.4t$$
$$30 - 18.5 = 1.4t$$
$$11.5 = 1.4t$$
$$\therefore t = \frac{11.5}{1.4} = 8.2$$

The car will have a velocity of 30 m/s after 8.2 seconds (to 1 decimal place).

Segmented linear graphs

Sometimes we need to break the data into sections and to model each section separately. This gives us what we call a **segmented graph**.

Example 12 Constructing a segment graph model

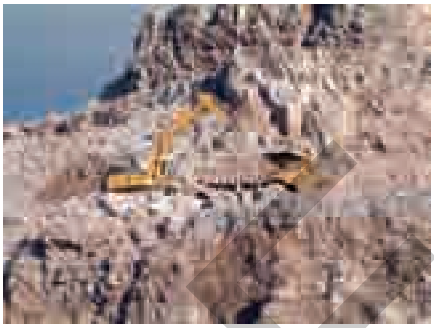
The amount, C dollars, charged to supply and deliver x m^3 of crushed rock is given by the equations

$$C = 50 + 40x \qquad (0 \le x < 3)$$
$$C = 80 + 30x \qquad (3 \le x \le 8)$$

a Use the appropriate equation to determine the cost to supply and deliver the following amounts of crushed rock.

i 2.5 m^3 **ii** 3 m^3 **iii** 6 m^3

b Use the equations to construct a segmented graph for $0 \le x \le 8$.



Solution

a

1 Write the equations.

$C = 50 + 40x \ (0 \le x < 3)$

$C = 80 + 30x \ (0 \le x \le 8)$

2 Then, in each case:
- Choose the appropriate equation.
- Substitute the value of x and evaluate.
- Write your answer.

a i When $x = 2.5$,

$C = 50 + 40(2.5) = 150$

Cost for 2.5 m^3 of crushed rock is \$150.

ii When $x = 3$,

$C = 80 + 30(3) = 170$

Cost for 3 m^3 of crushed rock is \$170.

iii When $x = 6$,

$C = 80 + 30(6) = 260$

Cost for 6 m^3 of crushed rock is \$260.

b

The graph has two line segments

1 Determine the coordinates of the end points of both lines.

b

$x = 0$: $C = 50 + 40(0) = 50$

$x = 3$: $C = 50 + 40(3) = 170$

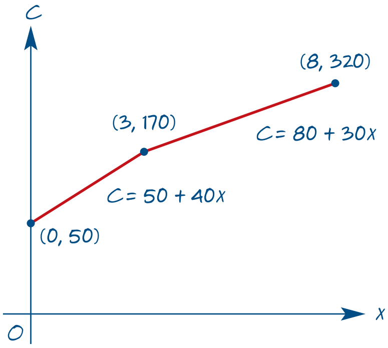
$x = 8$: $C = 80 + 30(8) = 320$

$x = 3$: $C = 80 + 30(3) = 170$

2 Draw a set of labelled axes and mark in the points with their coordinates.

3 Join up the end points of each segment with a straight line.

4 Label each segment with its equation.



Exercise 3G

1 A phone company charges a monthly service fee, plus the cost of calls. The graph opposite gives the total monthly charge, C dollars, for making n calls. This includes the service fee.

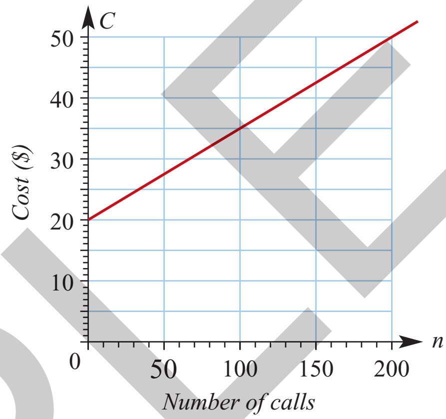
a How much is the monthly service fee ($n = 0$)?

b How much does the company charge if you make 100 calls a month?

c Find the equation of the line in terms of C and n.

d Use the equation to calculate the cost of making 300 calls in a month.

e How much does the company charge per call?



2 The graph opposite shows the volume of saline solution, V mL, remaining in the reservoir of a saline drip after t minutes.

a How much saline solution was in the reservoir at the start?

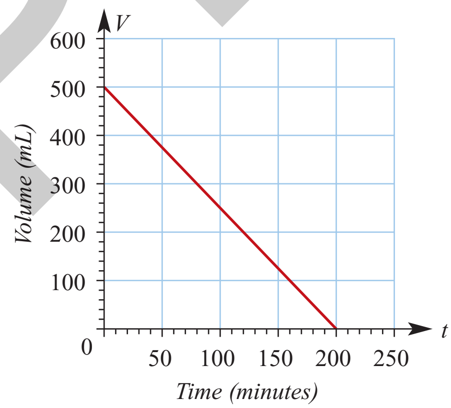
b How much saline solution remains in the reservoir after 40 minutes?

c How long does it take for the reservoir to empty?

d Find the equation of the line in terms of V and t.

e Use the equation to calculate the amount of saline solution in the reservoir after 115 minutes.

f At what rate (in mL/minute) is the saline solution flowing out of the drip?

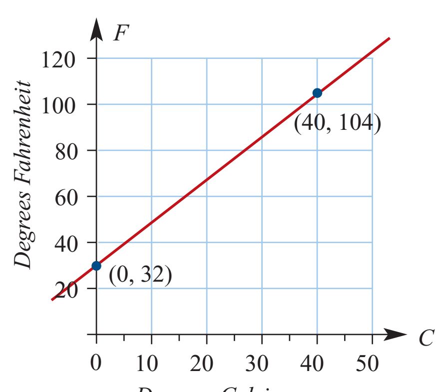


3 The graph opposite can be used to convert temperatures in degrees Celsius (C) to temperatures in degrees Fahrenheit (F).

a Find the equation of the line in terms of F and C.

b Use the equation to predict the temperature in degrees Fahrenheit when the temperature in degrees Celsius is:

i 50°C **ii** 150°C **iii** −40°C.



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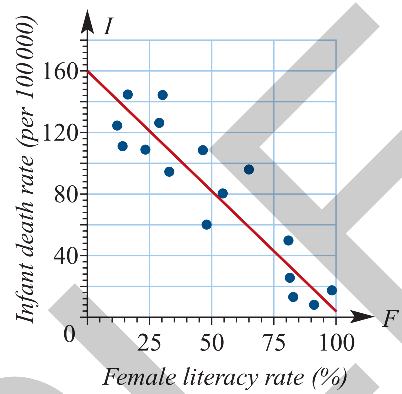
c Complete the following sentence by filling in the box:
When the temperature in Celsius increases by 1 degree, the temperature in Fahrenheit increases by $\boxed{\quad\quad}$ degrees.

4 A straight line has been fitted to a plot of infant death rate I (per 100 000 people) against female literacy rate F (%) for a number of countries.



a Determine the equation of the line in terms of I and F.

b Use the equation to predict the infant death rate in a country with a female literacy level of:

i 40% **ii** 60% **iii** 95%.

c Complete the following sentence by filling in the box:
When the female literacy rate increases by 1%, the infant death rate decreases by $\boxed{\quad\quad}$ per 100 000.

5 An empty tank is being filled from a mountain spring. For the first 30 minutes, the equation giving the volume, V, of water in the tank (in litres) at time t minutes is

$$V = 15t \quad (0 \le t \le 30)$$

After 30 minutes, the flow from the spring slows down. For the next 70 minutes, the equation giving the volume of water in the tank at time t is

$$V = 150 + 10t \quad (30 < t \le 100)$$

a Use the appropriate equation to determine the volume of water in the tank after:

i 20 minutes **ii** 30 minutes **iii** 60 minutes **iv** 100 minutes.

b Use the equations to construct a segmented graph for $0 \le t \le 100$.

MC TEST D&D TEST

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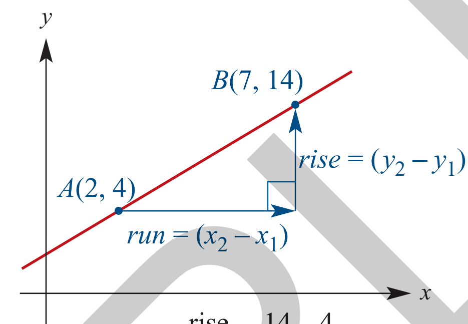
Key ideas and chapter summary

Slope of a straight-line graph

The **slope** of a straight line graph is defined to be

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

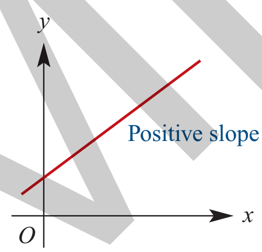
where (x_1, y_1) and (x_2, y_2) are two points on the line.
For example, for the line below:



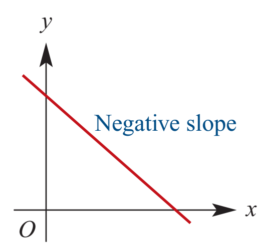
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{14-4}{7-2} = 2$$

Positive and negative slope

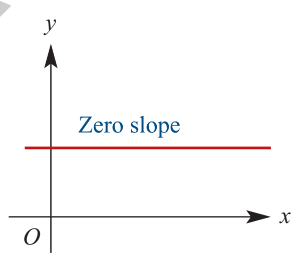
If the line rises to the right, the slope is **positive**.



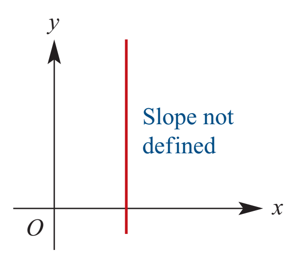
If the line falls to the right, the slope is **negative**.



If the line is horizontal, the slope is **zero**.



If the line is vertical, the slope is **undefined**.



Equation of straight-line graph: the intercept–slope form

The **equation of a straight line** can take several forms. The intercept–slope form is

$$y = a + bx$$

where a is the ***y*-intercept** and b is the **slope** of the line.

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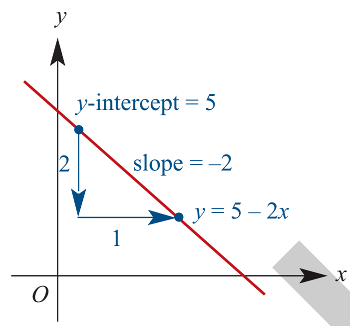
For example, for the graph with the equation

$$y = 5 - 2x$$

The y-intercept $= 5$ and the slope $= -2$.



When the graph of a straight line shows the intercept, the equation can be found by reading the value of the y-intercept from the graph and calculating the slope.

Equation of a straight line: the two-point formula

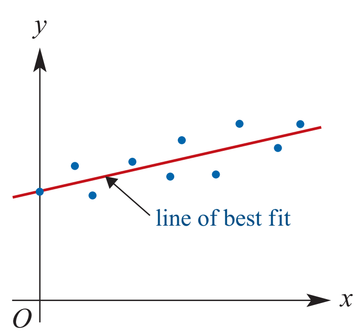
The **two-point formula** is needed when the y-intercept cannot be read off the graph.
If the points $A(x_1, y_1)$ and $B\,(x_2, y_2)$ lie on a straight line, the equation of the line is

$$y - y_1 = \text{slope} \times (x - x_1) \;\text{ where }\; \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Alternatively, a graphics calculator can be used to determine the equation.

Line of best fit

A **line of best fit** is used to approximately model the linear relationship between two variables.
This is needed when the data values do not lie exactly on a straight line.

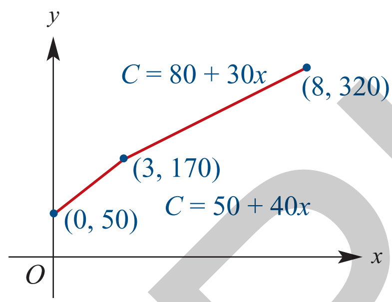


Segmented linear graphs

Segmented linear graphs are used in practical situations where more than one linear equation is needed to model the relationship between two variables.

For example, the segmented linear graph below represents the model defined by the equations

$$C = 50 + 40x \quad (0 \le x < 3)$$
$$C = 80 + 30x \quad (3 \le x \le 8)$$



Skills check

Having completed this chapter you should be able to:

- recognise a linear equation written in intercept–slope form
- determine the intercept and slope of a straight line graph from its equation
- determine the slope of a straight line from its graph
- determine the y-intercept of a straight line from its graph (if shown)
- determine the equation of a straight line, given its graph
- analyse and interpret straight-line graphs used to model practical situations
- analyse and interpret a line of best fit
- construct a segmented linear graph used to model a practical situation.

Multiple-choice questions

1 The equation of a straight line is $y = 4 + 3x$. When $x = 2$, $y =$

A 2 **B** 3 **C** 4 **D** 6 **E** 10

2 The equation of a straight line is $y = 5 + 4x$. The y-intercept is:

A 2 **B** 3 **C** 4 **D** 5 **E** 20

3 The equation of a straight line is $y = 10 - 3x$. The slope is:

A -3 **B** 0 **C** 3 **D** 7 **E** 10

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4 The equation of a straight line is $y - 2x = 3$. The slope is:

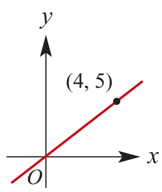
A -3 **B** -2 **C** 0 **D** 2 **E** 3

5 The slope of the line passing through the points (5, 8) and (9, 5) is:

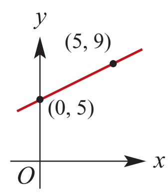
A -1.3 **B** -1 **C** -0.75 **D** 0.75 **E** 1.3

6 The graph of $y = 4 + 5x$ is:

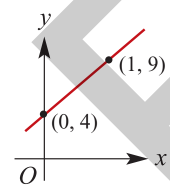
A



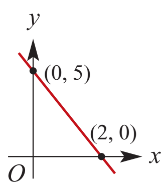
B



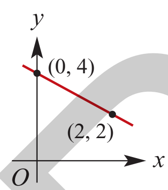
C



D

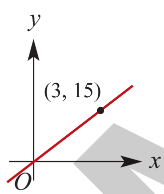


E

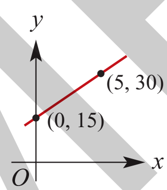


7 The graph of $y = 15 - 3x$ is:

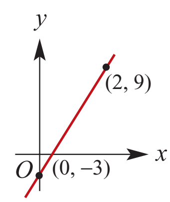
A



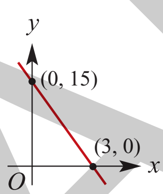
B



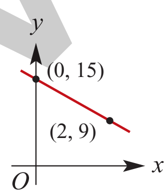
C



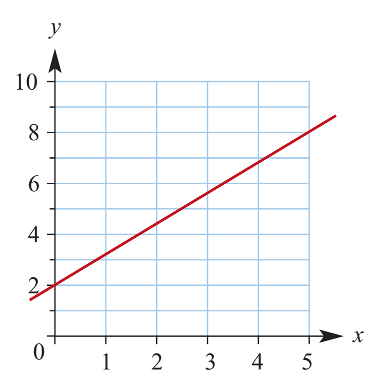
D



E



Questions 8 and 9 relate to the following graph



8 The y-intercept is:

A -2 **B** 0 **C** 2

D 5 **E** 8

9 The slope is:

A 1.6 **B** 1.2 **C** 2

D 5 **E** 8

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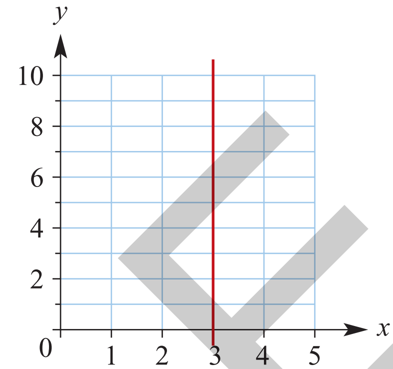
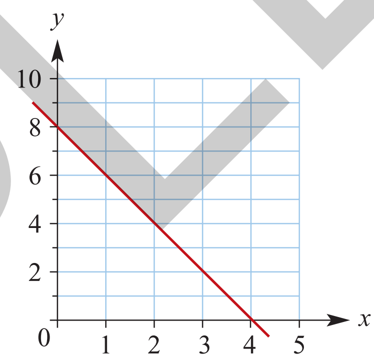
10 The slope of the line in the graph shown opposite is:

A negative **B** zero **C** positive
D three **E** not defined



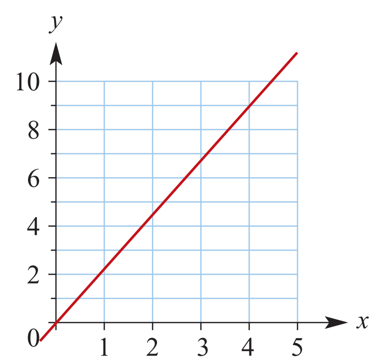
11 The equation of the graph shown opposite is:

A $y = -2 + 8x$ **B** $y = 4 - 2x$
C $y = 8 - 2x$ **D** $y = 4 + 2x$
E $y = 8 + 2x$



12 The equation of the graph shown opposite is:

A $y = -2.25x$ **B** $y = 2.25x$
C $y = -9x$ **D** $y = 9x$
E $y = 1 + 2x$

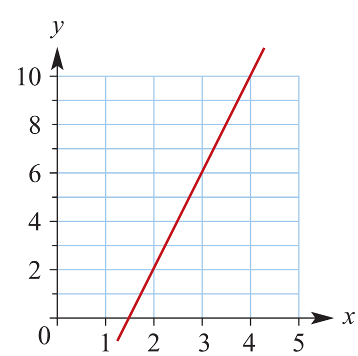


13 Which of the following points lies on the line $y = -5 + 10x$?

A (1, −5) **B** (1, 5) **C** (1, 15) **D** (2, 20) **E** (2, 25)

14 The equation of the graph shown opposite is:

A $y = -6 + 4x$ **B** $y = 6 - 4x$
C $y = -4 - 4x$ **D** $y = 2.5 - 4x$
E $y = -4 + 2.5x$



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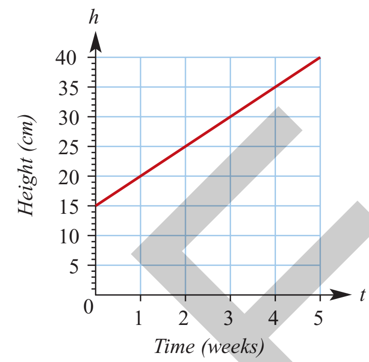
15 The graph opposite shows the height of a small sapling, h, as it increases with time, t.
Its growth rate is closest to:

A 1 cm/week **B** 3 cm/week
C 5 cm/week **D** 8 cm/week
E 15 cm/week



Short-answer questions

1 Plot the graphs of these linear relations by first forming a table of values of y using $x = 0, 1, 2, 3, 4$.

a $y = 2 + 5x$ **b** $y = 12 - x$ **c** $y = -2 + 4x$

2 For each of the linear relations given, use a graphics calculator to do the following:

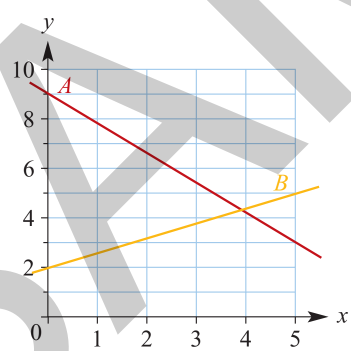
i Plot a graph in the window given.
ii Generate a table of values.

a $y = 5 - x$
$-10 \leq x \leq 10$
$-10 \leq y \leq 10$

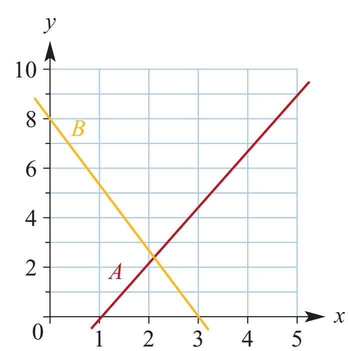
b $y = 5x$
$-0.5 \leq x \leq 5$
$-0.5 \leq y \leq 20$

c $y = 2 + 7x$
$-0.5 \leq x \leq 5$
$-0.5 \leq y \leq 40$

3 Find the slope of each of the lines A and B shown on the graph below.



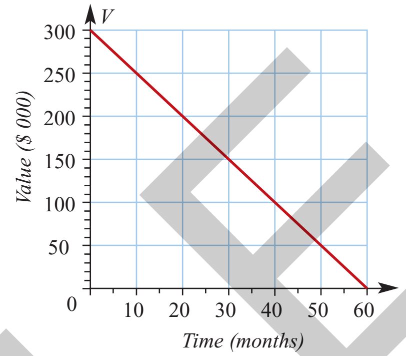
4 Find the slope of each of the lines A and B shown on the graph below.



Review

Extended-response questions

1 A new piece of machinery is purchased by a business for \$300 000. Its value is then depreciated each month using the graph opposite.



a What is the value of the machine after 20 months?

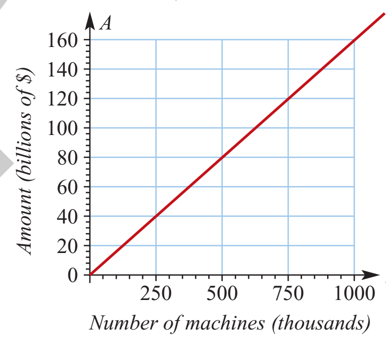
b When does the line predict that the machine will have no (zero) value?

c Find the equation of the line in terms of value V and time t.

d Use the equation to predict the value of the machine after 3 years.

e By how much does the machine depreciate in value each month?

2 The amount of money transacted through ATMs has increased with the number of ATMs available. The graph charts this increase.



a What was the amount of money transacted through ATMs when there were 500 000 machines?

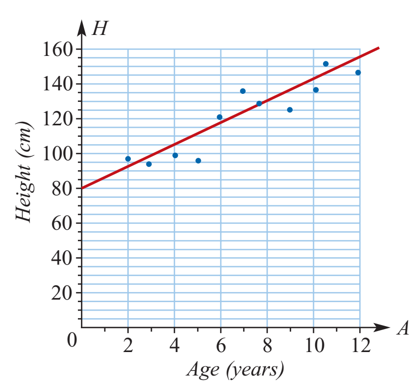
b Find the equation of the line in terms of amount of money transacted, A, and number of ATMs, N. (Leave A in billions and N in thousands).

c Use the equation to predict the amount of money transacted when there were 600 000 machines.

d If the same rule applies, how much money is predicted to be transacted through ATM machines when there are 1500 000 machines?

e By how much does the amount of money transacted through ATMs increase with each 1000 extra ATMs?

3 The heights, H, of a number of children are shown plotted against age, A. Also shown is a line of best fit.



a Find the equation of the line of best fit in terms of H and A.

b Use the equation to predict the height of a child aged 3.

c Complete the following sentence by filling in the box:
The equation of the line of best fit tells us that, each year, children's heights increase by $\boxed{}$ cm.

Review

4 To conserve water, one charging system increases the amount people pay as the amount of water used increases. The charging system is modelled by the following linear equations, where C is the charge in dollars, and x is the amount of water used in kilolitres (kL).

$$C = 5 + 0.40x \qquad (0 \le x < 30)$$
$$C = -31 + 1.6x \qquad (x \ge 30)$$

a Use the appropriate equation to determine the charge for using:
i 20 kL **ii** 30 kL **iii** 50 kL.

b How much does a kilolitre of water cost when you use:
i less than 30 kL? **ii** more than 30 kL?

c Use the equations to construct a segmented graph for $0 \le x \le 50$.