

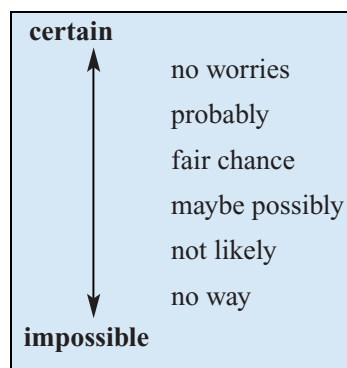
Probability

Objectives

- To understand the basic rules and notation of **set theory**.
- To understand the basic concepts and rules of **probability**.
- To use **Venn diagrams**, **tree diagrams**, and **Karnaugh maps** to determine probabilities for compound events.
- To understand and be able to apply the **addition rule**.
- To introduce the idea of mutually exclusive events.
- To review set notation and apply set notation to probability.

Uncertainty is involved in much of the reasoning we undertake every day of our lives. We make decisions based on the chances of this or that happening. More formally, chances are called **probability**. Some things which occur in the world can be predicted from our present store of knowledge, such as the time of the next high tide. Other things such as whether a head or tail will show when a coin is tossed, or the sex of a new baby are unpredictable. But even though the particular outcome is uncertain, the results are not entirely haphazard. There is a pattern that emerges in the long run which enables us to assign a numerical probability to each outcome, even though the individual outcome is unknown.

Probability has an everyday usage, giving some sort of rough gradation between the two extremes of *impossible* and *certain*, as indicated in the diagram.



10.1 Random experiments and events

In general, a **random experiment** is one in which the outcome of a single trial is uncertain, but is nonetheless an observable outcome. For example, consider the random experiment of tossing a coin and noting whether a head (H) or a tail (T) appears uppermost. The outcome observed must be either H or T , but on a particular toss it is not known which of H or T will be observed. The outcome observed must be one of a known set of possible outcomes, and the set of possible outcomes is called the **sample space** for the experiment. Set notation will be used in listing all the elements in the sample space in set brackets with each element separated by a comma. A single outcome of an experiment is often referred to as a sample point. For example, the sample space for the tossing of a coin would be written as

$$\{H, T\}$$

where H indicates head and T indicates tail. Throughout this chapter the letter ε will be used to denote the sample space.

For example, the following table lists the sample spaces for each of the random experiments described.

<i>Random experiment</i>	<i>Sample space</i>
The number of brown eggs in a carton of 12 eggs	$\varepsilon = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
The result when two coins are tossed	$\varepsilon = \{HH, HT, TH, TT\}$
The number of calls to your phone in the next two hours	$\varepsilon = \{0, 1, 2, 3, 4, \dots\}$
The time, in hours, it takes to complete your homework	$\varepsilon = \{t : t \geq 0\}$
The actual quantity of lemonade in a 1 litre bottle	$\varepsilon = \{q : 0 \leq q \leq 1\}$

An **event** may consist of a single outcome, or it may be several outcomes. For example, when rolling a die, the event of interest may be ‘getting a six’. In this case, since the event consists of just one outcome, it is called a **simple event**. However, the event ‘getting an odd number’ can be achieved by rolling a 1, a 3 or a 5. As there is more than one outcome which defines this event, it is called a **compound event**.

It is sometimes convenient to use set notation to list the elements of the event. In general we use capital letters, A, B, C, \dots to denote sets. Naturally, the set which is associated with an event will be a subset of the sample space. That is, every element of the set is also contained in the sample space.

For example, the following table lists the experiments shown earlier and describes the sample space and an event for each one.

Sample space	An event
<ul style="list-style-type: none"> ■ The number of brown eggs in a carton of 12 eggs. $\varepsilon = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ 	‘More than half brown’ = $\{7, 8, 9, 10, 11, 12\}$
<ul style="list-style-type: none"> ■ The result when two coins are tossed $\varepsilon = \{HH, HT, TH, TT\}$ 	‘Two heads’ = $\{HH\}$
<ul style="list-style-type: none"> ■ The number of calls to your phone in the next two hours $\varepsilon = \{0, 1, 2, 3, 4, \dots\}$ 	‘Fewer than two phone calls’ = $\{0, 1\}$
<ul style="list-style-type: none"> ■ The time in hours it takes to complete your homework $\varepsilon = \{t: t \geq 0\}$ 	‘More than two hours’ = $\{t: t > 2\}$
<ul style="list-style-type: none"> ■ The actual quantity of lemonade in a 1 litre bottle $\varepsilon = \{q: 0 \leq q \leq 1\}$ 	‘Less than half full’ = $\{q: 0 \leq q < 0.5\}$

Example 1

A bag contains 7 marbles numbered from 1 to 7 and a marble is withdrawn.

- a Give the sample space for this experiment.
- b List the sample points (elements) of the event ‘a marble with an odd number is withdrawn’.

Solution

- a $\{1, 2, 3, 4, 5, 6, 7\}$ b $\{1, 3, 5, 7\}$

Some of the experiments given in this example are **multi-stage experiments**. That is, they are concerned with the experiments which could be considered to take place in more than one stage. For example, when considering the outcomes from tossing two coins we should consider the possible outcomes in two stages: the outcome from coin 1, followed by the outcome from coin 2. In such cases, it is helpful to list the elements of the sample space systematically by means of a tree diagram.

Example 2

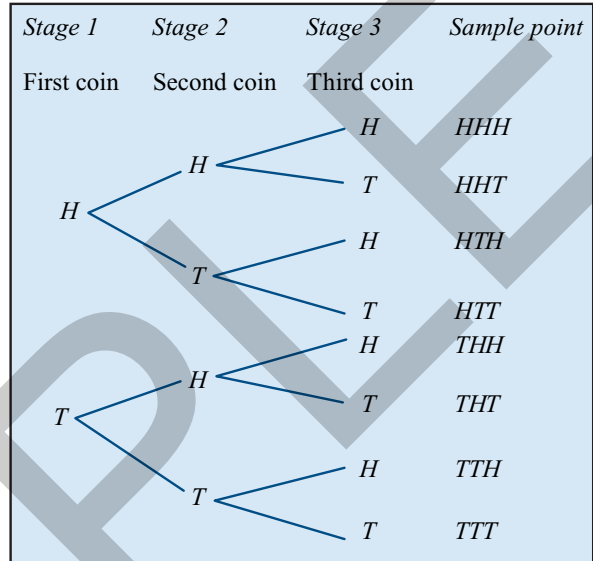
Three coins are tossed and the outcomes noted. List the sample space for this experiment.

Solution

A tree diagram is constructed to list the elements of the sample space.

Each path along the branches of the tree gives a sample point.

Thus the required sample space is $\varepsilon = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

**Exercise 10A****Example 1**

- 1 List the sample space for the toss of a coin.
- 2 List the sample space for the outcomes when a die is rolled.
- 3 Answer the following for a normal deck of playing cards:
 - a How many cards are there?
 - b How many suits are there?
 - c What are the suits called?
 - d Which suits are red and which suits are black?
 - e How many cards are there in each suit?
 - f Which cards are known as the 'picture cards'?
 - g How many aces are there in the deck?
 - h How many 'picture cards' cards are there in the deck?

Example 2

- 4 List the sample spaces for the following experiments:
 - a Two balls are chosen from a bag containing two black and two red balls.
 - b A coin is tossed and then a die is rolled.
 - c Three students are chosen for a committee from a class of 10 male and 10 female students, and the sex of each student noted.

- 5 List the sample spaces for the following experiments:
- the number of picture cards in a hand of five cards
 - the number of female children in a family with six children
 - the number of female students on a committee of three students chosen from a class of 10 male and 10 female students
- 6 List the sample spaces for the following experiments:
- the number of cars which pass through a particular intersection in a day
 - the number of people on board a bus licensed to carry 40 passengers
 - the number of times a die is rolled before a six is observed
- 7 List the outcomes (sample points) associated with the following events:
- 'an even number' when a die is rolled
 - 'more than two female students' when three students are chosen for a committee from a class of 10 male and 10 female students
 - 'more than four aces' when five cards are dealt from a standard pack of 52 cards
- 8 An experiment is defined as follows: A coin is tossed and the outcome observed. If the coin shows a head it is tossed again, but if it shows a tail a die is rolled. Use a tree diagram to list the sample space for the experiment.
- 9 A spinner is divided into four equal parts numbered 1, 2, 3 and 4. The spinner is spun twice.
- Use a tree diagram to list the sample space for the experiment.
 - Circle the outcomes associated with the event 'the sum of the numbers is equal to 6'.
- 10 Spinner *A* is divided into three equal parts which are numbered 1, 2 and 3. Spinner *B* is divided into four equal parts which are numbered 1, 2, 3 and 4. Spinner *A* is spun, and the result noted. Then spinner *B* is spun and the result noted.
- Use a tree diagram to list the sample space for the experiment.
 - List the outcomes associated with the event 'same number shows on both spinners'.
- 11 A die is rolled twice and the number uppermost recorded.
- Use a tree diagram to list the sample space for the experiment.
 - List the outcomes associated with the event 'the first number is odd and the second is even'.
- 12 Michael has a drawer containing three red, two black and seven white socks. He decides to choose socks with his eyes closed until he has a pair of socks.
- Use a tree diagram to list the sample space for the experiment.
 - What is the minimum number of socks he must draw in order to ensure he has a pair?

- 13 Cassie deals cards from a pack of 52 until she has two of the same suit.
- Use a tree diagram to illustrate the event ‘a spade on the first and a heart on the second’.
 - What is the minimum number of cards she must deal in order to ensure she has two of the same suit?

10.2 Determining empirical probabilities



In theory, every event has a true and constant probability or chance of occurring. The problem is that strategies are needed to help to determine the value of that probability. Sometimes, the probability is assigned a value just on the basis of experiences. For example, a newspaper sports journalist suggesting that Essendon has a 60% chance of winning its next game is relying purely on his or her own experiences. Another journalist might well assign this probability an entirely different value. Such probabilities are called **subjective probabilities**.

In practice, many probabilities are estimated by **experimentation**, by performing the random experiment leading to the outcome of interest many times and recording the results. This information can then be used to estimate the chances of it happening again in the future. Such probabilities are called **empirical probabilities**.

For example, consider the event ‘obtaining a head’ when a coin is tossed. Suppose one tosses the coin many times and counts the number of times a head is observed. The proportion of trials that resulted in a head is called the **relative frequency** of that event. That is:

$$\text{relative frequency of a head occurring} = \frac{\text{number of heads observed}}{\text{number of trials}}$$

The mathematics of probability recognises that in the long run the relative frequency of the occurrence of a particular event will settle down to a constant value, the true value of the probability. That is, if the number of trials, n , is sufficiently large, then the observed **relative frequency** becomes close to the **probability** $\Pr(A)$, or

$$\Pr(A) \approx \frac{\text{number of times the event occurs}}{\text{number of trials}} \quad \text{for large } n$$

This rule can be illustrated by repeating a simple experiment, such as the tossing of a coin, many times. On one historical occasion in about 1900, English statistician Karl Pearson tossed a coin 24 000 times, resulting in 12 012 heads and a relative frequency of 0.5005!

Example 3

Suppose that an actual observation of 100 rolls of two dice gave the following results:

<i>Event</i>	<i>Same numbers on the dice</i>	<i>Different numbers on the dice</i>
<i>Number of occurrences</i>	19	81

Use these results to estimate the probability of the same number appearing on both dice, and different numbers appearing on both dice.

Solution

We can calculate the proportion of the number of occurrences of each outcome to the number of tosses. In this example we divide each of the results by 100:

<i>Event</i>	<i>Same numbers on the dice</i>	<i>Different numbers on the dice</i>
<i>Relative frequency</i>	0.19	0.81

These proportions, or relative frequencies, are **estimates** of the chances of getting a particular number of ‘doubles’ on two rolls of the dice. If this experiment was repeated, it would generally be found that the results were slightly different. One might conclude that relative frequency is not a very good way of estimating probability. Sometimes, however, experiments are the **only** way to get at an unknown probability. One of the most valuable lessons to be learnt is that such estimates are not exact. Our best estimates of the probabilities will result from using as many trials as possible.

Example 4

In order to investigate the probability that a drawing pin lands point up, Katia decides to toss it 50 times and to count the number of favourable outcomes, which turns out to be 33. Mikki repeats the experiment, but she tosses the same drawing pin 100 times and counts 62 favourable outcomes. What is Katia’s estimate of the probability of the drawing pin landing point up? What is Mikki’s estimate? Which of these is the preferred estimate of the probability from these experiments? Based on the information available, what would be the best estimate of the probability?

Solution

From Katia’s information $\Pr(\text{point up}) \approx \frac{33}{50} = 0.66$

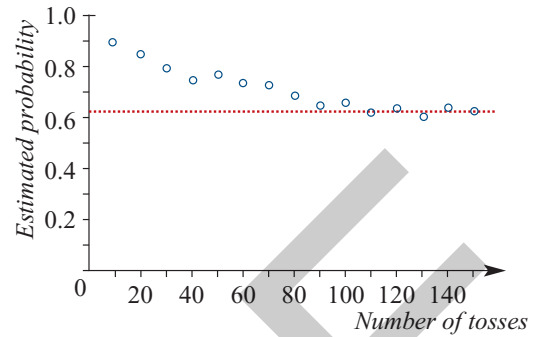
From Mikki’s information $\Pr(\text{point up}) \approx \frac{62}{100} = 0.62$

Since Mikki has estimated the probability from a larger number of trials, her estimate would be preferred to Katia’s.

Based on the information available, the best estimate of the probability would be found by combining the data from both experiments, and so maximising the number of trials. In total, 95 favourable outcomes were observed in 150 tosses, and this gives a ‘best’ estimate of the probability of $\frac{95}{150} = 0.63$.

Consider how the estimated probability of a drawing pin landing with the point up actually alters as the number of tosses used to estimate it is changed. The following graph shows the results of tossing a drawing pin 150 times.

The probability of the drawing pin landing point up was estimated every 10 throws. From the graph it may be seen that as the number of trials increases the estimated probability converges to a value and then stays fairly stable.



Thus, one interpretation of probability is as the proportion of times that event will occur in the long run. This interpretation also defines the minimum and maximum values of probability as 0 (the event never occurs) and 1 (the event always occurs). On analysing the frequency concept of probability, we see that other conditions must also hold which form the basis of our definition of probability. These are:

- $\Pr(A) \geq 0$. Since the relative frequency of occurrence of any event must be greater than or equal to zero, then so too must the probability associated with that event.
- $\Pr(\epsilon) = 1$. Since the relative frequency of the whole sample space must be unity then so too is the corresponding probability.
- The sum of the probabilities of all the outcomes of a random experiment must be equal to 1.

If the probability of an event is zero, then this event is said to be **impossible**, as it cannot occur. Conversely, an event associated with a probability of 1 must occur and so is said to be **certain**.

Example 5

A random experiment may result in 1, 2, 3 or 4. If $\Pr(1) = \frac{1}{13}$, $\Pr(2) = \frac{2}{13}$, $\Pr(3) = \frac{3}{13}$ find the probability of obtaining a 4.

Solution

The sum of the probabilities is 1, therefore

$$\begin{aligned}\Pr(4) &= 1 - \left(\frac{1}{13} + \frac{2}{13} + \frac{3}{13} \right) \\ &= 1 - \frac{6}{13} = \frac{7}{13}\end{aligned}$$

Example 6

A random experiment may result in A, B, C, D, E , where A, B, C, D are equally likely and E is twice as likely as A . Find $\Pr(E)$.

Solution

Let $\Pr(A) = \Pr(B) = \Pr(C) = \Pr(D) = x$. Then $\Pr(E) = 2x$.

The sum of the probabilities is 1, therefore

$$x + x + x + x + 2x = 1$$

$$6x = 1$$

$$x = \frac{1}{6}$$

Thus: $\Pr(E) = 2x = \frac{1}{3}$

Example 7

Consider the drawing pin tossing experiment in Example 4. What is the probability of the drawing pin landing point down?

Solution

Let U represent the event that the drawing pin lands point up, and D represent the event that the pin lands point down.

When two events U and D are mutually exclusive, and such that together they make up the entire sample space, they are said to be **complementary** events and

$$\Pr(U) + \Pr(D) = 1$$

Thus: $\Pr(D) = 1 - \Pr(U) = 1 - 0.63 = 0.37$

In general, the complement of any event A is denoted A' and $\Pr(A') = 1 - \Pr(A)$.

Exercise 10B**Example 3**

- 1 Estimate the probability of the event specified occurring, use the data given:
 - a $\Pr(\text{head})$ if a coin is tossed 100 times and 34 heads observed
 - b $\Pr(\text{ten})$ if a spinner is spun 200 times and lands on the 'ten' 20 times
 - c $\Pr(\text{two heads})$ of two coins are tossed 150 times and two heads are observed on 40 occasions
 - d $\Pr(\text{three sixes})$ if three dice are rolled 200 times and three sixes observed only once

Example 4

- 2 A student decides to toss two coins and notes the results.
 - a Do you think relative frequencies obtained from 20 trials would make for a good estimate of the probabilities?
 - b Perform the experiment 20 times and estimate $\Pr(\text{two heads})$, $\Pr(\text{one head})$, and $\Pr(\text{no heads})$.
 - c Combine your results with those of your friends, so that you have results from at least 100 trials. Use these results to again estimate the probabilities.
 - d Do you think the 100 trials data give better estimates of the probabilities?
 - e How many trials would you need to find the probabilities exactly?

- 3 A student decides to roll two dice and record their sum.
- Do you think the relative frequencies obtained from 20 trials would provide a good estimate of the probabilities?
 - Perform the experiment 20 times and estimate $\Pr(\text{total} = 4)$, $\Pr(\text{total} = 7)$, and $\Pr(\text{total} = 10)$.
 - Combine your results with those of your friends, or carry out four more sets of twenty trials, so that you have results from at least 100 trials of the experiment. Use these results to again estimate the probabilities.
 - Do you think the 100 trials data give better estimates of the probabilities?
 - How many trials would you need to find the probabilities exactly?
- 4 Two misshapen six-sided dice were used for the following experiment. The first die was thrown 500 times and 78 sixes were observed. The second die was thrown 700 times and 102 sixes were observed. If you wished to throw a six, which die would you choose to throw, and why?
- 5 A coin is known to be biased in favour of heads. To get some idea of the probability of the coin showing heads it is tossed 2000 times. The following figures, which were the only records kept, give the number of heads after various numbers of tosses:

<i>Number of tosses</i>	100	500	1000	2000
<i>Number of heads</i>	62	361	712	1404

- Based on the 2000 tosses, what would be your best guess for the probability of the coin coming up heads?
- How would this guess change if the coin had been tossed only 500 times?
- Suppose that you found out that immediately after the 100th toss a chip had come off the coin. Explain what effect this might have.
- Use the above data to obtain the best estimate of the probability of the coin coming up heads after it has been chipped.

Example 5 6 A random experiment results in 1, 2, 3, 4, 5, or 6. If $\Pr(1) = \frac{1}{12}$, $\Pr(2) = \frac{1}{6}$, $\Pr(3) = \frac{1}{8}$, $\Pr(5) = \frac{1}{6}$, $\Pr(6) = \frac{1}{8}$, find the probability of obtaining a 4.

Example 6 7 When a biased six-sided die was tossed a large number of times, it was observed that the numbers 2, 3, 4 and 5 were equally likely to occur. The number 6 was noted to occur twice as often as the number 2, whereas the number 1 was noted to occur half as often as the number 2. Find the probability of each of the possible outcomes.

- 8 A target is divided into five regions labelled A, B, C, D, E . The probability of hitting the regions A, B, C and D are equal. If $\Pr(E) = 0.1$, find the probability of hitting region A , $\Pr(A)$.

Example 7 9 For the target described in Question 8, find the probability of not hitting region A , $\Pr(A')$.

10.3 Determining probabilities by symmetry

Performing an experiment to estimate the probability of an event occurring is a time-consuming exercise. There must be other ways of predicting the probability of an event, using our knowledge of the situation – that is, to predict the probability of an outcome **before** the event takes place. This is certainly true when we are rolling a perfectly fair die. Since six outcomes are possible, it seems reasonable to claim that the probability of any one of them occurring on a roll is equal. That is:

$$\Pr(1) = \Pr(2) = \Pr(3) = \Pr(4) = \Pr(5) = \Pr(6) = \frac{1}{6}$$

This claim is made on the assumption that the outcomes from rolling the die are all equally likely to occur.

Whenever an experiment has equally likely outcomes, we may define the probability of an event E as:

$$\Pr(E) = \frac{\text{number of outcomes favourable to } E}{\text{total number of possible outcomes}} = \frac{n(E)}{n(\mathcal{E})}$$

where the notation $n(E)$ is used to represent the number of elements in set E .

Recall that the set of possible outcomes from a random experiment is called the sample space for the experiment, and is written as a list of items separated by commas and surrounded by curly brackets, thus $\{a, b, c\}$. In the two dice examples discussed in the previous section, there is the possibility of $\{1, 2, 3, 4, 5, 6\}$ on die 1, and $\{1, 2, 3, 4, 5, 6\}$ on die 2. So the sample space for this experiment looks like this:

Die 1 \ Die 2	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The events discussed in the previous section correspond to subsets of the sample space.

For example,

$$\text{'double'} = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

Thus, the probability of rolling a double can be determined:

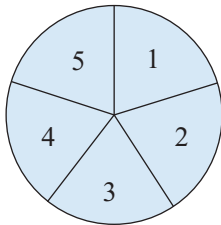
$$\Pr(\text{double}) = \frac{6}{36} = \frac{1}{6} = 0.167$$

From the previous section you can see that the experimental results are quite close to these, but not exactly the same. The probabilities determined by symmetry predict what would happen to the experimental probabilities in the long run. Like the experimental approach to probability, this definition defines the minimum and maximum values of probability as 0 (the event never occurs) and 1 (the event always occurs), and it can be readily verified that the other rules for probability still hold here where probabilities are calculated using symmetry.

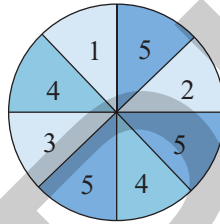
Example 8

Find the probability that each of the possible outcomes is observed for the following spinners.

a



b



Solution

a On spinner **a** there are five equally likely outcomes, so

$$\Pr(1) = \Pr(2) = \Pr(3) = \Pr(4) = \Pr(5) = 0.2.$$

b On spinner **b** the areas allotted to each of the outcomes are not all equal, so the associated probabilities will not all be the same. The probabilities will be equal to the fraction of the whole circle that each outcome defines.

Thus:

$$\Pr(1) = \Pr(2) = \Pr(3) = \frac{1}{8} = 0.125$$

$$\Pr(4) = \frac{2}{8} = \frac{1}{4} = 0.25$$

$$\Pr(5) = \frac{3}{8} = 0.375$$

Note that in both these cases $\Pr(1) + \Pr(2) + \Pr(3) + \Pr(4) + \Pr(5) = 1$.

If the sample space for an experiment contains n elements, all of which are equally likely to occur, we assign a probability of $\frac{1}{n}$ to each of these points. The probability of any event A , which contains m of these sample points, occurring is then the ratio of the number of elements in A , $n(A)$, to the number of elements in ϵ , $n(\epsilon)$. That is:

$$\Pr(A) = \frac{n(A)}{n(\epsilon)} = \frac{m}{n}$$

Symmetry can also be used with areas of regions.

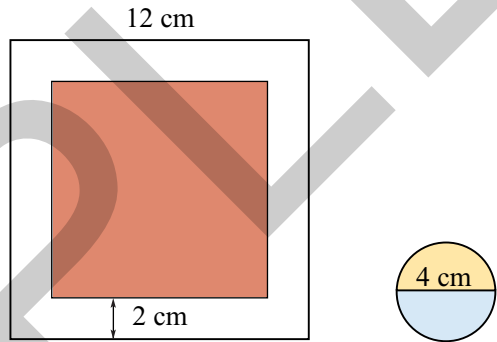
Example 9

A square with side length 12 cm is drawn on a table.

- a** A circular disc of radius 2 cm is placed on the table so that it lies completely inside the square. Draw a diagram of the square and indicate the region in which the centre of the disc can lie if the disc must be inside the square. (Touching an edge is accepted.)
- b** A game consists of throwing the disc described above onto the table. A point is scored if the disc lies completely within the square. (Touching an edge is accepted.) If a player always gets the centre of the disc into the square, what is the probability of scoring a point?

Solution

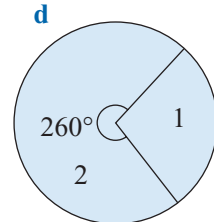
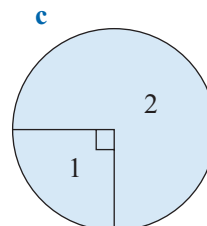
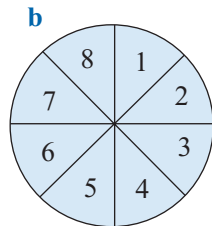
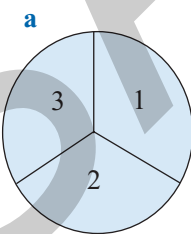
- a** The shaded square indicates the region in which the centre of the disc can lie if the disc is to lie within the square.



- b** There is a 2 cm wide border surrounding the red square. The area of the initial square is 144 cm^2 . The area of the red region is 64 cm^2 . Therefore the probability of the disc lying entirely within the square is $\frac{64}{144} = \frac{4}{9}$.

Exercise 10C**Example 8**

- 1** Consider the following spinners. In each case, what is the chance of the pointer stopping in region 1?

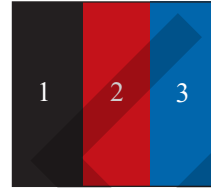


- 2** In 1989 the number of university enrolments in Victoria, by location, were as follows:

University	Number enrolled
Deakin	8262
La Trobe	13491
Monash	14847
Melbourne	21819

- 10** A flag is made up of three sections, as shown in the diagram. Four colours are available for the flag: red, yellow, black and blue. However, each segment of the flag must be of a different colour.

- a** Draw a tree diagram to show all the possible outcomes, when colouring sections 1, 2 and 3 in succession.
b Find the probability that:
i red is not chosen **ii** red and black are chosen
iii section 1 is black and section 2 is red.

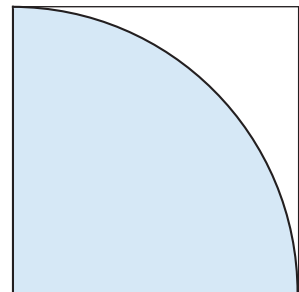


- 11** A bag contains five red balls, two blue balls and six white balls. A ball is chosen at random and its colour noted. Find the probability that the ball chosen is:
a red **b** not blue
- 12** A bag contains five balls, numbered 1 to 5. A ball is chosen at random, the number noted and the ball replaced. A second ball is then chosen at random and its number noted.
a Draw up a table of ordered pairs to show the sample space for the experiment.
b Find the probability that:
i the sum of the two numbers is 5 **ii** the two numbers are different
iii the second number is two more than the first.

Example 9

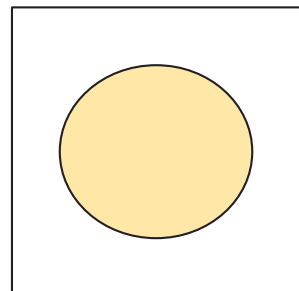
- 13** A square with side length 5 cm is drawn on a table.
a A circular disc of radius 1 cm is placed on the table so that it lies completely inside the square. Draw a diagram of the square and indicate the region in which the centre of the disc can lie if the disc must be inside the square. (Touching an edge is accepted.)
b A game consists of throwing the disc described above onto the table. A point is scored if the disc lies completely within the square. (Touching an edge is accepted.) If a player always gets the centre of the disc into the square, what is the probability of scoring a point?

- 14** The square has side length 1 metre. A dart is thrown at the square. The blue region is a one-quarter of a circular disc centred at the bottom left vertex of the square. The dart is equally likely to hit any part of the square and it hits the square every time. Find the probability of hitting the blue region.



- 15** In a sideshow at a fete a dart is thrown at a square with side length 1 metre. The circle shown has a radius of 0.4 m. The dart is equally likely to hit any point on the square. Find the probability that the dart will hit:

- a** the shaded part of the square
b the unshaded part of the square.



10.4 The addition rule

Before proceeding with the discussion of probability a review of sets and set notation is necessary.

The **null** or **empty set**, denoted by \emptyset , is the set consisting of no elements. This is different from $\{0\}$, which is a set containing one element, 0.

Sets, and the relationships between sets, can be illustrated clearly by using **Venn diagrams**. ϵ is usually shown as a rectangle, and a subset of ϵ as a circle.

If A and B are any two sets, then the **union** of A and B , denoted $A \cup B$, is the set of all elements in A or B or both.

For example, if A is the set of students in a school who play hockey, and B the set of students who play tennis, then the union of A and B (shown on the Venn diagram by shading both sets A and B), would represent the set of students who play either hockey or tennis or both.

The **intersection** of A and B , denoted by $A \cap B$, is the set of elements that are in both A and B .

For example, the intersection of the sets previously described would represent the set of students who play both hockey and tennis, and is shown on a Venn diagram by shading only the area contained in both A and B .

As previously, note that the **complement** of A , denoted A' , is the set of points that are in ϵ but not in A .

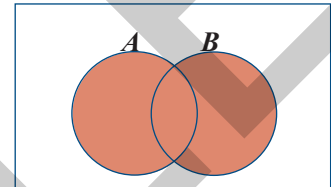
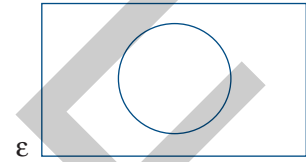
The complement of the set of students who play hockey in a school would represent the set of students who do not play hockey and is shown on a Venn diagram by shading only the area not contained in A .

Two sets, A and B , are said to be **disjoint** or **mutually exclusive** if they have no elements in common.

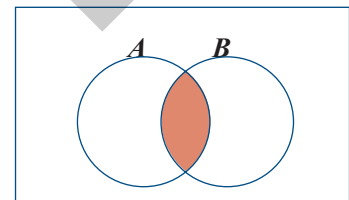
Thus, if A is the set of girls who play hockey in a school, and B is the set of boys who play hockey, then A and B are mutually exclusive, as no student can belong to both sets. The Venn diagram illustrates that the two sets are mutually exclusive. That is, $A \cap B = \emptyset$.

Finally, the number of elements in a set A is usually denoted $n(A)$. For example, if $A = \{a_1, a_2, a_3\}$ then $n(A) = 3$.

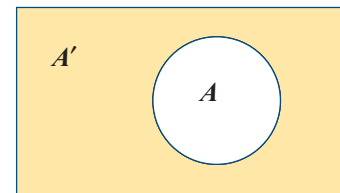
Venn diagrams can be used to help us solve practical problems involving sets.



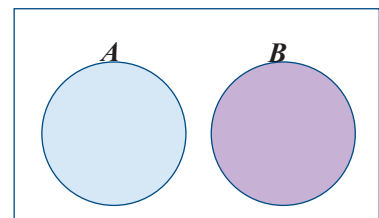
Union



Intersection



Complement



Disjoint sets

Example 10

Fifty teenagers were asked what they did on the weekends. A total of 35 said they went to football matches, the movies or both. Of the 22 who went to football matches, 12 said they also went to the movies. Show this information on a Venn diagram. How many teenagers went to the movies but not to football matches? How many did not go to either of these events?

Solution

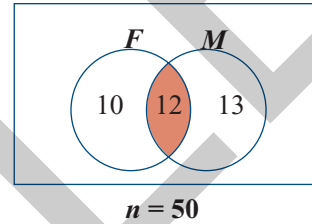
Let F denote the set of teenagers who attend football matches and M denote the set of teenagers who attend movies.

Hence from the information given

$$n(F \cup M) = 35, n(F) = 22 \text{ and } n(F \cap M) = 12.$$

Students who go to the movies but not to football matches are found in the region $F' \cap M$, and from the diagram $n(F' \cap M) = 13$. Those who attend neither event are found in the region $F' \cap M'$, and from the diagram.

$$n(F' \cap M') = 15$$



Venn diagrams can be used to illustrate a very important rule that will enable us to calculate probabilities for more complex events. If A and B are two events in a sample space ϵ , and $A \cap B \neq \emptyset$, then the relationship between them can be represented by a Venn diagram, as shown.

From the Venn diagram we can see that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

(As the intersection has been counted twice, in both $n(A)$ and in $n(B)$, we must subtract it.)

Dividing through by $n(\epsilon)$ gives:

$$\frac{n(A \cup B)}{n(\epsilon)} = \frac{n(A)}{n(\epsilon)} + \frac{n(B)}{n(\epsilon)} - \frac{n(A \cap B)}{n(\epsilon)}$$

Now, if each of the outcomes in ϵ is equally likely to occur, then each term in this expression is equal to the probability of that event occurring. This can be rewritten as:

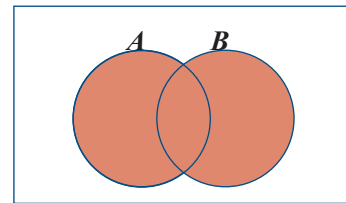
$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

So the probability of A or B or both occurring can be calculated using

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

This is called the **addition rule** for combining probabilities.

This rule can be used to help solve more complex problems in probability.



Example 11

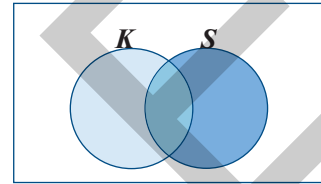
If one card is chosen at random from a well-shuffled deck, what is the probability that the card is either a king or a spade?

Solution

Let event K be 'a king'. Then $K = \{\text{king of spades, king of hearts, king of diamonds, king of clubs}\}$, and $n(K) = 4$.

Let event S be 'a spade'. Then $S = \{\text{ace of spades, king of spades, queen of spades, } \dots\}$, and $n(S) = 13$.

The event 'a king or a spade' corresponds to the union of sets K and S .



$$\Pr(K) = \frac{4}{52}$$

$$\Pr(S) = \frac{13}{52}$$

$$\Pr(K \cap S) = \frac{1}{52}$$

and so, using the addition rule, we find

$$\Pr(K \cup S) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = 0.3077 \quad (\text{correct to 4 decimal places})$$



Exercise 10D

Example 10 1 $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$. Show these sets on a Venn diagram and use your diagram to find:

- a** $A \cup B$ **b** $A \cap B$ **c** A'
d $A \cap B'$ **e** $(A \cap B)'$ **f** $(A \cup B)'$

2 $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, $A = \{\text{multiples of four}\}$, $B = \{\text{even numbers}\}$. Show these sets on a Venn diagram and use your diagram to find:

- a** A' **b** B' **c** $A \cup B$ **d** $(A \cup B)'$ **e** $A' \cap B'$

3 $\varepsilon = \{\text{different letters of the word MATHEMATICS}\}$

$A = \{\text{different letters of the word ATTIC}\}$

$B = \{\text{different letters of the word TASTE}\}$

Show ε , A and B on a Venn diagram, entering all the elements. Hence list the sets:

- a** A' **b** B' **c** $A \cup B$
d $(A \cup B)'$ **e** $A' \cup B'$ **f** $A' \cap B'$

- 4 In a survey of 100 university students, a market research company found that 70 students owned CD players, 45 owned cars and 35 owned CD players and cars. Use a Venn diagram to help you answer the following questions:
- a How many students owned neither a car nor a CD player?
 - b How many students owned either a car or a CD player, but not both?
- 5 A swimming team consists of 18 members. Each member performs in at least one of the three events, freestyle (F), backstroke (B) and diving (D). Every diver also races. Eleven of the team swim freestyle and ten swim backstroke. Two of the team swim backstroke and dive, but do not swim freestyle. Five of the team swim freestyle only, and seven of those who swim backstroke do not dive. Draw a Venn diagram and find:
- a $n(D)$
 - b $n(F \cap D \cap B)$
 - c $n(F \cup B)$
 - d $n(F \cap B \cap D)$
- 6 If $\epsilon = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\}$ and $B = \{3\}$ find:
- a $\Pr(A \cup B)$
 - b $\Pr(A \cap B)$
 - c $\Pr(A')$
 - d $\Pr(B')$

Example 11 7 Let $\epsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

If A is the event ‘an even number’ and B is the event ‘a multiple of three’, find:

- a $\Pr(A)$
- b $\Pr(B)$
- c $\Pr(A \cap B)$ and hence $\Pr(A \cup B)$

- 8 A letter is drawn at random from the word PROBABILITY. Let

A = the event ‘drawing a vowel’

B = the event ‘drawing a consonant’

C = the event ‘drawing an I’

D = the event ‘drawing a B’.

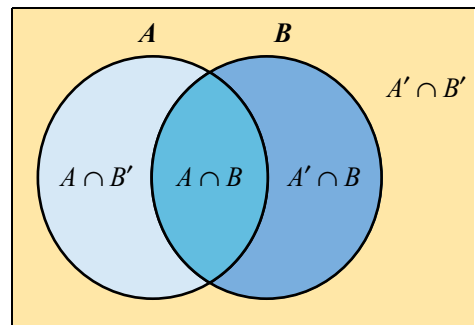
Find the probability of:

- a $A \cup B$
- b $A \cup C$
- c $B \cup C$
- d $A \cup D$
- e $B \cup D$
- f $C \cup D$

10.5 Probability tables and Karnaugh maps

The **probability table** is an alternative to a Venn diagram when illustrating a probability problem diagrammatically. Consider the Venn diagram which illustrates two intersecting sets A and B .

From the Venn diagram it can be seen that the sample space is divided by the sets into four disjoint regions: $A \cap B$, $A \cap B'$, $A' \cap B$ and $A' \cap B'$. These regions may be represented in a table as follows. Such a table is sometimes referred to as a **Karnaugh map**:



	B	B'
A	$A \cap B$	$A \cap B'$
A'	$A' \cap B$	$A' \cap B'$

In a probability table, the entries in each cell give the probabilities of each of these events occurring.

		Column 1	Column 2
		B	B'
Row 1	A	$\Pr(A \cap B)$	$\Pr(A \cap B')$
Row 2	A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$

Further, from the Venn diagram we can see that set A consists of the union of the section of set A that intersects with set B , and the section of set A which does not intersect with set B . That is:

$$A = (A \cap B) \cup (A \cap B')$$

Sets $A \cap B$ and $A \cap B'$ are mutually exclusive, therefore:

$$\Pr(A \cap B) + \Pr(A \cap B') = \Pr(A)$$

and thus summing the probabilities in row 1 gives $\Pr(A)$.

Similarly, $\Pr(A \cap B) + \Pr(A' \cap B) = \Pr(B)$

$$\Pr(A' \cap B) + \Pr(A' \cap B') = \Pr(A')$$

$$\Pr(A \cap B') + \Pr(A' \cap B') = \Pr(B')$$

Finally, since

$$\Pr(A) + \Pr(A') = 1 \quad \text{and} \quad \Pr(B) + \Pr(B') = 1$$

the totals for both column 3 and row 3 are equal to 1. Thus, the completed table becomes:

		Column 1	Column 2	Column 3
		B	B'	
Row 1	A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\Pr(A)$
Row 2	A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\Pr(A')$
Row 3		$\Pr(B)$	$\Pr(B')$	1

These tables can be useful when solving problems involving probability, as shown in the next three examples.

Example 12

If A and B are events such that $\Pr(A) = 0.7$, $\Pr(A \cap B) = 0.4$ and $\Pr(A' \cap B) = 0.2$, find:

- a** $\Pr(A \cap B')$ **b** $\Pr(B)$ **c** $\Pr(A' \cap B')$ **d** $\Pr(A \cup B)$

Solution

		Column 1	Column 2	Column 3
		B	B'	
Row 1	A	$\Pr(A \cap B) = 0.4$	$\Pr(A \cap B')$	$\Pr(A) = 0.7$
Row 2	A'	$\Pr(A' \cap B) = 0.2$	$\Pr(A' \cap B')$	$\Pr(A')$
Row 3		$\Pr(B)$	$\Pr(B')$	1

The given information has been entered in the table in red.

- a** From row 1: $\Pr(A \cap B') = \Pr(A) - \Pr(A \cap B) = 0.7 - 0.4 = 0.3$
b From column 1: $\Pr(B) = \Pr(A' \cap B) + \Pr(A \cap B) = 0.2 + 0.4 = 0.6$
c From column 3: $\Pr(A') = 1 - \Pr(A) = 1 - 0.7 = 0.3$
 From row 2: $\Pr(A' \cap B') = 0.3 - 0.2 = 0.1$
d The addition rule gives $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
 $= 0.7 + 0.6 - 0.4$
 $= 0.9$

The completed table is shown below:

		Column 1	Column 2	Column 3
		B	B'	
Row 1	A	$\Pr(A \cap B) = 0.4$	$\Pr(A \cap B') = 0.3$	$\Pr(A) = 0.7$
Row 2	A'	$\Pr(A' \cap B) = 0.2$	$\Pr(A' \cap B') = 0.1$	$\Pr(A') = 0.3$
Row 3		$\Pr(B) = 0.6$	$\Pr(B') = 0.4$	1

Example 13

Records indicate that, in Australia, 65% of secondary students participate in sport, and 71% of secondary students are Australian by birth. They also show that 53% of students are Australian by birth and participate in sport. Use this information to find:

- a** the probability that a person selected at random is not Australian by birth
b the probability that a person selected at random is not Australian by birth, and does not participate in sport.

Solution

The information in the question may be entered into a table as shown. (We will use A to represent ‘Australian’ and S to represent ‘participation in sport’.)

	S	S'	
A	0.53		0.71
A'			
	0.65		1

All the empty cells in the table may now be filled in by subtraction.

In column 1: $\Pr(A' \cap S) = 0.65 - 0.53 = 0.12$

In column 3: $\Pr(A') = 1 - 0.71 = 0.29$

In row 1: $\Pr(A \cap S') = 0.71 - 0.53 = 0.18$

In row 3: $\Pr(S') = 1 - 0.65 = 0.35$

In row 2: $\Pr(A' \cap S') = 0.35 - 0.18 = 0.17$

The completed table is as follows:

	S	S'	
A	0.53	0.18	0.71
A'	0.12	0.17	0.29
	0.65	0.35	1

The probability that a person selected at random is not Australian by birth is given by $\Pr(A') = 0.29$.

The probability that a person selected at random is not Australian by birth and does not participate in sport is given by $\Pr(A' \cap S') = 0.17$.

Example 14

Suppose that of the people in Australia approximately 6% are colourblind. Suppose also that 45% of Australians are male and, further, that 5% of males are colourblind. Use this information to find:

- the probability that a person selected at random is female and colourblind
- the probability that a person selected at random is male and not colourblind.

Solution

This information is summarised in the following table. (We will use M to represent ‘male’ and B to represent ‘colourblind’.)

	M	M'	
B	0.05		0.06
B'			
	0.45		1

Filling the empty cells by subtraction gives:

	M	M'	
B	0.05	0.01	0.06
B'	0.40	0.54	0.94
	0.45	0.55	1

Thus, reading from the table, the probability that a person selected at random is female (not male) and colourblind is $\Pr(M' \cap B) = 0.01$.

The probability that a person selected a random is male and not colourblind is given by $\Pr(M \cap B') = 0.40$.

**Exercise 10E****Example 12**

- If A and B are events such that $\Pr(A) = 0.6$, $\Pr(A \cap B) = 0.4$ and $\Pr(A' \cap B) = 0.1$, find:
 - $\Pr(A \cap B')$
 - $\Pr(B)$
 - $\Pr(A' \cap B')$
 - $\Pr(A \cup B)$
- If A and B are events such that $\Pr(A') = 0.25$, $\Pr(A' \cap B) = 0.12$ and $\Pr(B) = 0.52$, find:
 - $\Pr(A)$
 - $\Pr(A \cap B)$
 - $\Pr(A \cup B)$
 - $\Pr(B')$
- If C and D are events such that $\Pr(C \cup D) = 0.85$, $\Pr(C) = 0.45$ and $\Pr(D') = 0.37$, find:
 - $\Pr(D)$
 - $\Pr(C \cap D)$
 - $\Pr(C \cap D')$
 - $\Pr(C' \cup D')$
- If E and F are events such that $\Pr(E \cup F) = 0.7$, $\Pr(E \cap F) = 0.15$ and $\Pr(E') = 0.55$, find:
 - $\Pr(E)$
 - $\Pr(F)$
 - $\Pr(E' \cap F)$
 - $\Pr(E' \cup F)$
- If A and B are events such that $\Pr(A) = 0.8$, $\Pr(B) = 0.7$ and $\Pr(A' \cap B') = 0.1$, find:
 - $\Pr(A \cup B)$
 - $\Pr(A \cap B)$
 - $\Pr(A' \cap B)$
 - $\Pr(A \cup B')$

Example 13

- 6 In a recent survey of senior citizens, it was found that 85% favoured giving greater powers of arrest to police, 60% favoured longer sentences for convicted persons, and 50% favoured both propositions.
- What percentage favoured at least one of the two propositions?
 - What percentage favoured neither proposition?
- 7 Suppose a card is selected at random from an ordinary deck of 52 playing cards. Let
 A = event a picture card is selected (i.e. jack, queen, king or ace)
 C = event a heart is selected.
- List the event spaces corresponding to events A and C .
 - Determine the following probabilities and express your results in words:
 - $\Pr(A)$
 - $\Pr(C)$
 - $\Pr(A \cap C)$
 - $\Pr(A \cup C)$
 - $\Pr(A \cup C')$
- 8 The following information applies to a particular class:
- The probability that a student's name begins with M and that the student studies French is $\frac{1}{6}$.
 - The probability that a student's name begins with M is $\frac{3}{10}$.
 - The probability that a student does not study French is $\frac{7}{15}$.
- Find the probability that a student chosen at random from this class:
- studies French
 - has a name which does not begin with M
 - has a name which does begin with M, but does not study French
 - has a name which does not begin with M and does not study French.
- 9 A frame is chosen at random from a shop where picture frames are sold. It is known that in this shop:
- the probability that the frame is made of wood is 0.72
 - the probability that the frame is freestanding is 0.65
 - the probability that the frame is not made of wood and is not freestanding is 0.2.
- Find the probability that the randomly chosen frame is:
- made of wood or is freestanding
 - made of wood and is freestanding
 - not made of wood
 - not made of wood but is freestanding.
- 10 A book is chosen at random from a bookshop. It is known that in this bookshop:
- the probability that the book is a hardback but not a novel is 0.05
 - the probability that the book is not hardback but is a novel is 0.12
 - the probability that the book is not a novel is 0.19.

Find the probability that the randomly chosen book is:

- a** a novel
- b** a hardback novel
- c** a hardback
- d** a novel or a hardback

- 11** At a school camp consisting of 60 students, sailing was offered as an activity one morning, and bushwalking in the afternoon. Every student attended at least one activity. 32 students went sailing and 40 students went bushwalking, find the probability that a student chosen at random:

- a** undertook neither of these activities
- b** has sailed or bushwalked
- c** has sailed and bushwalked
- d** has sailed but not bushwalked

- 12** At a barbecue attended by 50 people, hamburgers and sausages were available. It was found that 35 hamburgers and 38 sausages were eaten, and six people were noted to have eaten neither a hamburger nor a sausage. If no person ate more than one hamburger or one sausage, find the probability that a person chosen at random ate:

- a** a hamburger or a sausage
- b** a hamburger and a sausage
- c** had only one serve of food
- d** had only a hamburger





Chapter summary

- Probability is a numerical measure of the chance of a particular event occurring and may be determined experimentally or by symmetry.
- Whatever method is used to determine the probability, the following rules will hold:
 - a $0 \leq \Pr(A) \leq 1$ for all events $A \subseteq \varepsilon$
 - b $\Pr(\varepsilon) = 1$
 - c $\Pr(\emptyset) = 0$
 - d $\Pr(A') = 1 - \Pr(A)$, where A' is the complement of A
 - e $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ (the **addition rule**).
- Probabilities associated with compound events are sometimes able to be calculated more easily from a probability table.
- Two events A and B are mutually exclusive if $A \cap B = \emptyset$
- If two events are mutually exclusive, $\Pr(A \cap B) = 0$ and $\Pr(A \cup B) = \Pr(A) + \Pr(B)$
- A sample space ε can be divided into four disjoint regions $A \cap B$, $A \cap B^1$, $A^1 \cap B$ and $A^1 \cap B^1$

Multiple-choice questions

- 1 If the probability of Chris scoring 50 or more marks in the exam is 0.7, then the probability he scores less than 50 marks is
 A 0 B 0.3 C 0.4 D 0.7 E 0.8
- 2 A spinner is coloured red, yellow, blue and green. When spun the probability that it lands on red is 0.1, yellow is 0.2 and blue is 0.4. What is the probability that it lands on green?
 A 0.1 B 0.2 C 0.3 D 0.4 E 0.5
- 3 Phillip is making a sign and has cut the letters of the word THEATRETTE out of wood, and placed them in his tool box. If a letter is selected at random from the toolbox, then the probability that it is a T is
 A $\frac{2}{5}$ B $\frac{3}{10}$ C $\frac{1}{5}$ D $\frac{1}{6}$ E $\frac{3}{5}$
- 4 Of a group of 25 people in a restaurant, three chose a vegetarian meal, five chose fish, ten chose beef and the rest chose chicken for their main course. What is the probability that a randomly chosen diner chose chicken?
 A $\frac{3}{25}$ B $\frac{6}{25}$ C $\frac{7}{25}$ D $\frac{2}{5}$ E $\frac{7}{18}$

- 5 Suppose that a card is chosen at random from a well-shuffled deck of 52 playing cards. What is the probability that the card is either a spade or a jack?

A $\frac{1}{4}$ B $\frac{1}{13}$ C $\frac{17}{52}$ D $\frac{4}{13}$ E $\frac{9}{26}$

- 6 Suppose that 57% of swimmers in a club are female (F), that 32% of the swimmers in the club swim butterfly (B), and that 11% of the swimmers in the club are females and swim butterfly. Which of the following probability tables correctly summarises this information?

A

	B	B'	
F	0.11	0.21	0.32
F'	0.46	0.22	0.68
	0.57	0.43	1

B

	B	B'	
F	0.04	0.53	0.57
F'	0.28	0.15	0.43
	0.32	0.68	1

C

	B	B'	
F	0.18	0.39	0.57
F'	0.14	0.29	0.43
	0.32	0.68	1

D

	B	B'	
F	0.11	0.32	0.43
F'	0.21	0.36	0.57
	0.32	0.68	1

E

	B	B'	
F	0.11	0.46	0.57
F'	0.21	0.22	0.43
	0.32	0.68	1

The following information relates to Questions 7 and 8.

A and B are events such that $\Pr(A) = 0.35$, $\Pr(A \cap B) = 0.18$ and $\Pr(B) = 0.38$.

- 7 $\Pr(A \cup B)$ is equal to
 A 0.73 B 0.133 C 0.15 D 0.21 E 0.55
- 8 $\Pr(A' \cap B)$ is equal to
 A 0.18 B 0.17 C 0.45 D 0.20 E 0.65
- 9 A square has side length of 4 metres. Inside the square is a circle of radius 1.5 metres. If a dart thrown at the square is equally likely to land at any point inside the square, then the probability that it will land outside the circle is closest to
 A 0.442 B 0.295 C 0.558 D 0.250 E 0.375

- 10 The following information applies to a particular class:
- The probability that a student studies Mathematics is $\frac{2}{3}$.
 - The probability that a student studies German is $\frac{3}{10}$.
 - The probability that a student studies Mathematics and does not study German is $\frac{7}{15}$.
- The probability that a randomly chosen student does not study either Mathematics or German is
- A $\frac{1}{5}$ B $\frac{7}{30}$ C $\frac{7}{15}$ D $\frac{7}{10}$ E $\frac{1}{3}$

Short-answer questions (technology-free)

- 1 Two six-sided dice are tossed. Find the probability that:
 - a the sum of the values of the uppermost faces is 7
 - b the sum is not 7
- 2 The probability that a computer chip is operational is 0.993. What is the probability that it is not operational?
- 3 A whole number between 1 and 300 (inclusive) is chosen at random. Find the probability that the number is:
 - a divisible by 3
 - b divisible by 4
 - c divisible by 3 or by 4
- 4 A drawer contains 30 red socks and 20 blue socks.
 - a If a sock is chosen at random, its colour noted, the sock replaced and a second sock withdrawn, what is the probability that both socks are red?
 - b If replacement doesn't take place, what is the probability that both socks are red?
- 5 Box A contains 5 pieces of paper numbered 1, 3, 5, 7, 9.
Box B contains 3 pieces of paper numbered 1, 4, 9.
One piece of paper is removed at random from each box. Find the probability that the two numbers obtained have a sum that is divisible by 3.
- 6 A three-digit number is formed by arranging digits 1, 5 and 6 in a random order.
 - a List the sample space.
 - b Find the probability of getting a number larger than 400.
 - c What is the probability that an even number is obtained?
- 7 A letter is chosen at random from the word STATISTICIAN.
 - a What is the probability that it is a vowel?
 - b What is the probability that it is a T?
- 8 If $\Pr(A) = 0.6$ and $\Pr(B) = 0.5$, can A and B be mutually exclusive? Why or why not?

- 9 Ivan and Joe are chess players. In any game the probabilities of Ivan beating Joe, Joe beating Ivan or the game resulting in a draw are 0.6, 0.1 or 0.3 respectively. They play a series of three games. Calculate the probability that:
- they win alternate games, with Ivan winning the first game
 - the three games are drawn
 - exactly two of the games are drawn
 - Joe does not win a game.
- 10 A die with 2 red faces and 4 blue faces is thrown three times. Each face is equally likely to face upward. Find the probability of obtaining the following:
- 3 red faces
 - a blue on the first, a red on the second and a blue on the third
 - exactly 1 red face
 - at least 2 blue faces

Extended-response questions

- 1 Let A and B be events in an event space ϵ , such that $\Pr(A) = \alpha$, $\Pr(B) = \beta$, $\Pr(A \cap B) = \gamma$. Find expressions for the probabilities of the following events in terms of α , β and γ :
- $A' \cap B$
 - $A \cap B'$
 - $A' \cap B'$
 - $A' \cup B'$
 - $(A \cup B)'$
 - $(A \cap B)'$
- 2 To have a stage production ready for opening night there are three tasks which must be done and, as the same people are involved in each task, these must be done in sequence. The following probabilities are estimated for the duration of the activities:

Task	6 days	7 days	8 days
Build scenery	0.3	0.3	0.4
Paint scenery	0.6	0.3	0.1
Print programs	0.4	0.4	0.2

- What is the probability that the building and painting of the scenery will together take exactly 15 days?
- What is the probability that all three tasks will together take exactly 22 days?
- Find the probability distribution of T , the total length of time taken to complete all three tasks.

- 3 On average, Emma goes home for dinner once a fortnight, and Sally goes home for dinner once a week.
- a What is the probability that either Emma or Sally or both of them go home to dinner on a particular day if it is known that, $\Pr(\text{Both girls go home to dinner on a particular day}) = \frac{1}{7} \times \frac{1}{14} = \frac{1}{98}$?
- b Establish inequalities for the probability that one or other or both of them go home to dinner on a particular day if the value of $\Pr(\text{Both girls go home to dinner on a particular day})$ is unknown.

- 4 A confectionary machine produces jellybeans of different shapes and colours in a random way and pours them into a giant vat. The jellybeans are produced in the proportions indicated in the table. There are millions of jellybeans in the vat.

<i>Cross-sectional shape</i>	<i>Purple</i>	<i>Green</i>	<i>Red</i>
Circular	0.4	0.2	0.1
Oval	0.2	0.1	0

- a What is the probability that a randomly chosen jellybean is purple?
- b What is the probability that a randomly chosen jellybean has an oval cross-section, given that it is purple? (Consider sample space of purple jellybeans.)
- c What is the probability of obtaining a green jellybean, given that it has a circular cross-section. (Consider sample space of jellybeans with circular cross-section.)
- d Three jellybeans are withdrawn from the vat. What is the probability of obtaining one of each colour. (Assume replacement.)
- e Ten jellybeans are taken from the vat and put in a jar. The number of each type of jellybean is indicated in the table.

<i>Cross-sectional shape</i>	<i>Purple</i>	<i>Green</i>	<i>Red</i>
Circular	4	2	1
Oval	2	1	0

Three jellybeans are removed from the jar without replacement. What is the probability that there is one of each colour?