CHAPTER

20

MODULE 4

Principles of financial mathematics

- How do we determine the new price when discounts or increases are applied?
- How do we determine the percentage discount or increase applied when given the old and new price?
- How do we determine the old price when given the new price and the percentage discount or increase applied?
- What is the formula for simple interest relating principal, interest rate, time and future value?
- How do we apply the simple interest formula to calculate the value of the fourth variable when the other three are known?
- What is the formula for compound interest that relates principal, interest rate, time and future value?
- How do we apply the compound interest formula to calculate the value of the fourth variable when the other three are known?
- How do we calculate the balance of a loan when periodic payments are being made?

20.1 Percentage change

When a shop advertises a sale they often give a standard discount on all goods, such as ‘25% discount off every item in the store’. Sometimes the goods are advertised as ‘marked down by 25%’, which means the same thing. This means that every item will have its purchase price reduced by 25% or, equivalently, the new price of the item will be 75% of the original price; that is:

$$\begin{aligned}
&\text{discount} &&= 25\% \text{ of original price} = 0.25 \times \text{original price}\\
\text{and}\quad &\text{new price} &&= 100\% \text{ of old price} - 25\% \text{ of old price}\\
& &&= 75\% \text{ of old price}\\
& &&= 0.75 \times \text{old price}
\end{aligned}$$

If an r% discount is applied then:

$$\text{discount} = \frac{r}{100} \times \text{original price}$$

$$\text{new price} = \text{original price} - \text{discount} = \frac{(100 - r)}{100} \times \text{original price}$$

Example 1 Calculating the amount of the discount and the new price

a How much is saved if a 25% discount is offered on an item marked $8.00?

b What is the new discounted price of this item?

Solution

a Evaluate the discount.

Discount $= 0.25 \times 8.00 = \$2.00$

b Evaluate the new price by either:

- subtracting the discount from the original price, or

New price = original price − discount
$= 8.00 - 2.00 = \$6.00$

- directly calculating 75% of the original price.

New price $= 0.75 \times 8.00 = \$6.00$

Of course prices are not always decreased. Sometimes they are increased, or 'marked up'. If a price is increased by 25% then:

increase = 25% of original price = $0.25 \times$ original price

and new price = 100% of old price + 25% of old price
= 125% of old price
= $1.25 \times$ old price

If an r% increase is applied then:

$$\text{increase} = \frac{r}{100} \times \text{original price}$$

$$\text{new price} = \text{original price} + \text{increase} = \frac{(100 + r)}{100} \times \text{original price}$$

Example 2 Calculating the amount of the increase and the new price

a If petrol prices increase by 10%, what is the amount of the increase when the price is 99.0 cents per litre?

b What is the new price per litre for petrol?

Solution

a Evaluate the increase.

Increase $= 0.10 \times 99.0 = 9.9$

b Evaluate the new price by either:

- adding the increase to the original price, or

New price = original price + increase
$= 99.0 + 9.9 = 108.9$ cents per litre

- directly calculating 110% of the original price.

New price $= 1.10 \times 99.0 = 108.9$ cents per litre

Given the original price and the new price of an item, we can work out the percentage change. To do this, the amount of the decrease or increase is determined and then converted to a percentage of the original price.

$$\text{percentage } \mathbf{discount} = \frac{\text{discount}}{\text{original price}} \times \frac{100}{1}$$

or

$$\text{percentage } \mathbf{increase} = \frac{\text{increase}}{\text{original price}} \times \frac{100}{1}$$

Example 3 Calculating the percentage discount or increase

a If the price of a book is reduced from \$25 to \$20, what percentage discount has been applied?

b If the price of a book is increased from \$20 to \$25, what percentage increase has been applied?

Solution

a **1** Determine the amount of the discount.

Discount = original price − new price
$= 25.00 - 20.00 = \$5.00$

2 Express this amount as a percentage of the original price.

Percentage discount $= \frac{5.00}{25.00} \times \frac{100}{1} = 20\%$

b **1** Determine the amount of the increase.

Increase = new price − original price
$= 25.00 - 20.00 = \$5.00$

2 Express this amount as a percentage of the original price.

Percentage increase $= \frac{5.00}{20.00} \times \frac{100}{1} = 25\%$

Sometimes we are given the new price and the percentage increase or decrease ($r\%$), and asked to determine the original price. Since we know that:

For a discount $\quad \text{new price} = \frac{(100 - r)}{100} \times \text{original price}$

For an increase $\quad \text{new price} = \frac{(100 + r)}{100} \times \text{original price}$

We can rearrange these formulas to give rules for determining the original price, as follows:

When an r% **discount** has been applied: original price $=$ new price $\times \dfrac{100}{(100-r)}$

When an r% **increase** has been applied: original price $=$ new price $\times \dfrac{100}{(100+r)}$

Example 4 Calculating the original price

Suppose that Steve has a $60 gift voucher from his favourite shop.

a If the store has a '25% off' sale, what is the original value of the goods he can now buy?

b If the store raises its prices by 25%, what is the original value of the goods he can now buy?

Solution

a Substitute new price $= 60$ and $r = 25$ in the formula for an r% discount.

$$\text{Original price} = 60 \times \frac{100}{75} = \$80$$

b Substitute new price $= 60$ and $r = 25$ in the formula for an r% increase.

$$\text{Original price} = 60 \times \frac{100}{125} = \$48$$

Exercise 20A

1 For each of the following find:

i the mark-down **ii** the new price

a $20.00 discounted by 10%
b $3.60 discounted by 25%
c $75.00 marked down by 30%
d $40.00 discounted by 50%
e $2.99 discounted by 20%
f $14.50 marked down by 15%
g $42.99 discounted by 12.5%

2 For each of the following find:

i the mark-up **ii** the new price

a $20.00 increased by 10%
b $3.60 increased by 25%
c $75.00 marked up by 30%
d $40.00 increased by 50%
e $2.99 increased by 20%
f $14.50 marked up by 15%
g $42.99 increased by 12.5%

3 Find the percentage discount that has been applied in each of the following situations:

a $20.00 reduced to $15.00
b $30.00 reduced to $27.00
c $39.99 reduced to $19.99
d $22 450 reduced to $19 082.50

4 Find the percentage increase that has been applied in each of the following situations:

a $15.00 increased to $20.00
b $9.95 increased to $11.94
c $1000 increased to $1050
d $2000 increased to $2250

5 Find the original price of the items that have been marked down as follows:

a Marked down by 15%, now priced \$20.00
b Marked down by 10%, now priced \$44.96
c Marked down by 25%, now priced \$50.00
d Marked down by 8%, now priced \$18 880

6 Find the original price of the items that have been marked up as follows:

a Marked up by 15%, now priced \$20.00
b Marked up by 12.5%, now priced \$24.00
c Marked up by 5%, now priced \$65.95
d Marked up by 2.5%, now priced \$12 500

7 Trinh has a part-time job at the local service station, where he is entitled to a staff discount of 10% on petrol and 15% on food and drink. On a particular day he buys 20 litres of petrol, which has a retail price of 97.8 cents per litre, and a can of coke and a packet of lollies, which cost \$1.70 cents and \$3.05 respectively. How much was his bill, after discount, for that day?

8 Raj is planning a holiday overseas. When he makes the initial enquiry his airfare is \$1850 and the accommodation is \$550. When he goes to book the holiday he finds that the airfare has increased in price by 2.5% and the accommodation by 7%. What is the total increase in the price of the holiday?

9 The Costless Sporting Goods store advertises all its goods as being at least 25% cheaper than either of the other two stores in town. Shopping around for some tennis equipment, Katia obtains prices for identical items from the three stores.

Item	*Costless*	*Sporties*	*Goodalls*
Racquet	\$78.99	\$94.99	\$92.99
Shoes	\$45.98	\$64.50	\$72.99
Skirt	\$23.50	\$33.98	\$32.99
T-shirt	\$24.99	\$33.75	\$33.99

Calculate whether Costless Sporting Goods store's claim is justified for each item.

10 A jacket that Chris wants to buy has an original price of \$240. It is marked down by 15% at the start of the sale, and on the day that he arrives to buy it he finds that it is marked down a further 15% on the reduced price.

a How much does he pay for the jacket?
b What is the total discount that Chris receives on the original price?

11 Geoff is saving for a car which is priced at \$24 950 in January. The price rises by 3% in June, and then by another 3.5% in September.

a What is the price of the car after the second rise in price?
b What is the total percentage increase in the price of the car between January and the end of September?

12 Georgia has a card which entitles her to a discount of 10% on petrol.

a If the price of petrol is \$1.15 per litre, how much does Georgia pay for 50 litres of petrol?

b If the price of petrol rises by 10%, what is the price per litre Georgia pays after her discount has been applied?

c What percentage discount or increase is this on the original price of $1.15 per litre?

13 Jim receives a $100 gift voucher from his family to spend at the local record store. Luckily for him, the store is having a sale.

a If the store is having a '20% off' sale, what value of goods can Jim now buy?

b If the store is having a '30% off' sale, what value of goods can Jim now buy?

14 At a sale at the jewellery shop, Cara buys a bracelet for $153.75, which had been discounted by 25%. What was the original price of the bracelet?

15 After a 12.5% price rise, the cost of a bottle of soft drink is $2.35. What was the price of the soft drink before the price rise?

20.2 Simple interest

If you borrow money, you must pay for the use of that money. If you lend money, you will be paid for doing so. The price of borrowing or lending money is called **interest**. The simplest type of interest is called **simple interest**.

Suppose we borrow $500 from a friend, and agree to pay him 10% interest for one year, after which time both the amount borrowed and the interest will be repaid. What does this mean? It means we will pay our friend $500, plus 10% of the amount borrowed, or

$$\$500 \times \frac{10}{100} = \$50$$

at the end of the year giving a total of $550.

If the money is borrowed or invested for several years, then interest may be paid or charged more than once. For example, if interest of 10% is charged **per annum**, then 10% of the amount borrowed is charged each year until the loan is repaid.

To calculate simple interest, we need to know the amount invested or borrowed, the interest rate and the length of time for which the money is invested or borrowed.

Example 5 **Calculating simple interest from first principles**

How much interest would you earn if you invested $5000 at 10% interest per annum for 3 years?

Solution

1 Calculate the interest for the first year. $\text{Interest} = 5000 \times \frac{10}{100} = 500$

2 Calculate the interest for the second year. $\text{Interest} = 5000 \times \frac{10}{100} = 500$

3 Calculate the interest for the third year. $\text{Interest} = 5000 \times \frac{10}{100} = 500$

4 Calculate the total interest. $\text{Interest for three years} = 500 + 500 + 500 = \$1500$

Back to Menu >>>

Since the amount of simple interest earned is the same every year, we can apply a general rule.

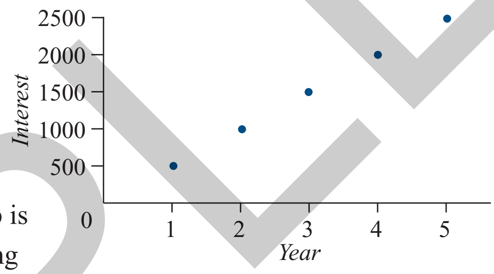
Simple interest

$$\text{Interest} = \frac{\text{amount invested} \times \text{interest rate (per annum)} \times \text{length of time (in years)}}{100}$$

or $$I = \frac{P \times r \times t}{100} = \frac{Prt}{100}$$

Here the amount invested or borrowed (P) is known as the principal, r is the interest rate and t is the time (in years).

How does this relationship look graphically? Suppose we were to borrow $5000 at 10% per annum simple interest for a period of years. A plot of **interest** against **time** is shown.



From this graph we can see that the relationship is linear, with the amount of interest paid or due being directly proportional to the time for which the money is borrowed or invested. The slope or gradient of a line which could be drawn through these points is numerically equal to the interest rate.

To determine the amount of the investment, the interest earned is added to the amount initially invested.

The amount of an investment, A, is the principal plus the amount of interest earned.

$$A = P + I = P + \frac{Prt}{100}$$

Here P is the amount invested or borrowed, r is the interest rate and t is the time (in years).

If the money is invested for more or less than one year, the amount of interest payable is proportional to the length of time for which it is invested.

Example 6 **Calculating the simple interest for a period other than one year**

How much interest would be due on a loan of $5000 at 10% per annum for six months?

Solution

Apply the formula with $P = 5000$, $r = 10\%$ and $t = 6/12$ since the investment is only for 6 months and the interest rate is for the whole year.

$$I = \frac{Prt}{100} = 5000 \times \frac{10}{100} \times \frac{6}{12}$$
$$= \$250$$

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Example 7 **Calculating the total amount borrowed or invested**

Find the total amount owed on a loan of \$10 000 at 12% per annum simple interest at the end of two years.

Solution

1 Apply the formula with $P = 10\,000$, $r = 12\%$ and $t = 2$ to find the interest.

$$I = \frac{Prt}{100} = 10\,000 \times \frac{12}{100} \times 2$$
$$= \$2400$$

2 Find the total owed by adding the interest to the principal.

$$A = P + I = 10\,000 + 2400 = \$12\,400$$

The formula given for simple interest can be rearranged to find any of the variables when the values of the other three variables are known.

Finding the interest rate

To find the interest rate per annum r, given the values of P, I and t:

$$r = \frac{100I}{Pt}$$

where P is the principal, I is the amount of interest and t is the time in years.

Example 8 **Calculating the interest rate of the loan or investment**

Find the rate of simple interest charged per annum if a loan of \$20 000 incurs interest of \$12 000 after eight years.

Solution

1 Apply the formula with $P = 20\,000$, $I = 12\,000$ and $t = 8$ to find the value of r.

$$r = \frac{100I}{Pt} = \frac{100 \times 12\,000}{20\,000 \times 8}$$
$$= 7.5\%$$

2 Since the unit of time was years, the interest rate can be written as the interest per annum.

Interest rate $= 7.5\%$ per annum

Finding the term

To find the number of years or term of an investment, t years, given the values of P, I and r:

$$t = \frac{100I}{Pr}$$

where P is the principal, I is the amount of interest and r is the interest rate per annum.

Back to Menu >>>

Example 9 Calculating the time period of the loan or investment

Find the length of time it would take for \$50 000 invested at an interest rate of 8% per annum to earn \$10 000 interest.

Solution

Apply the appropriate formula with $P = 50\,000$, $I = 10\,000$ and $r = 8$ to find the value of t.

$$t = \frac{100I}{Pr} = \frac{100 \times 10\,000}{50\,000 \times 8} = 2.5 \text{ years}$$

Finding the principal

To find the value of the principal P, given the values of I, r and t:

$$P = \frac{100I}{rt}$$

where I is the amount of interest, r is the interest rate per annum and t is the time is years.

To find the value of the principal P, given the values of A, r and t:

$$P = \frac{A}{\left(1 + \frac{rt}{100}\right)}$$

where A is the amount of investment, r is the interest rate per annum and t is the time is years.

Example 10 Calculating the principal of the loan or investment

a Find the amount that should be invested in order to earn \$1350 interest over 3 years at an interest rate of 4.5% per annum.

b Find the amount that should be invested at an interest rate of 5% per annum if you require \$5500 in two years' time.

Solution

a Since we are given the value of the interest I here we will use the first formula with $I = 1350$, $r = 4.5$, and $t = 3$ to find the principal, P.

$$P = \frac{100I}{rt} = \frac{100 \times 1350}{4.5 \times 3} = \$10\,000$$

b Here we are not given the value of the interest I, but the value of the total investment A. We will use the second formula with $A = 5500$, $r = 5$, and $t = 2$ to find the principal, P.

$$P = \frac{A}{\left(1 + \frac{rt}{100}\right)} = \frac{5500}{\left(1 + \frac{5 \times 2}{100}\right)} = \$5000$$

The graphics calculator enables us to investigate simple interest problems using both the tables and graphing facilities of the calculator.

Back to Menu >>>

How to use the graphics TI-Nspire to solve simple interest problems

Suppose we wish to know the length of time it would take for \$40 000 invested at 6.25% interest per annum earn \$20 000 interest.

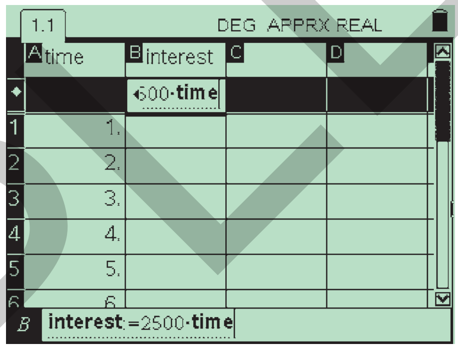
Steps

1 Substitute $P = 40\,000$, $r = 6.25$ in the formula for interest rate.

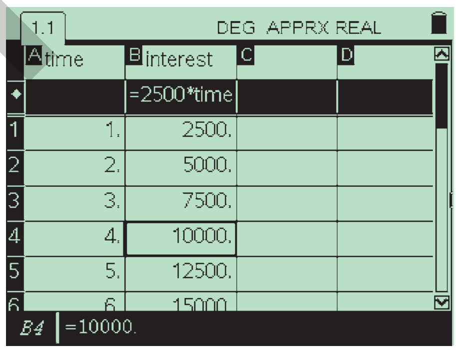
$$I = \frac{Prt}{100} = \frac{40\,000 \times 6.25 \times t}{100}$$
$$= 2500t$$

Method 1: Form a table

2 Start a new document by pressing ctrl + N and select **3:Add Lists & Spreadsheet**. Name the lists *time* (to represent time in years) and *interest*.
Enter the data **1** to **10** into the list named *time*, as shown.
Note: You can also use the sequence command to do this.

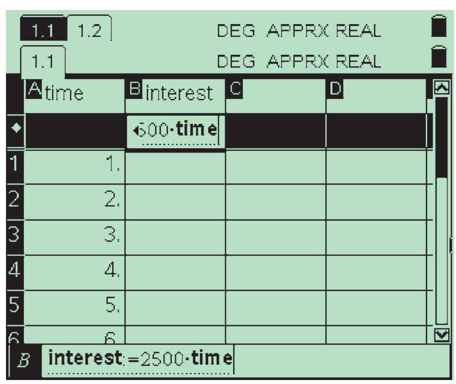


3 Place the cursor in the grey formula cell in the list named *interest* and type $=$ **2500** $\times$ **time**.
Note: You can also use the h key and paste *time* from the variable list
Press enter to display the values as shown.
Scrolling down the table we can see that it takes 4 years to earn \$10 000 interest.



Method 2: Draw a graph

4 Press c/**5:Add Data & Statistics** and construct a scatterplot of *interest* against *time*, as shown.



Notes:

1 To connect the data points, press ctrl + menu and select **2:Connect Data Points**.

2 To display a value, place the cursor on the data point and hold . Press esc before moving to another data point.

We can see that it takes 4 years to earn \$10 000 interest. It is also worth noting that the slope of the line is equal to the interest earned per year.

Note: You can also graph this example in the **Graphs & Geometry** application and use the **Function Table** to answer key questions.

Back to Menu >>>

How to use the graphics ClassPad to solve simple interest problems

Suppose we wish to know the length of time it would take for \$40 000 invested at 6.25% interest per annum earn \$20 000 interest.

Steps

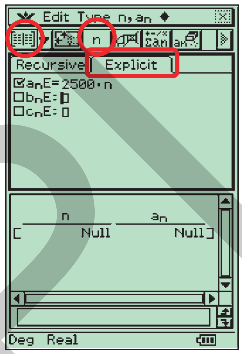
1 Substitute $P = 40\,000$, $r = 6.25$ in the formula for interest rate.

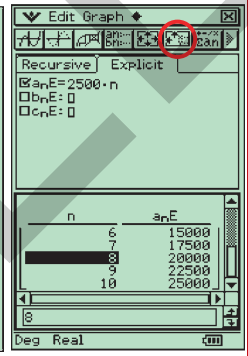
$$I = \frac{Prt}{100} = \frac{40\,000 \times 6.25 \times t}{100}$$
$$= 2500t$$

Method 1: Form a table

2 From the application menu, locate and open the **Sequence** (Sequence) application. Select the **Explicit** tab and type the rule **2500*n*** adjacent to a_n**E**: and press EXE. Tap from the toolbar to view a table of values.

Note: ***n*** is found in the toolbar (n).





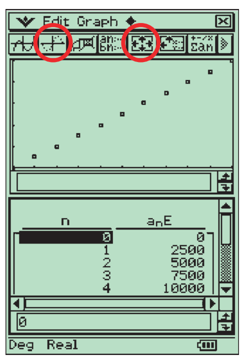
3 Scrolling down the table, it can be seen that the interest of \$20 000 will have been earned after 8 years.

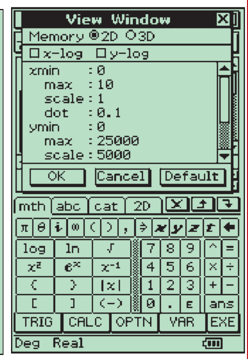
Note: If the table does not give enough values, tap **Sequence TableInput** ( ) and adjust the **Start** and **End** values as required.

Method 2: Draw a graph

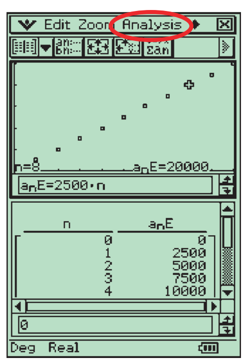
4 Select **Sequence Grapher** ( ) from the toolbar to graph the sequence of simple interest values.

Note: To define the graph window scale select **View Window** ( ) from the toolbar and set the values as shown. Tap **OK** to confirm your settings. Leave the **xdot** and **ydot** settings as they are. These values control the trace increment of the cursor.





5 From the menu bar, select **Analysis** and then **Trace**. This will place a marker on the graph at the first value in the table. Use the cursor arrow keys (i.e. ◀, ▶) to move from one table value to the next.



We can see that it takes 8 years to earn \$20 000 interest. It is also worth noting that the slope of line is equal to the interest earned per year.

Exercise 20B

1 Calculate the simple interest on the following amounts:

a $2000 invested at 6% per annum for four years
b $10 000 invested at 12% per annum for five years
c $8000 invested at 12.5% per annum for three years

2 Calculate the simple interest on the following amounts:

a $10 000 invested at 6% per annum for eight months
b $3500 invested at 10% per annum for 54 months
c $12 000 invested at 12% per annum for $1\frac{1}{4}$ years

3 Find the simple interest that is earned on the following investments:

a $1000 invested for one year at 6% per annum
b $5400 invested for three years at 7% per annum
c $875 invested for three-and-a-half years at 5% per annum

4 A sum of $8500 was invested in a fixed-term deposit account for three years. Calculate the simple interest earned if the rate of interest is 7.9% per annum.

5 Find the amount of interest paid on a personal loan of $7000 taken out at a simple interest rate of 14% per annum over a period of:

a 18 months b two years c three years and 150 days

6 Ben decides to invest his savings of $1850 from his holiday job for five years at 13.25% per annum simple interest.

a How much will he have at the end of this period?
b Use your graphics calculator to sketch a graph of the simple interest earned against time (in years).

7 A loan of $900 is taken out at a simple interest rate of 16.5% per annum.

a How much is owing after four months have passed?
b Use your graphics calculator to sketch a graph of the simple interest paid against time (in years).

8 For each of the following, calculate the interest payable and the balance of the account after one year:

a $500 deposited in a savings account at 3.5% per annum
b $1200 invested in a fixed term deposit account at 5.1% per annum
c $4350 transferred to an advantage saver account at 7% per annum

9 Find the rate of simple interest per annum used in the following investments:

a \$5300 invested for five years and earning \$2119 interest

b \$620 invested for one year and earning \$24 interest

c \$200 500 invested of two-and-a-half years and earning \$30 075 interest

10 To buy his first car, Gary took out a personal loan for \$3500. He paid it back over a period of two years and this cost him \$1085 in interest. At what simple interest rate was he charged?

11 If John invests \$20 000 at 10% per annum until he has \$32 000, for how many years will he have to invest the money?

12 How long will it take for \$17 000 invested at 15% per annum to grow to \$32 300?

13 Mikki decides to put \$6000 in the bank and leave it there until it doubles. If the money is earning simple interest at a rate of 11.5% per annum, how long will this take, to the nearest month?

14 Find the time taken for the following investments to earn the stated amounts of simple interest:

a \$2400 at 12% per annum earns \$175 interest

b \$700 at 4.9% per annum earns \$43 interest

15 Jodie deposited \$1250 in a fixed-term deposit account with a simple interest rate of $6\frac{3}{4}\%$ per annum. She withdrew her money after it had earned \$141.36 in interest. For how many months had her money been invested?

16 Over a period of five years an investment earned \$1070.25 at a simple interest rate of 5.45% per annum. What was the original amount deposited?

17 How much money needs to be invested in order to produce \$725 in interest calculated at 7.5% per annum simple interest over four years?

18 How much money needs to be invested at an interest rate of 3.5% per annum simple interest if you require \$10 000 in three years' time?

19 How much money needs to be invested at an interest rate of 4.25% per annum simple interest if you require \$22 500 in five years' time?

20 How long will it take, to the nearest month, for \$2200, invested at 12. 75% per annum, to double in value?

21 The local store advertises a stereo for \$1095, or \$100 deposit and \$32 per week for two years.

a How much does the stereo end up costing under this scheme?

b What equivalent rate of simple interest is being charged over the two years?

22 A personal loan of \$10 000 over a period of three years is repaid at the rate of \$400 per month.

- a How much money will be repaid in total?
- b What equivalent rate of simple interest is being charged over the three years?

23 Vicki invested \$25 000 in bonds, which return monthly interest at the simple interest rate of 12.0% per annum.

- a What rate is paid each month?
- b How much interest does Vicki receive each month?
- c How much interest does Vicki receive each year?
- d How long does it take for the deposit to pay out \$7500 interest?
- e How much interest does Vicki receive after a period of 10.5 years?

24 Maryanne invested \$50 000 in a bank account which pays annual interest at the simple interest rate of 7% per annum.

- a Draw the graph of the interest earned each year against time (in years).
- b If the interest is paid into the same account, draw the graph of the amount in the account against time (in years).

20.3 Compound interest

Most interest calculations are not as straightforward as simple interest. The more usual form of interest is **compound interest**. It is called compound interest because the interest accumulated each year is added to the principal, and for each subsequent year interest is earned on this total of principal and interest. The interest thus compounds. Consider the situation where \$5000 is invested at 10% interest per annum, and the interest is credited to the account annually:

In the first year:

$$\text{interest} = \$5000 \times 10\% \times 1 = \$500$$

and so at the end of the first year the amount of money in the account is:

$$\$5000 + \$500 = \$5500$$

In the second year:

$$\text{interest} = \$5500 \times 10\% \times 1 = \$550$$

and so at the end of the second year the amount of money in the account is:

$$\$5500 + \$550 = \$6050$$

In the third year:

$$\text{interest} = \$6050 \times 10\% = \$605$$

and so at the end of the third year the amount of money in the account is:

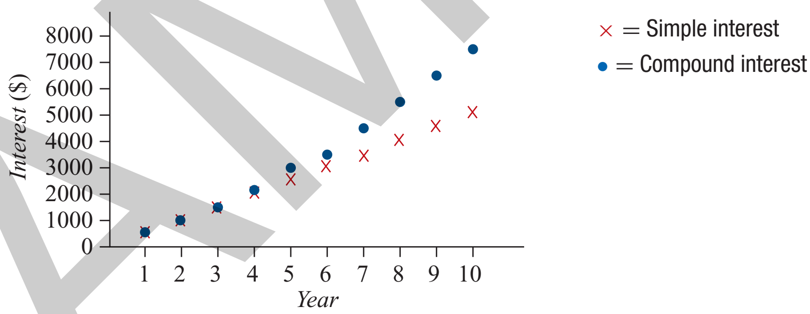
$$\$6050 + \$605 = \$6655$$

Back to Menu >>>

Continuing in this way we can build the following table of values. For comparison, the simple interest values are also shown.

Year (n)	*Simple interest (total)*	*Compound interest (total)*
1	\$500.00	\$500.00
2	\$1000.00	\$1050.00
3	\$1500.00	\$1655.00
4	\$2000.00	\$2320.50
5	\$2500.00	\$3052.55
6	\$3000.00	\$3857.81
7	\$3500.00	\$4743.59
8	\$4000.00	\$5717.94
9	\$4500.00	\$6789.74
10	\$5000.00	\$7968.71

Comparing the interest earned from simple interest to that earned from compound interest we see that after the first year, compound interest is higher, and the advantage to the investor of compound interest over simple interest becomes more obvious as the length of time increases. This advantage can be seen clearly from the graph, which also shows that the shape of the plot of interest against time when the interest is compound is curved, suggesting that this relationship is not linear.



In fact, the amount of money that would accrue in an investment in which interest is compounded follows a geometric sequence, and an expression for this amount can be readily generated using the formula for the nth term of a geometric sequence.

The amount of money (\$$A$) that would result from investing \$$P$ at r% per annum compounded annually for a time period of t years is:

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

To find the amount of interest earned, we need to subtract the initial investment (P) from the final amount (A).

Back to Menu >>>

The interest ($$I$) that would result from investing $$P$ at r% per annum compounded annually for a time period of t years is:

$$I = A - P = P \times \left(1 + \frac{r}{100}\right)^t - P$$

Example 11 **Calculating the investment and interest with interest compounded annually**

a Determine the amount of money accumulated after four years if $10 000 is invested at an interest rate of 9% per annum, compounded annually, giving your answer to the nearest dollar.

b Determine the amount of interest earned.

Solution

a Substitute $P = \$10\,000$, $t = 4$, $r = 9$ in the formula giving the amount of the investment.

$$A = P \times \left(1 + \frac{r}{100}\right)^t = 10\,000 \times \left(1 + \frac{9}{100}\right)^4$$
$$= 10\,000 \times 1.4116$$
$$= \$14\,116 \text{ to the nearest dollar}$$

b Subtract the principal from this amount to determine the interest earned.

$$I = A - P = 14\,116 - 10\,000$$
$$= \$4116$$

Another way of determining compound interest is to enter the appropriate formula into the graphics calculator, and examine the interest earned using the table and graph facilities of the calculator.

How to investigate compound interest problems using the TI-Nspire CAS

Determine the amount of money accumulated after 4 years if $10 000 is invested at an interest rate of 9% per annum, compounded annually. Give your answer to the nearest dollar.

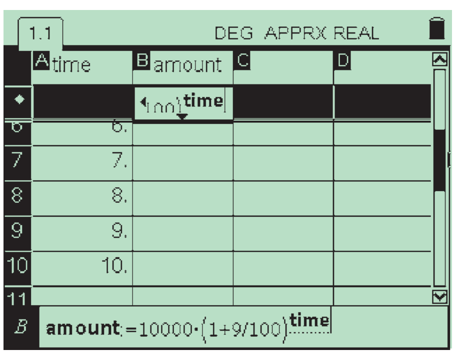
Steps

1 Substitute $P = 10\,0000$, $r = 9$ in the compound interest formula.

$$A = 10\,000 \times \left(1 + \frac{9}{100}\right)^t$$

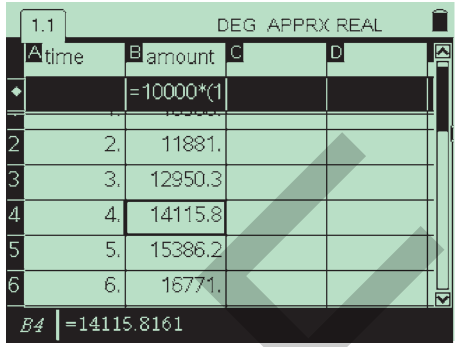
Method 1: Form a table

2 Start a new document by pressing ctrl + N and select **3:Add Lists & Spreadsheet**. Name the lists *time* (to represent time in years) and *amount*. Enter the data **1** to **10** into the list named *time*, as shown.



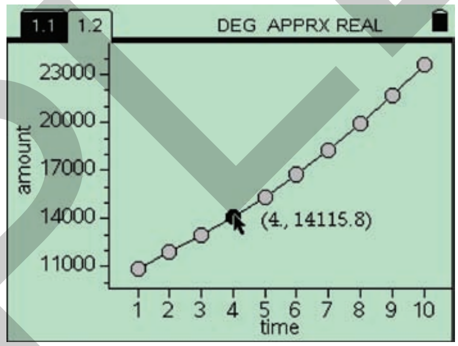
Back to Menu >>>

3 Place the cursor in the grey formula cell in the list named *amount* and type $= \mathbf{10\,0000 \times (1 + 9 \div 100)^{\wedge}}$ time.
Press (enter) to display the values as shown.
Scrolling down the table we can see the amount of money accumulated after 4 years is \$14 116.



Method 2: Draw a graph

4 Press (home) /**5:Add Data & Statistics** and construct a scatterplot of *amount* against *time*, as shown.



Notes:

1 To connect the data points, press (ctrl) + (menu) and select 2:Connect Data Points.

2 To display a value, place the cursor on the data point and hold (click). Press (esc) before moving to another data point.

3 You can use (ctrl) + (menu) and select **1:Zoom/1:Window Settings** and set the **Ymin** to 0 if you prefer.

From the graph, we can see that the amount of money accumulated after 4 years is \$14 116.

How to investigate compound interest problems using the ClassPad

Determine the amount of money accumulated after 4 years if \$10 000 is invested at an interest rate of 9% per annum, compounded annually. Give your answer to the nearest dollar.

Steps

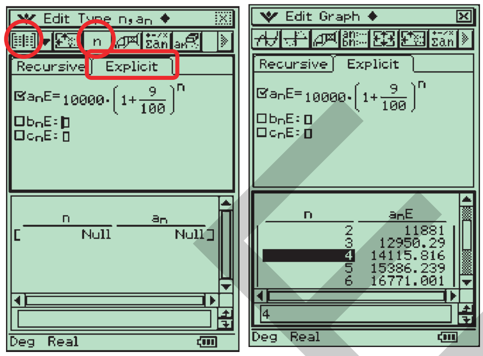
1 Substitute $P = 10\,0000$, $r = 9$ in to the formula for compound interest.

$$A = 10\,000 \times \left(1 + \frac{9}{100}\right)^{t}$$

Back to Menu >>>

Method 1: Form a table

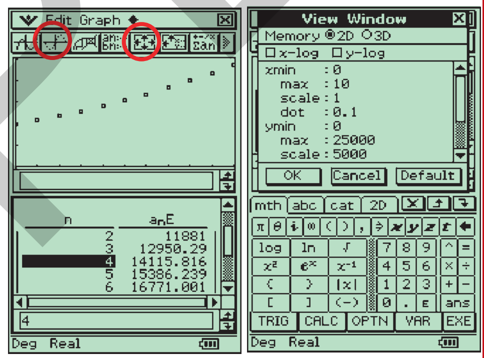
2 From the application menu, locate and open the **Sequence** application. Select the **Explicit** tab and type the rule $\mathbf{10\,000 \times (1 + 9/100)^n}$ adjacent to $\boldsymbol{a_n}$**E**: and press (EXE).
Tap [table icon] from the toolbar to view a table of values.
Notes: ***n*** is found in the toolbar ([n]).



3 Scrolling down the table, it can be seen that, correct to the nearest dollar, an amount of $14 116 will have accumulated after 4 years.

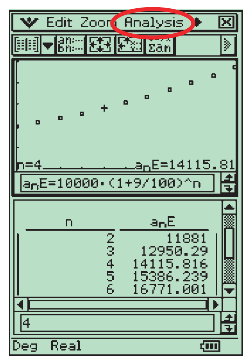
Method 2: Draw a graph

4 **4** Select **Sequence Grapher** ([graph icon]) from the toolbar to graph the sequence of compound interest values.
Notes: To define the graph window scale select **View Window** ([window icon]) from the toolbar and set the values as shown. Tap **OK** to confirm your settings. Leave the ***xdot*** and ***ydot*** settings as they are. These values control the trace increment of the cursor.



5 **5** From the menu bar, select **Analysis** and then **Trace**. This will place a marker on the graph at the first value in the table. Use the cursor arrow keys (i.e. ◀, ▶) to move from one table value to the next.

From the graph, we can see that, correct to the nearest dollar, the amount of money accumulated after 4 years is $14 116.



How would our answer to the previous example change if, instead of adding in the interest at the end of each year (called **compounded annually**), the interest is added to the account every six months (called **compounded half-yearly**)? Compounding 9% half-yearly means that the interest rate for the six-month period is reduced to 4.5%, but the number of time periods increases from four to eight.

Back to Menu >>>

This situation is shown in the table below.

Year	*Compounding annually*	*Compounding half-yearly*
0.5	\$10 000	\$10 450
1	\$10 900	\$10 920
1.5	\$10 900	\$11 412
2	\$11 881	\$11 925
2.5	\$11 881	\$12 462
3	\$12 950	\$13 023
3.5	\$12 950	\$13 609
4	\$14 116	\$14 221

The final entry in the table can be determined by evaluating:

$$10\,000 \times (1 + 0.045)^8 = 14\,221$$

Thus, we need to modify the formula previously stated to take into account situations where interest is compounded, or adjusted, other than annually.

Compound interest

In general:

$$A = P \times \left(1 + \frac{r/n}{100}\right)^{nt}$$

Where $\$A$ = the amount of the investment after t years
$\$P$ = the initial amount
r = the interest rate per annum
n = the number of times per year interest is compounded
t = the number of years

Example 12 Calculating the amount of the investment with interest compounded monthly

Determine the amount accrued if \$2700 is invested at an interest rate of 6% per annum for a period of two years and interest is compounded monthly.

Solution

Substitute $P = 2700$, $r = 6$, $n = 12$ and $t = 2$ in the formula giving the amount of the investment.

$$A = P \times \left(1 + \frac{r/n}{100}\right)^{nt}$$

$$= 2700 \times \left(1 + \frac{6/12}{100}\right)^{12\times 2}$$

$$= \$3043.33$$

Example 13 Calculating the interest on the investment or loan

Determine the interest earned in Example 12.

Solution

Subtract the principal from the amount of the investment to find the interest.

$I = A - P = 3043.33 - 2700$
$= \$343.33$

As was the case with simple interest, we often use the formula for compound interest to find the value of any of the variables in the equation when the values of the other variables are known. However, since the compound interest formula is quite complex, the easiest way to do this is to use the Equation Solver function of the graphics calculator.

How to solve for any variable in the compound interest formula using the TI-Nspire CAS

Suppose an investment of \$2000 has grown to \$2123.40 after 12 months invested at r% per annum compound interest, compounded monthly. Find the value of r, correct to 1 decimal place.

Steps

1 The compound interest formula is

$$A = P\left[1 + \frac{\frac{r}{n}}{100}\right]^{nt}$$

Substitute $P = 2000$, $A = 2123.40$, $n = 12$ and $t = 1$ into this formula.
Use the **solve(** command to solve for r, the annual interest rate.

$$A = P \times \left(1 + \frac{\frac{r}{n}}{100}\right)^{nt}$$

$$2123.40 = 2000 \times \left(1 + \frac{\frac{r}{12}}{100}\right)^{12\times 1}$$

or

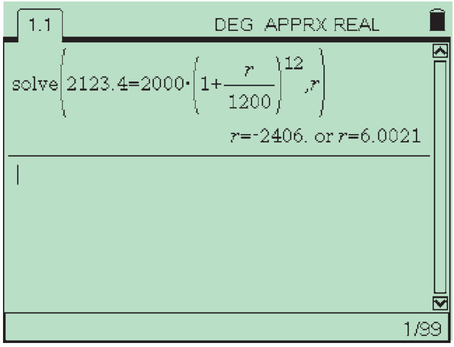
$$2123.40 = 2000 \times \left(1 + \frac{r}{1200}\right)^{12}$$

2 Start a new document by pressing (ctrl) + (N).

   a Select **1:Add Calculator** and press (menu)/**3: Algebra/1:Solve** to paste in the **solve (command**.

   b Complete the command by typing in the following equation to be solved and the unknown (r):

$$\mathbf{2123.40 = 2000\,(1 + r \div 1200)^{12}\,,r}$$



That is, **solve** $\left(\mathbf{2123.40 = 2000\,(1 + r \div 1200)^{12}\,, r}\right)$ Press (enter) to execute the command and display the answer.

Note: Use the ▶ arrow after typing in the **12** to return to the base line to finish typing the entry.

3 Note there are two solutions, $r = -2406$ and $r = 6.002$. The interest rate cannot be negative, so $r = 6.0\%$.

How to solve for any variable in the compound interest formula using the ClassPad

Suppose an investment of \$2000 has grown to \$2123.40 after 12 months invested at r% per annum compound interest, compounded monthly. Find the value of r, correct to 1 decimal place.

Steps

1 The compound interest formula is

$$A = P \times \left[1 + \frac{\frac{r}{n}}{100}\right]^{nt}$$

Substitute $P = 2000$, $A = 2123.40$, $n = 12$ and $t = 1$ into this formula.

Use the **solve(** command to solve for r, the annual interest rate.

$$A = P \times \left(1 + \frac{\frac{r}{n}}{100}\right)^{nt}$$

$$2123.40 = 2000 \times \left(1 + \frac{\frac{r}{12}}{100}\right)^{12\times 1}$$

or

$$2123.40 = 2000 \times \left(1 + \frac{r}{1200}\right)^{12}$$

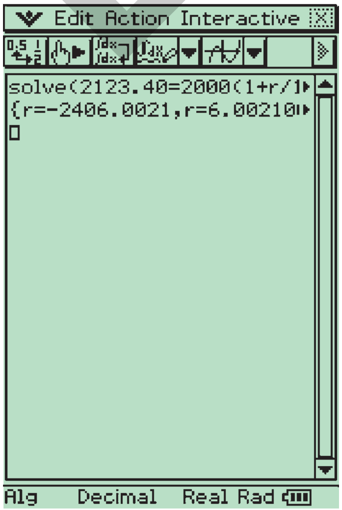
2 Open the built-in **Main** application and tap Keyboard to display the keypad.

Tap **Action/Advanced/solve** in the top bar to paste the **solve(** command onto the screen and complete as shown

solve $(2123.40 = 2000 \times (1 + r/1200)^{12}$

Alternatively, type (and/or tap) in the whole expression directly.

Press EXE to solve.



3 There are two solutions, $r = -2406$ and $r = 6.002$. The interest rate cannot be negative, so $r = 6.0\%$.

Example 14 **Using the solve command to find the initial investment**

How much money must you deposit at 7% per annum compound interest, compounding yearly, if you require \$10 000 in 3 years' time? Give your answer to the nearest dollar.

Back to Menu >>>

Solution

The compound interest formula is

$$A = P \times \left(1 + \frac{\frac{r}{n}}{100}\right)^{nt}$$

1 Substitute $A = 10\,000$, $r = 7$, $n = 1$ and $t = 3$ into this formula.

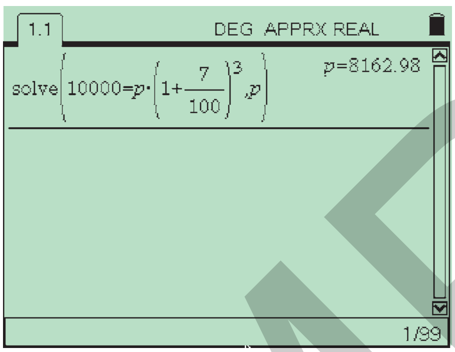
$$A = P \times \left(1 + \frac{\frac{r}{n}}{100}\right)^{nt}$$

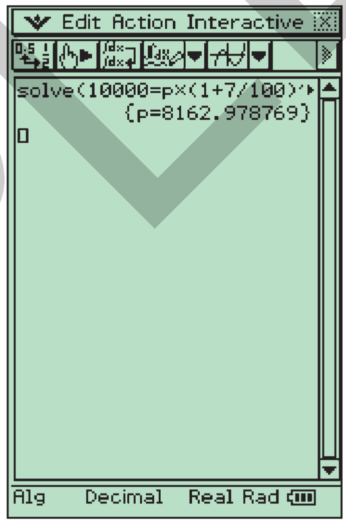
$$10\,000 = P \times \left(1 + \frac{\frac{7}{1}}{100}\right)^{1\times 3}$$

or

$$10\,000 = P \times \left(1 + \frac{7}{100}\right)^{3}$$

2 Use the **solve(** command to solve for P (the principal).





3 Write the answer.

Answer: Deposit $8163

Example 15 Using the solve command to find a time period

How long, to the nearest year, will it take for an investment of $1000 to reach $1873 if it is invested at 9% per annum compounded monthly?

Solution

The compound interest formula is

$$A = P \times \left(1 + \frac{\frac{r}{n}}{100}\right)^{nt}$$

$$A = P \times \left(1 + \frac{\frac{r}{n}}{100}\right)^{nt}$$

1 Substitute $A = 1873$, $P = 1000$, $r = 9$, and $n = 12$ into the formula.

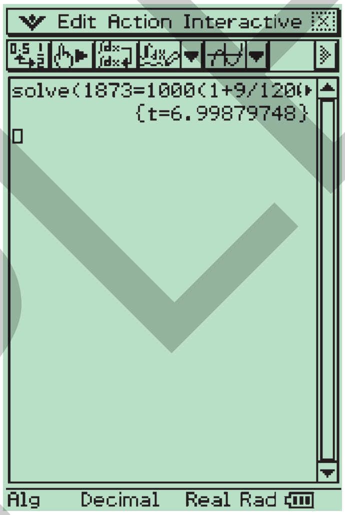
$$1873 = 1000 \times \left(1 + \frac{\frac{9}{12}}{100}\right)^{12 \times t}$$

or

$$1873 = 1000 \times \left(1 + \frac{9}{1200}\right)^{12t}$$

2 Use the **solve (** command to solve for t.

Hint for ClassPad:
When entering the $12t$ term, place brackets around it $(12t)$; hence, enter the expression as
solve(1873 = 1000 × (1 + 9/1200)$^{(12t)}$)



3 Write the answer.

Answer: Invest for 7 years.

Exercise 20C

1 Calculate the compound interest for the following:
- **a** \$2000 invested at 6% per annum for four years
- **b** \$10 000 invested at 12% per annum for five years
- **c** \$8000 invested at 12.5% per annum for three years

2 How much money would be in an account after five years if \$3000 is invested at 10% per annum compounded annually?

3 How much interest is earned if \$3300 is invested for 10 years at 7.5% per annum compounded annually?

4 Compare the interest earned with both simple and compound interest if \$4500 is invested at 11% per annum for six years. What is the amount of the difference?

5 **a** Find the amount of money in the following accounts after one year if the initial investment of $1000 is invested at 7% per annum, compounded:

i annually **ii** quarterly **iii** monthly **iv** weekly **v** daily

b What do you notice about your answers in **a**?

6 Calculate the amount of compound interest earned (to the nearest cent) if:

a $7000 is invested for four years at 8% per annum compounded annually

b $2995 is invested for 20 years at 7.2% per annum compounded monthly

7 **a** Find the total amount owed on a loan of $850 borrowed at 13.25% per annum compound interest adjusted weekly for six months.

b Use your graphics calculator to construct a graph of the amount owed against time (in years).

8 Philip wishes to invest $1500 for five years. Which of the following is his best option?

a 11% per annum, interest adjusted weekly

b 11.75% per annum, interest adjusted quarterly **c** 12.5% per annum, simple interest

9 Ilana uses her credit card to buy a dress costing $300, knowing that she will not be able to pay it off for some time. If she is charged interest on the amount for six months, how much will the dress finally cost her? (Assume interest is charged at 18% per annum, compounded monthly, and that no payments are made during the 6-month period.)

10 A CD player, priced at $1299, is advertised in a sale with a 20% discount. Unfortunately, Sam does not have the money to buy it, so he puts it on his credit card.

a If Sam pays interest on the CD player for four months, how much will it cost him? (Assume interest is charged at 18% per annum, compounded monthly and that no payments are made during the 4-month period.)

b Would Sam have been better off to save and buy the CD player at the full price?

11 Peter places $3000 in an account that pays 5.65% per annum compounded weekly.

a How much will he have in the account in five years' time?

b Use your graphics calculator to construct a graph of the amount owed against time (in years).

12 How much money must you deposit at a fixed rate of 6.8% per annum compounded yearly if you require $12 000 in four years' time?

13 Singh wants to buy a boat in five years' time. He estimates that it will cost him $15 000. His bank offers him an interest rate of 6.25% per annum compounded yearly for the 5-year term.

a How much money should he invest now in order to have sufficient funds to buy the boat in five years' time?

b If the interest was added daily, how much less would he need to invest?

14 What initial investment is required to produce a final amount of \$30 000 in 30 months' time, given that an interest rate of 9.7% per annum compounded quarterly is guaranteed?

15 On the birth of his granddaughter a man invests a sum of money at a fixed rate of 11.65% per annum compounded twice a year. On her 21st birthday he gives all the money in the account to his granddaughter. If she receives \$2695.55, how much did her grandfather invest?

16 Sarah invested \$3500 at 6.75% per annum compound interest adjusted monthly. If the investment now amounts to \$5241.61, for how many years was it invested?

17 If a principal of \$6000 is invested at 5.25% per annum, compounded half-yearly, and the amount due is \$7774.69, for how many years was it invested?

18 How long would it take for \$200 to exceed \$20 000 if it was invested at 4.75% per annum compounded yearly?

19 In which year will \$1000 first exceed \$2000 if invested at:

a 10% per annum, compounded quarterly? **b** 12% per annum, compounded quarterly?

20 How long (to the nearest year) will money take to double if it is invested at:

a 5% per annum compounded monthly? **b** 6% per annum compounded monthly?

21 Suppose an investment of \$1000 has grown to \$1051.16 after 12 months invested at r% per annum compound interest compounded monthly. Find the value of r.

22 Geoff invests \$18 000 in an investment account. After 2 years the investment account contains \$19 299.27. If the account pays r% interest per annum compounded quarterly, find the value of r, to one decimal place.

23 Bob is trying to plan for his retirement. He is looking for an investment. Investing in alpacas has been suggested as a possible alternative, and the advertising literature suggest that Bob will double his investment in 3 years.

a Suppose Bob has \$30 000 to invest. What annual interest rate would Bob need to get from the bank to achieve the same return that is suggested from the alpaca investment, to the nearest whole number? (Assume that the bank would pay interest compounded monthly.)

b Suppose Bob has \$50 000 to invest. Does this change your answer to part **a**?

Back to Menu >>>

20.4 Reducing balance loans

Excel
Excel
PowerPoint

In the previous section we looked at the effect of compound interest on the value of a loan over a period of time. These calculations were based on the premise that no repayments were made during the period of the loan, and that the whole sum was due at end of the time period. Of course this is not how most loans are actually established. Usually, regular repayments are made so that the loan, plus interest, is repaid over a specified time period. This is illustrated in the following example.

Example 17 **Determining the balance when regular payments are made**

Jim borrows $4000 from the bank, at an interest rate of 12% per annum compounded monthly. He makes regular monthly repayments of $500. How much of the principal has he repaid after three months?

Solution

We will set up a table to help solve this problem.

End of month	Interest ($)	Repayment ($)	Balance of loan ($)
1	40.00	500	3540.00
2	35.40	500	3075.40
3	30.75	500	2606.15

After three months Jim has paid $ 4000 - $ 2606.15 = $ 1393.85 off the principal.

Exercise 20D

Skillsheet
TEACHER

1 A loan of $12 000 is to be repaid in instalments of $2700 per year with an interest rate of 16% per annum being charged on the unpaid balance at the end of each year. After four repayments, calculate:

a the amount still owing   b the interest charged to date

2 Interest is charged on a loan of $6450 at the rate of 1.5% per month on the outstanding balance before monthly repayments of $225 are made. After five repayments calculate:

a the amount still owing   b the interest charged to date

3 Jack takes out a personal loan of $25 000 to help finance the building of his holiday house. The terms of his loan specify monthly repayments of $595 over five years, with 15% per annum interest calculated monthly. By calculating the amount still owing on the loan after each month find:

a the amount still owing after five months
b the interest charged over the 5-month period

Back to Menu >>>

4 For the loan described in Question 3, find the amount still owing after five months and the interest charged if Jack instead pays monthly repayments of:

a i $400 ii $700

b Which is the better repayment plan for Jack? Explain your answer.

5 A family borrows $110 000 to buy a house. The monthly repayments on the loan are $840 over 20 years, with 7.5% per annum interest calculated monthly. Find:

a the amount still owing after four months

b the interest charged over the 4-month period

6 For the loan described in Question 5, find the amount still owing after four months and the interest charged if the family instead pays monthly repayments of:

a $750 b $900

7 Shelly buys some track shoes for $149.99. She puts the shoes on her credit card because she cannot afford to pay for them outright.

a If Shelly pays interest on the shoes for three months, how much are they now costing her? (Assume the interest is charged at 18% per annum, compounded monthly, and that no payments are made over the 3-month period.)

b If Shelly had paid $50 off the principal amount at the end of the first and second months, after the interest had been added, how much would the final payment have been at the end of the third month? How much would Shelly save by doing this, rather than by adopting the first method?

MC TEST D&D TEST

Review

Key ideas and chapter summary

Percentage increase or decrease

A **percentage increase** or **decrease** is the amount of the increase or decrease expressed as a percentage of the original value.

$$\text{percentage change} = \frac{\text{amount of change}}{\text{original value}} \times \frac{100}{1}$$

Simple interest

Simple interest is the interest (I) paid on an investment or loan on the basis of the original amount invested or borrowed, which is called the principal (P). The amount of the simple interest is constant from year to year, and thus is linearly related to the term of the investment. It is given by

$$I = \frac{Prt}{100}$$

where P is the amount invested or borrowed, r is the interest rate per annum and t is the time in years.

Amount of the investment or loan (simple interest)

The total **amount** owed or invested (\$$A$) after the interest has been added is given by:

$$A = P + I = P + \frac{Prt}{100}$$

where P is the amount invested of borrowed, r is the interest rate and t is the time (in years).

Compound interest

Compound interest is where the interest paid on a loan or investment is credited or debited to the account and the interest for the next period based on the sum of the principal and previous interest. The amount of the compound interest increases each year, and thus there is a non- linear relationship between compound interest and the term of the investment. The interest is given by:

$$I = A - P = P \times \left(1 + \frac{r/n}{100}\right)^{nt} - P$$

where \$$A$ is the amount of the investment after t years, \$$P$ is the principal, r is the interest rate per annum and n is the number of times per year interest is compounded.

Amount of the investment or loan (compound interest)

The total **amount** owed or invested (\$$A$) after the interest has been added is given by:

$$A = P \times \left(1 + \frac{r/n}{100}\right)^{nt}$$

(cont'd.)

	where \$$A$ is the amount of the investment after t years, \$$P$ is the principal, r is the interest rate per annum and n is the number of times per year interest is compounded.
Reducing balance loan	A **reducing balance loan** is a loan which attracts compound interest, but where regular repayments are also made, so that in most instances the amount of the loan and interest are eventually repaid in full.

Skills check

Having completed this chapter you should be able to:

- calculate the amount of the discount and the new price when an r% discount is applied
- calculate the amount of the increase and the new price when an r% increase is applied
- calculate the percentage discount or increase applied given the old and new prices
- calculate the original price given the new price and the percentage discount or increase which has been applied
- use a form of the formula for simple interest to find the value of any one of the variables I, P, r, or t when the values of the other three are known
- calculate the amount of an investment after simple interest has been added
- plot the value of the simple interest (I) against time (t) to show a linear relationship
- use Equation Solver to find the value of any variable in the compound interest formula when the values of the other four are known
- plot the value of the compound interest (I) against time (t) to show a non-linear relationship
- use a table to determine the outstanding balance of a loan which is attracting compound interest from first principals when a small number of payments have been made

Multiple-choice questions

1 If the price of a TV set is reduced from \$598 to \$485 in a sale, then the percentage discount which has been applied is closest to:

A 19% **B** 81% **C** 23% **D** 77% **E** 113%

2 The original price of an item which is priced at \$375 after having been marked down by 25% is closest to:

A \$281.25 **B** \$500 **C** \$300 **D** \$1500 **E** \$214.29

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Back to Menu >>>

3 If interest of \$4687.50 is paid on an investment of \$25 000 after 5 years then the rate of simple interest paid per annum is closest to:

A 3.75% **B** 18.75% **C** 3.44% **D** 2.5% **E** 4.5%

4 Susan invests \$3000 at 4.9% simple interest per annum for 3 years. The amount of interest she will earn is

A \$147.00 **B** \$3441.00 **C** \$474.02 **D** \$3474.02 **E** \$441.00

5 In 2004 a company charged \$55.00 per hour for accounting services. In 2005 the company increased their fees by 15%. By how much did this increase the price of a 2-hour consultation?

A \$8.25 **B** \$63.25 **C** \$126.50 **D** \$16.50 **E** \$1.65

6 Sandra invests \$6000 at 4.75% per annum compounding monthly. After two years, the value of her investment will be closest to:

A \$570 **B** \$6570 **C** \$6597 **D** \$597 **E** \$6584

7 The amount which should be invested at 5% per annum simple interest, if you require \$20 000 in three years' time, is closest to:

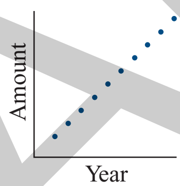
A \$17 391 **B** \$19 265 **C** \$17 230 **D** \$23 215 **E** \$17 235

8 The amount of money which should be invested at 5% per annum compound interest, compounding quarterly, if you require \$20 000 in three years' time, is closest to:

A \$17 391 **B** \$19 265 **C** \$17 230 **D** \$23 215 **E** \$17 235

9 An investment of \$50 000 is made at a fixed rate of interest compounding quarterly over a number of years. Which graph best represents the amount of the investment at the end of each year?

A



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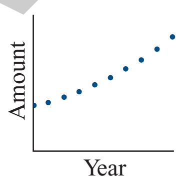

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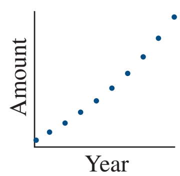

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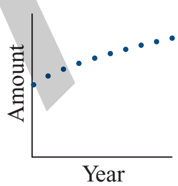
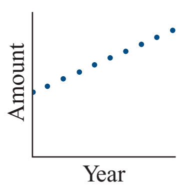


E



Questions 10 and 11 relate to the following information

Lauren wants to buy a dress which is priced at \$250. The day she goes to buy the dress the store is having a sale, and everything is marked down by 20%. She is also entitled to a further staff discount of 12.5% on the reduced price.

10 The price she pays for the dress that day is closest to:

A \$168.75 **B** \$217.50 **C** \$231 **D** \$225 **E** \$175

11 The total discount that Lauren receives on the original price is closest to:

A 32.5% **B** 30% **C** 10% **D** 7.5% **E** 2.5%

12 The number of years it would take $4500 at 5.75% per annum compound interest adjusted monthly to grow to $10 000 is closest to:

A 6 **B** 7 **C** 13 **D** 14 **E** 21

13 Sandra invests $10 000 at 5.3% per annum compounding monthly. The amount of interest she earns in the 6th year of the investment is closest to:

A $707.43 **B** $13 026.71 **C** $13 734.14 **D** $543.07 **E** $3734.14

Questions 14 and 15 relate to the following information

Interest is charged on a loan of $17540 at the rate of 0.75% per month on the outstanding balance before monthly repayments of $225 are made. The table summarises the situation after three payments have been made:

End of month	*Interest* ($)	*Repayment* ($)	*Balance of loan* ($)
1	131.55	225	17 446.55
2	130.85	225	17 352.40
3	a	225	b

14 The value of a in the above table is closest to:

A $130.14 **B** $130.85 **C** $131.55 **D** $130.10 **E** $121.47

15 The value of b in the above table is closest to:

A $17 222.26 **B** $17 447.26 **C** $17 257.54 **D** $17 127.40 **E** $16 997.26

Extended-response questions

1 **a** The wholesale price of a lounge suite is $650. The maximum profit a retailer is allowed to make when selling this particular suite is 85% of the wholesale price. Calculate the maximum retail price of the suite.

b Suppose that the wholesale price of the lounge suite increases at 5% per annum simple interest for the next five years.

i By how much will the wholesale price have increased at the end of five years?

ii What is the new wholesale price of the lounge suite?

c Suppose that the price of a different lounge suite is $1500. If the retailer is making 65% profit on the suite, what is its wholesale price (give your answer to the nearest dollar)?

2 **a** Meredith wanted to buy a television set on sale for $940. She worked out that she could repay the purchase price, plus a total interest charge of $220, in eight equal quarterly instalments.

i How much will she have to repay per quarter?

ii What is the annual simple interest rate that she has been charged? Give your answer correct to one decimal place.

b Meredith decides instead to borrow the \$940 from a friend. She agrees to repay the money, plus interest compounded quarterly, in one lump sum at the end of two years. If the interest charge amounted to \$220, calculate, correct to one decimal place, the quarterly compound interest rate charged by her friend.

3 Sergei has \$5000 he wishes to invest for 5 years.

a Company A offers him 6.3% per annum simple interest. How much will he have at the end of the five years under this plan?

b Company B offers him 6.1% per annum compound interest compounding monthly. How much will he have at the end of the five years under this plan?

c Find, correct to one decimal place, the simple interest rate Company A should offer if the two investments are to be equal after five years?

4 The car that Andrew wants to buy has a retail price of \$45 000. He arranges to buy the car for 24 equal monthly instalments of \$2125.

a What is the total price that Andrew pays for the car?

b What is the annual simple interest rate that he has been charged on the purchase price? Give your answer correct to one decimal place.

c What is the annual compound interest rate that he has been charged on the purchase price, if interest is compounded per annum? Give your answer correct to one decimal place.

5 Suppose that you have \$50 000 to invest, and there are two alternative plans for investment:

Plan A offers 6.4% per annum simple interest.

Plan B offers 6.0% per annum compound interest compounding annually.

a Use your graphics calculator to construct a graph of the interest earned under Plan A against time.

b On the same axis, use your graphics calculator to construct a graph of the interest earned under Plan B against time.

c If you wish to invest for three years, which of the plans would you choose, A or B?

6 Brett takes out a personal loan of \$36 000 to buy a boat. The terms of his loan specify monthly repayments of \$595, with 8% per annum interest calculated monthly.

a By calculating the amount still owing on the loan after each month find:

i the amount still owing after three months

ii the interest charged over the 3-month period

iii the amount still owing after three months and the total interest charged over the three months if Brett instead pays monthly repayments of \$200.

b Which is the better repayment plan? Explain your answer.