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C H A P T E R

14

MODULE 2

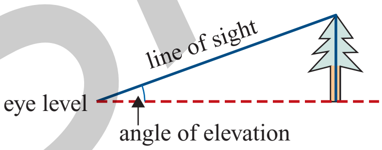
Applications of geometry and trigonometry

- How do we apply trigonometric techniques to problems involving angles of depression and elevation?
- How do we apply trigonometric techniques to problems involving bearings?
- How do we apply trigonometric techniques to three-dimensional problems?
- How do we draw and interpret contour maps?
- How do we draw and interpret scale drawings?

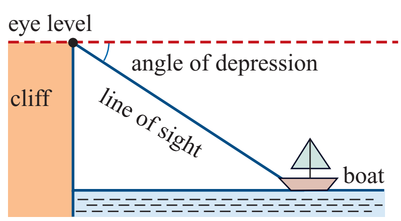
14.1 Angles of elevation and depression, bearings, and triangulation

Angles of elevation and depression

The **angle of elevation** is the angle between the horizontal and a direction above the horizontal.



The **angle of depression** is the angle between the horizontal and a direction below the horizontal.



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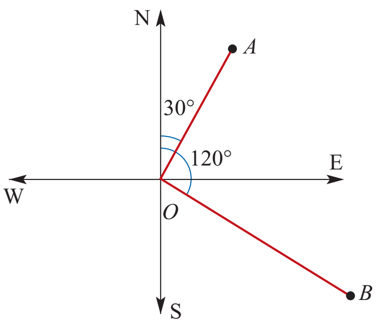
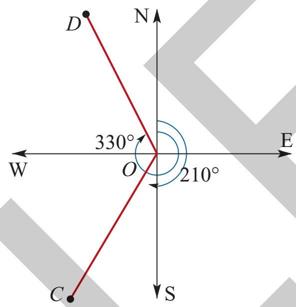
Bearings

The **three-figure bearing** (or compass bearing) is the direction measured clockwise from north.

The bearing of A from O is 030°.
The bearing of B from O is 120°.

The bearing of C from O is 210°.
The bearing of D from O is 330°.

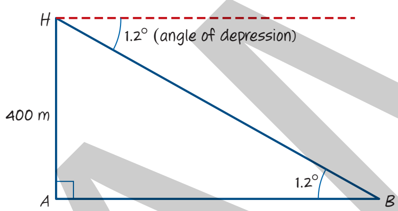




Example 1 Angle of depression

The pilot of a helicopter flying at 400 m observes a small boat at an angle of depression of 1.2°. Draw a diagram and calculate the horizontal distance of the boat to the helicopter correct to the nearest 10 metres.

Solution



$$\frac{AH}{AB} = \tan 1.2°$$

$$\therefore \frac{400}{AB} = \tan 1.2°$$

$$AB = \frac{400}{\tan 1.2}$$

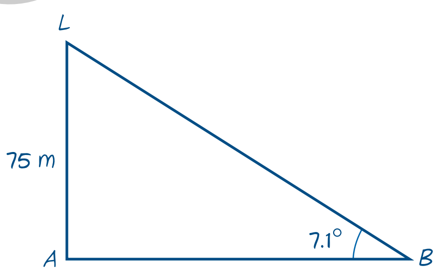
$$AB = 19\,095.80056\ldots$$

The horizontal distance is 19 100 m to the nearest 10 metres.

Example 2 Angle of elevation

The light on a cliff-top lighthouse, known to be 75 m above sea level, is observed from a boat at an angle of elevation of 7.1°. Draw a diagram and calculate the distance of the boat from the lighthouse to the nearest metre.

Solution



$$\frac{75}{AB} = \tan 7.1°$$

$$\therefore AB = \frac{75}{\tan(7.1°)}$$

$$= 602.135\ldots$$

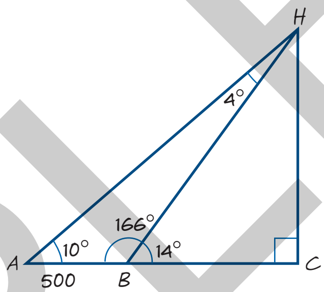
The distance of the boat from the lighthouse is 602 m to the nearest metre.

Example 3 Applying geometry and trigonometry with angle of elevation

From a point A, a man observes that the angle of elevation of the summit of a hill is 10°. He then walks towards the hill for 500 m along flat ground. The summit of the hill is now at an angle of elevation of 14°. Draw a diagram and find the height of the hill above the level of A to the nearest metre.

Solution

1 Draw a diagram.



2 Find all the unknown angles that will be required. This is done using properties of angles discussed in Chapter 12.

The magnitude of angle HBA = (180 − 14)° = 166°.
The magnitude of angle AHB = 180 − (166 + 10) = 4°.

3 You choose to work in particular triangles. In this case it is triangle ABH.

Using the sine rule in triangle ABH:

$$\frac{500}{\sin 4^\circ} = \frac{HB}{\sin 10^\circ}$$

$$\therefore HB = \frac{500 \times \sin 10^\circ}{\sin 4^\circ} = 1244.67\ldots$$

4 The information found in triangle ABH is the length HB. This can now be used to find HC in triangle BCH.

In triangle BCH:

$$\frac{HC}{HB} = \sin 14^\circ$$

$$\therefore HC = HB \sin 14^\circ = 301.11\ldots$$

5 Write down your answer.

The height of the hill is 301 m to the nearest metre.

Example 4 Bearings and Pythagoras' theorem

The road from town A runs due west for 14 km to town B. A television mast is located due south of B at a distance of 23 km. Draw a diagram and calculate the distance of the mast from the centre of town A to the nearest kilometre. Find the bearing of the mast from the centre of the town.

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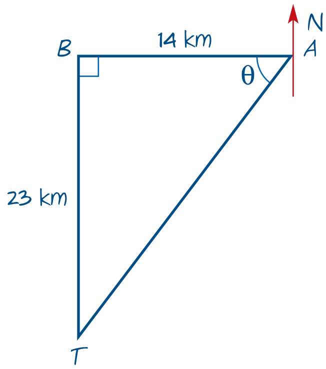
Solution



$$\tan\theta = \frac{23}{14}$$

$$\therefore\ \theta = 58.67^\circ \text{ (to two decimal places)}$$

$$\therefore\ \text{bearing} = 180^\circ + (90 - 58.67)^\circ$$

$$= 211.33^\circ$$

By Pythagoras' theorem

$$AT^2 = AB^2 + BT^2$$

$$= 14^2 + 23^2$$

$$= 725$$

$$\therefore\ AT = 26.925\ldots$$

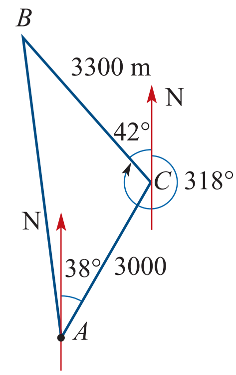
$\therefore$ The mast is 27 km from the centre of town A (to the nearest kilometre) and on a bearing of 211.33°.

Example 5 Bearings and the cosine and sine rules

A yacht starts from a point A and sails on a bearing of 038° for 3000 m. It then alters its course to a bearing of 318°, and after sailing for 3300 m it reaches a point B.

a Find the distance AB correct to the nearest metre.

b Find the bearing of B from A correct to the nearest degree.

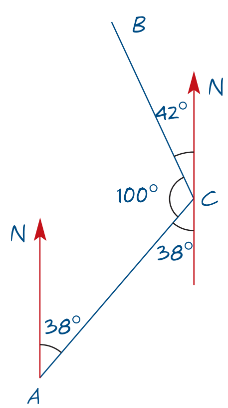


Solution

1 To find the distance AB, the magnitude of angle ACB needs to be determined so that the cosine rule can be applied in triangle ABC.

The magnitude of angle ACB
$= [180 - (38 + 42)]^\circ = 100^\circ$

a



2 Apply the cosine rule in triangle *ABC*.

$AB^2 = 3000^2 + 3300^2 - 2 \times 3000 \times 3300 \times \cos(100^\circ) = 23\,328\,233.92$

$\therefore AB = 4829.93104\ldots$

3 Write down your answer.

The distance of B from A is 4830 m (to the nearest metre).

To find the bearing of *B* from *A*, the magnitude of angle *BAC* must first be found. The sine rule can be used.

b $\dfrac{3300}{\sin A} = \dfrac{AB}{\sin 100^\circ}$

$\therefore \sin A = \dfrac{3300 \times \sin 100^\circ}{AB}$

$\therefore \sin A = 0.6728\ldots$

$\therefore A = (42.288\ldots)^\circ$

$\therefore$ The bearing of B from A

$= 360^\circ - (42.29^\circ - 38^\circ)$

$= 355.71^\circ$

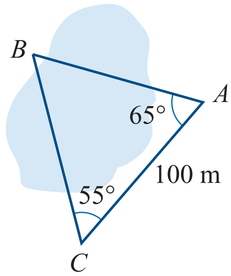
The bearing of B from A is 356° to the nearest degree.

Triangulation

Surveyors sometimes need to measure distances to inaccessible points, or measure lengths that are impossible to measure directly. The method of **triangulation** involves using the theory of solving triangles developed in this module.

Example 6 Triangulation

Two points *A* and *B* are on opposite sides of a lake so that the distance between them cannot be measured directly. A third point, *C*, is chosen at a distance of 100 m from *A* and with angles *BAC* and *BCA* of 65° and 55° respectively. Calculate the distance between *A* and *B* correct to two decimal places.



Solution

The magnitude of angle ABC is 60°. Using the sine rule for triangle ABC

$\dfrac{100}{\sin 60^\circ} = \dfrac{AB}{\sin 55^\circ}$

$\therefore AB = \dfrac{100}{\sin 60^\circ} \times \sin 55^\circ$

$= 94.587\ldots$

The length of AB is 94.59 m (correct to two decimal places).

Skillsheet TEACHER

Exercise 14A

1 The angle of elevation of the top of an old chimney stack at a point 40 m from its base is 41°. Find the height of the chimney.

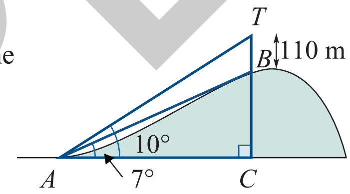
2 A ship sails 10 km north and then 15 km east. What is its bearing from the starting point?

3 From the top of a vertical cliff 130 m high the angle of depression of a buoy at sea is 18°. What is the distance of the buoy from the foot of the cliff?

4 The bearing of a point A from a point B is 207°. What is the bearing of B from A?

5 A man standing on top of a mountain observes that the angle of depression to the foot of a building is 41°. If the height of the man above the foot of the building is 500 m, find the horizontal distance from the man to the building.

6 A tower 110 m high stands on the top of a hill. From a point A at the foot of the hill the angle of elevation of the bottom of the tower is 7°, and that of the top is 10°.

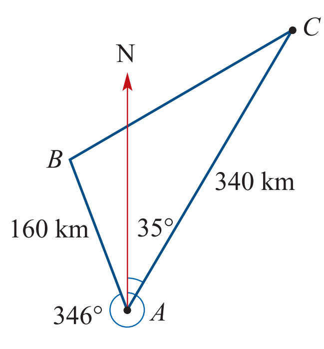


- **a** Find the magnitude of angles TAB, ABT and ATB.
- **b** Use the sine rule to find the length AB.
- **c** Find CB, the height of the hill.

7 The bearing of a ship S from a lighthouse A is 055°. A second lighthouse B is due east of A. The bearing of S from B is 302°. Find the magnitude of angle ASB.

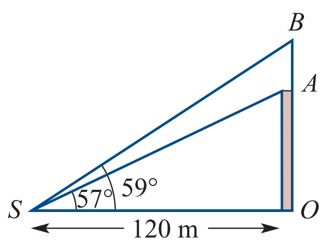
8 A yacht starts from L and sails 12 km due east to M. It then sails 9 km on a bearing of 142° to K. Find the magnitude of angle MLK.

9 The bearing of C from A is 035°.
The bearing of B from A is 346°.
The distance of C from A is 340 km.
The distance of B from A is 160 km.



- **a** Find the magnitude of angle BAC.
- **b** Use the cosine rule to find the distance of B to C.

10 Point S is at a distance of 120 m from the base of a building.
On the building is an aerial, AB.
The angle of elevation from S to A is 57°.
The angle of elevation from S to B is 59°.



- **a** Find the distance OA.
- **b** Find the distance OB.
- **c** Find the distance AB.

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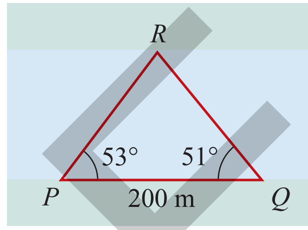
11 A man lying down on top of a cliff 40 m high observes the angle of depression to a buoy in the sea below to be 20°. If he is in line with the buoy, calculate the distance between the buoy and the foot of the cliff, which may be assumed to be vertical.

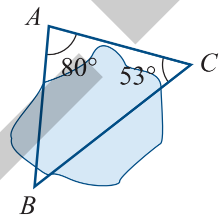
12 *P* and *Q* are points on the bank of a river. A tree is at a point *R* on the opposite bank such that ∠*QPR* is 53°. and ∠*RQP* is 51°.



a Find: **i** *RP* **ii** *RQ*

b *T* is a point between *P* and *Q* such that ∠*PTR* is a right angle. Find *RT* and hence the width of the river correct to two decimal places.

13 Two points *A* and *B* are on opposite sides of a lake so that the distance between them cannot be measured directly. A third point, *C*, is chosen at a distance of 300 m from *A* and with angles *BAC* and *BCA* of 80° and 53° respectively. Calculate the distance between *A* and *B*.



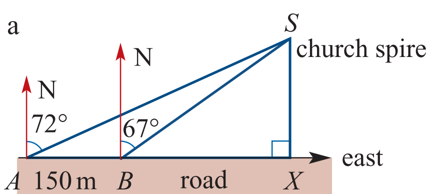
14 A man standing on top of a cliff 50 m high is in line with two buoys whose angles of depression are 18° and 20°. Calculate the distance between the buoys.

15 From a ship *S*, two other ships *P* and *Q* are on bearings 320° and 075° respectively. The distance *PS* = 7.5 km and the distance *QS* = 5 km. Find the distance *PQ*.

16 A ship leaves port *A* and steams 15 km due east. It then turns and steams for 22 km due north.

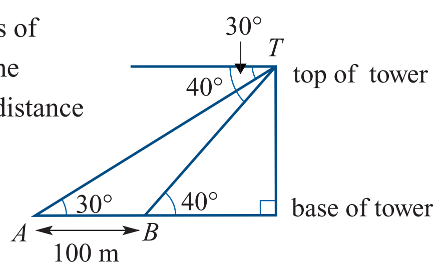
a What is the bearing of the ship from *A*?
b What is the bearing of port *A* from the ship?

17 A man walking due east along a level road observes a church spire from point *A*. The bearing of the spire from *A* is 072°. He then walks 150 m to point *B* where the bearing is 067°.



a Find the distance of the church spire from *B* (i.e. *BS*).
b Find the distance of the church spire from the road (i.e. *SX*).

18 From the top of a communications tower, the angles of depression of two points *A* and *B* on a horizontal line through the foot of the tower are 30° and 40°. The distance between the points is 100 m. Find:



a the distance *AT*
b the distance *BT*
c the height of the tower.

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19 A yacht starts from point A and sails on a bearing of 035° for 2000 m. It then alters its course to one in a direction with a bearing of 320° and after sailing for 2500 m it reaches point B.

a Find the distance AB. **b** Find the bearing of B from A.

14.2 Problems in three dimensions

Problems in three dimensions are solved by picking out triangles from a main figure and finding lengths and angles through these triangles.

Example 7 Applications in three dimensions

$ABCDEFGH$ is a cuboid. Find:

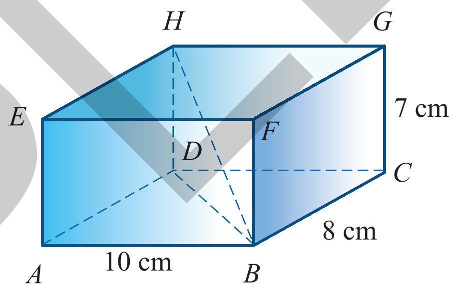
a distance DB

b distance HB

c the magnitude of angle HBD

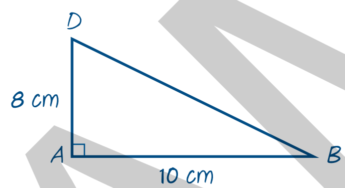
d distance HA

e the magnitude of angle HBA



Solution

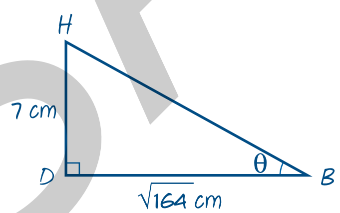
a *Strategy:* You always work in triangles to obtain values. Here we start with triangle ABD.



$$DB^2 = 8^2 + 10^2$$
$$= 164$$
$$\therefore DB = \left(\sqrt{164}\right)$$
$$\therefore DB = 12.806\ldots$$

The length of DB is 12.81 cm correct to two decimal places.

b *Strategy:* Now use the information from **a** in triangle BDH.



$$HB^2 = HD^2 + DB^2$$
$$= 7^2 + \left(\sqrt{164}\right)^2$$
$$= 213$$
$$\therefore HB = \sqrt{213}$$
$$= 14.59\ldots$$

The length of HB is 14.59 cm correct to two decimal places.

c *Strategy*: Triangle BDH is again used.

$$\text{Tan } \theta = \frac{HD}{BD} = \frac{7}{\sqrt{164}} = 0.5466\ldots$$

$\theta = 28.66°$ correct to two decimal places.

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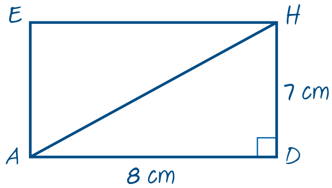
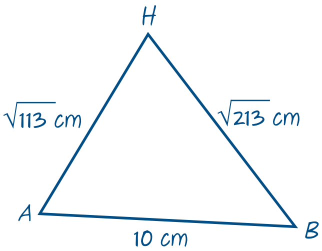
d *Strategy*: Triangle *ABH* is used to find *HA*.



$$HA^2 = 8^2 + 7^2$$
$$= 64 + 49$$
$$= 113$$
$$\therefore\ HA = \sqrt{113}$$

e *Strategy*: Apply the cosine rule to triangle *ABH* to find the magnitude of angle *HBA*.



$$HA^2 = AB^2 + HB^2 - 2 \times AB \times HB \cos B$$
$$113 = 100 + 213 - 2 \times 10 \times \sqrt{213} \cos B$$
$$\therefore\ \cos B = \frac{-200}{-20\sqrt{213}}$$
$$\therefore\ \cos B = 0.68518\ldots$$
$$B = 46.75^\circ \text{ (correct to two decimal places)}$$

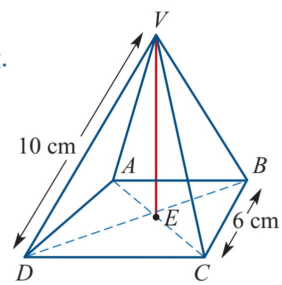
Example 8 Using Pythagoras' theorem in three dimensions

The diagram shows a pyramid with a square base. The base has sides 6 cm long and the edges *VA*, *VB*, *VC*, *VD* are each 10 cm long.

a Find the length of *DB*. **b** Find the length of *BE*.

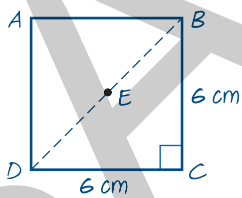
c Find the length of *VE*. **d** Find the magnitude of angle *VBE*.

Give all answers correct to two decimal places.



Solution

a



$$DB^2 = 6^2 + 6^2$$
$$= 72$$
$$\therefore\ DB = 8.4852\ldots$$

The length of DB is 8.49 cm correct to two decimal places.

b $BE = \frac{1}{2}DB$

$$\therefore\ BE = \frac{1}{2}\sqrt{72} = 4.2426\ \ldots$$

The length of BE is 4.24 cm correct to two decimal places.

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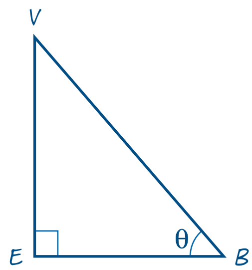
c



$$VE^2 = VB^2 - EB^2$$
$$= 100 - \frac{1}{4} \times 72$$
$$= 100 - 18 = 82$$
$$\therefore\ VE = \sqrt{82} = 9.0553\ldots$$

The length of VE is 9.06 cm correct to two decimal places.

d $\sin\theta = \dfrac{VE}{VB} = \dfrac{\sqrt{82}}{10}$

$= 0.9055\ldots$

$\therefore\ \theta = 64.90^\circ$

The magnitude of angle VBE is 64.90° correct to two decimal places.

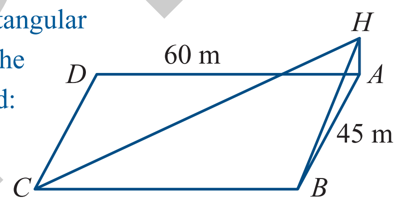
Example 9 Using Pythagoras' theorem and tan in three dimensions

A communications mast is erected at the corner A of a rectangular courtyard $ABCD$ whose sides measure 60 m and 45 m. If the angle of elevation of the top of the mast from C is 12°, find:

a the height of the mast

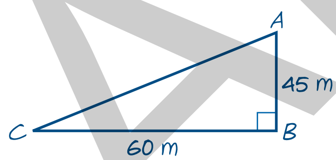
b the angle of elevation of the top of the mast from B (where $AB = 45$ m)

Give answers correct to two decimal places.

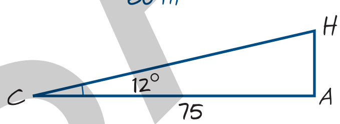


Solution

a



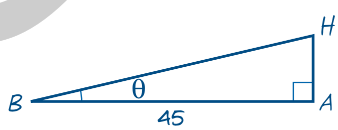
$$AC^2 = AB^2 + CB^2$$
$$= 45^2 + 60^2 = 5625$$
$$\therefore\ AC = 75$$



$$\frac{HA}{75} = \tan 12^\circ$$
$$\therefore\ HA = 75\tan 12^\circ$$
$$= 15.9417$$

The height of the mast is 15.94 m correct to two decimal places.

b



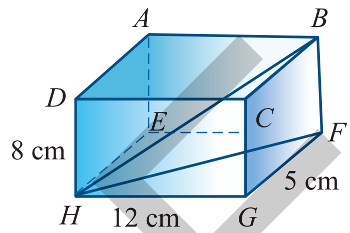
$$\tan\theta = \frac{HA}{45}$$
$$= 0.3542\ldots$$
$$\therefore\ \theta \approx 19.51^\circ$$

The angle of elevation of the top of the mast, H, from B is 19.51° correct to two decimal places.

Skillsheet TEACHER

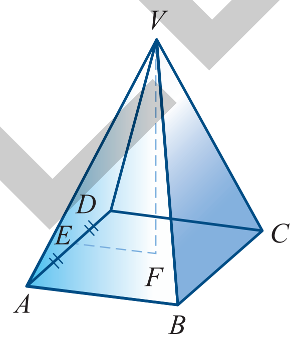
Exercise 14B

1 *ABCDEFGH* is a cuboid with dimensions as shown.



a Find the length of *FH*.
b Find the length of *BH*, correct to two decimal places.
c Find the magnitude of angle *BHF*, correct to one decimal places.
d Find the magnitude of angle *BHG*, correct to two decimal places.

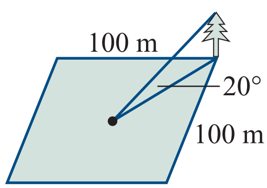
2 *VABCD* is a right pyramid with a square base. The sides of the base are 8 cm in length. The height, *VF*, of the pyramid is 12 cm.



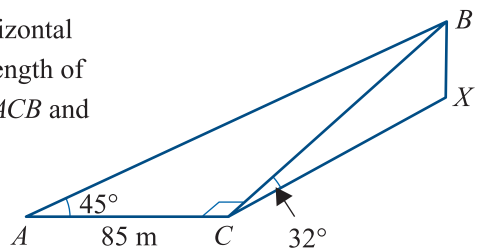
a Find *EF*.
b Find the magnitude of angle *VEF*.
c Find the length of *VE*.
d Find the length of a sloping edge.
e Find the magnitude of angle *VAD*.
f Find the surface area of the pyramid.

Where decimals are involved, give all answers correct to two decimal places.

3 A tree stands at the corner of a square playing field. Each side of the square is 100 m long. At the centre of the field a tree subtends an angle of 20°. What angle does it subtend at each of the other three corners of the field, correct to the nearest degree?



4 Suppose that *A*, *C* and *X* are three points in a horizontal plane and *B* is a point vertically above *X*. If the length of $AC = 85$ m and the magnitudes of angles *BAC*, *ACB* and *BCX* are 45°, 90° and 32° respectively, find:



a the distance *CB* correct to the nearest metre
b the height *XB* correct to the nearest metre

5 Standing due south of a tower 50 m high, the angle of elevation of the top is 26°. What is the angle of elevation after walking a distance 120 m due east?

6 From the top of a cliff 160 m high two buoys are observed. Their bearings are 337° and 308°. Their respective angles of depression are 3° and 5°. Calculate the distance between the buoys, correct to the nearest metre.

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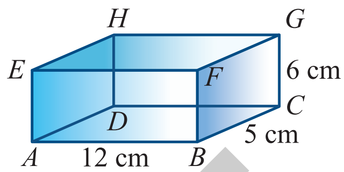
7 Find the magnitude of each of the following angles for the cuboid shown:

a *ACE* **b** *HDF* **c** *ECH*



8 From a point A due north of a tower the angle of elevation to the top of the tower is 45°. From a point B, 100 m on a bearing of 120° from A the angle of elevation is 26°. Find the height of the tower.

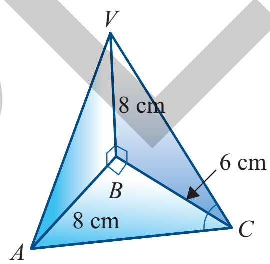
9 A and B are two positions on level ground. From an advertising balloon at a vertical height of 750 m, A is observed in an easterly direction and B at a bearing 160°. The angles of depression of A and B as viewed from the balloon are 40° and 20° respectively. Find the distance between A and B.

10 Angles *VBC*, *VBA* and *ABC* are right angles.

a Find the distance *VA*. **b** Find the distance *VC*.

c Find the distance *AC*.

d Find the magnitude of angle *VCA*.



11 A right pyramid, height 6 cm, stands on a square base of side 5 cm. Find:

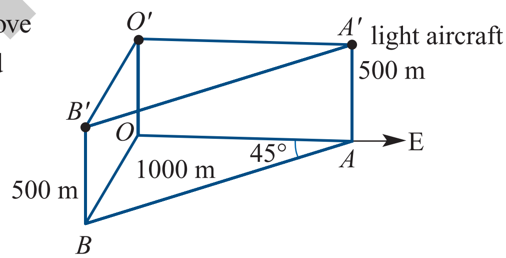
a the length of a sloping edge **b** the area of a triangular face

12 A light aircraft flying at a height of 500 m above the ground is sighted by an observer stationed at a point O on the ground, measured to be 1 km from the plane. The aircraft is flying south west (along $A'B'$) at 300 km/h.

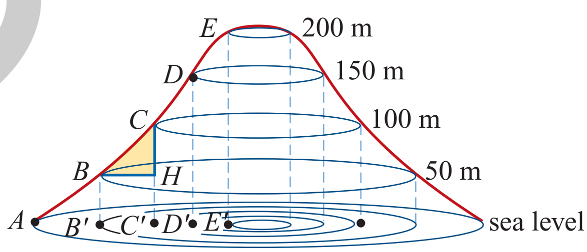
a How far will it travel in one minute?

b Find its bearing from $O(O')$ at this time.

c What will be its angle of elevation from O at this time?

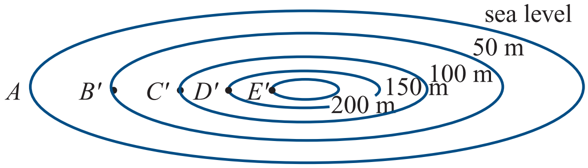


14.3 Contour maps



This diagram shows a hill over 200 m high rising from sea level. B is a point on the hill 50 m above sea level. A line drawn through B passes through all other points that are 50 m above sea level. This is called the 50 m **contour line**.

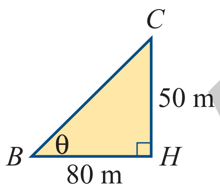
Similarly, C, D and E mark the levels of the 100 m, 150 m and 200 m contour lines respectively. Imagine that the contour lines are all painted black and then projected onto the base as shown below.



This is called a **contour map**.

Note that the map does not give the actual distance between B and C but gives the horizontal distance.

In order to find the distance between B and C, first determine from the diagram (drawn to a scale) the horizontal distance $B'C'$. Suppose this distance is 80 m. Then triangle BCH in the first diagram can be used to find the distance between B and C and the **average slope** between B and C.



$$\begin{aligned} BC^2 &= BH^2 + CH^2 \\ &= 6400 + 2500 \\ &= 8900 \\ BC &= \sqrt{8900} \\ &= 94.3398\ldots \end{aligned}$$

The distance BC is 94 m to the nearest metre.

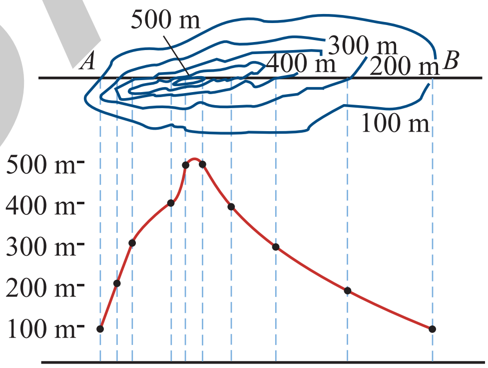
The **average slope** $= \dfrac{CH}{BH} = \dfrac{50}{80} = 0.625$

and $\tan\theta = 0.625$

which implies $\theta = 32.00^\circ$ correct to two decimal places

The angle of elevation of C from B is 32° to the nearest degree.

A cross-sectional profile can be drawn from a contour map for a given cross-section AB. This is illustrated below.



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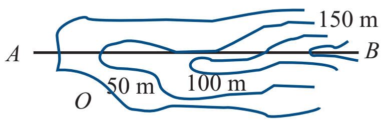
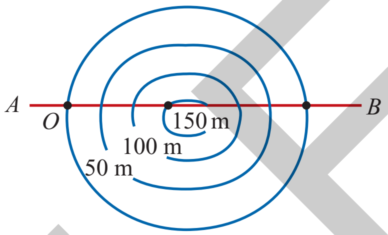
Skillsheet TEACHER

Exercise 14C

1 Draw a cross-sectional profile for each of the following maps with the given cross-section *AB*.

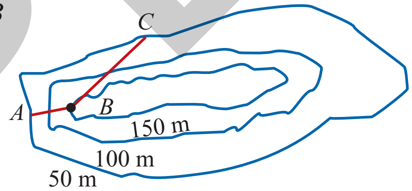
a


b


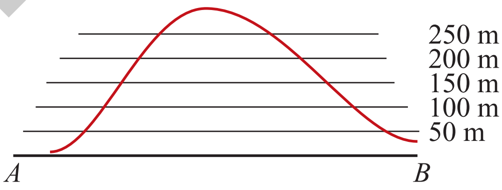
2 Two places on a map are 5 cm apart. One is on a 50 m contour and the other on a 450 m contour. If the scale of the map is 1 cm to 1 km, what is the angle of elevation from the first to the second place?

3 a For this diagram the horizontal distance from *A* to *B* is 400 m. Find:
  i the distance from *A* to *B*
  ii the angle of elevation of *B* from *A*

b The horizontal distance from *B* to *C* is 1 km. Find:
  i the distance of *B* from *C*
  ii the angle of elevation of *B* from *C*



4 Draw a possible contour map to match the given cross-section.



MC TEST D&D TEST

Key ideas and chapter summary

Angle of elevation	The **angle of elevation is the angle** between the horizontal and a direction above the horizontal. (See p. 370)
Angle of depression	The **angle of depression is the angle** between the horizontal and a direction below the horizontal. (See p. 370)
Three-figure bearing	The **three-figure bearing** is the direction measured from north clockwise (also called compass bearing).
Contour line/diagram/map	A contour line joins all the points that are the same distance above sea level. (See p. 381) A contour diagram is made up of a number of contour lines showing the whole feature, e.g. a hill. A contour map is produced when the contour lines are projected onto the base of the diagram.

Skills check

Having completed this chapter you should be able to:

- apply the idea of angle of depression
- apply the idea of angle of elevation
- apply the idea of a bearing
- use, construct and interpret contour maps

Multiple-choice questions

1 A man walks 5 km due east followed by 7 km due south. The bearing he must take to return to the start is:

A 036° **B** 306° **C** 324° **D** 332° **E** 348°

2 A boat sails at a bearing of 215° from A to B. The bearing he would take from B to return to A is:

A 035° **B** 055° **C** 090° **D** 215° **E** 250°

3 From a point on a cliff 500 m above sea level, the angle of depression to a boat is 20°. The distance from the foot of the cliff to the boat to the nearest metre is:

A 182 m **B** 193 m **C** 210 m **D** 1374 m **E** 1834 m

4 A tower 80 m high is 1.3 km away from a point on the ground. The angle of elevation to the top of the tower from this point, correct to the nearest degree is:

A 1° **B** 4° **C** 53° **D** 86° **E** 89°

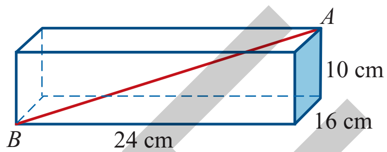
5 A boat sails from a harbour on a bearing of 035° for 100 km. It then takes a bearing of 190° for 50 km. The distance from the harbour, correct to the nearest km, is:

A 51 km **B** 58 km **C** 59 km **D** 108 km **E** 3437 km

6 A hiker walks 5.2 km on a bearing of 160° and then takes a bearing of 035° and walks 7 km. The bearing he must take to return directly to the start is:

A 047° **B** 215° **C** 223° **D** 235° **E** 262°

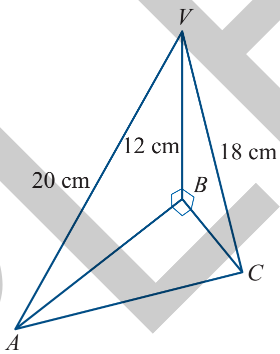
7 The diagram shows a rectangular prism (cuboid). The dimensions are as marked. The length of the diagonal is closest to:



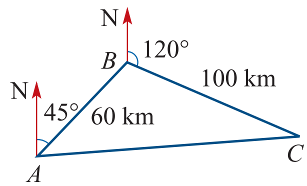
A 26 cm **B** 30 cm **C** 31 cm
D 34 cm **E** 39 cm

8 *VABC* is a pyramid. *V* is vertically above *B* and triangle *ABC* has a right angle at *B*. The distance between points *A* and *C* is closest to:

A 12 cm **B** 21 cm **C** 26 cm
D 30 cm **E** 36 cm

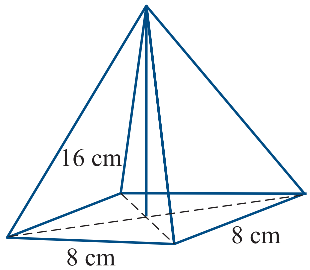


9 A ship travels from a point *A* to *B* on a bearing of 045° for a distance of 60 km. It then travels to a point *C* on a bearing of 120° for a distance of 100km. The distance of *C* to *A* is closest to:



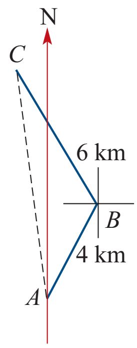
A 123 km **B** 125 km **C** 127 km
D 129 km **E** 131 km

10 A right pyramid with a square base is shown in the diagram. Each edge of the square base has length 8 centimetres and the height of the pyramid is 16 cm. The length of a sloping edge of the pyramid in centimetres is:



A $\sqrt{288}$ **B** $\sqrt{155}$ **C** $\sqrt{125}$
D $\sqrt{324}$ **E** $\sqrt{425}$

11 A hiker walks 4 km from point *A* on a bearing of 030° to point *B* and then 6 km on a bearing of 330° to point *C*. The distance *AC* in kilometres is:



A $\dfrac{4}{\sin 30°}$ **B** $\sqrt{6^2 + 4^2 - 48\cos 120°}$

C $\sqrt{6^2 + 4^2 + 48\cos 120°}$ **D** $6\sin 60°$

E $\sqrt{52}$

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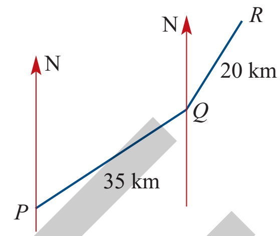
12 A ship travels from P to Q on a course of 050° for 35 km and then from Q to R on a course of 020° for 20 km. How far east of P is R in kilometres?



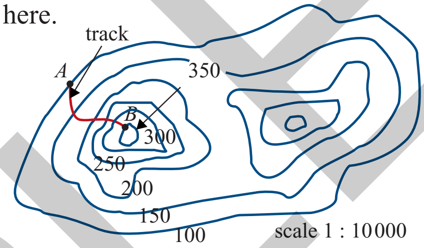
A $35\sin 50^\circ + 20\sin 20^\circ$ **B** $35\cos 50^\circ + 20\cos 20^\circ$

C $\dfrac{35}{\sin 40^\circ} + \dfrac{20}{\sin 70^\circ}$ **D** $\dfrac{35}{\sin 40^\circ} + \dfrac{20}{\cos 70^\circ}$

E $55\cos 110^\circ$

13 The contour map of a region of a hill is shown here.

A hiker walks up the hill from a point A on the 100 m contour line to a point B on the 350 m contour line along the track shown. From the contour map the steepest part of the track is between the:



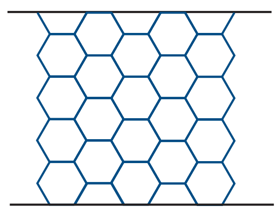
A 100 m and 150 m contour lines **B** 150 m and 200 m contour lines

C 200 m and 250 m contour lines **D** 250 m and 300 m contour lines

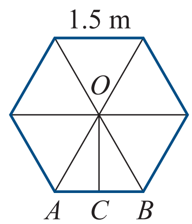
E 300 m and 350 m contour lines

Extended-response questions

1 Aristotle valued the aesthetic appearance of the Greek city wall. He was disturbed at the earlier polygonal walls, which were made of irregular shapes. He convinced Philip, King of Macedonia, that the walls of Miea should be constructed with regular hexagonal blocks as shown here.



The diagram opposite shows a hexagonal face of one of the blocks. A and B mark two adjacent corners on the face of the block while O marks the centre of the hexagonal face. The length of each edge of each hexagon is 1.5 m. Point C is midway between A and B.

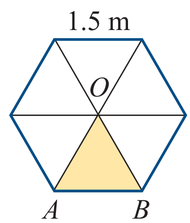


a Explain why the length of OA is 1.5 m.

b Taking the length of OA to be 1.5 m, find the length of OC in metres. Give your answer correct to three decimal places.

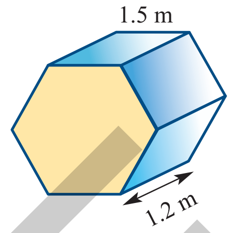
c The wall is ten blocks high. Find the height of the wall in metres. Give your answer correct to the nearest metre.

d Find the area of the shaded triangle (see diagram) drawn on a hexagonal face of one of the blocks. Give your answer in square metres correct to three decimal places.



e Find the area of the hexagonal face of one of the blocks. Give your answer in square metres correct to one decimal place.

f The hexagonal blocks are 1.2 m deep. Find the volume of one of the hexagonal blocks. Give your answer in cubic metres correct to one decimal place.

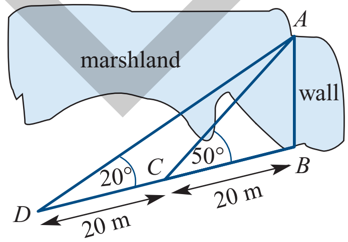


Aristotle wanted to see a scale model of a section of the wall before it was built. The scale he chose was 1 : 25.

g What would be the length of an edge of a hexagonal face of a block for the model? Give your answer in centimetres.

h What is the ratio of the volume of a block in the model to the volume of a block in the actual wall?

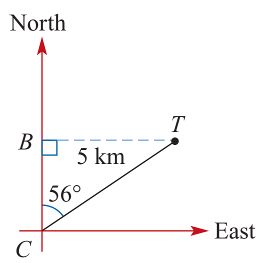
A part of the wall is to cross a marshland. Aristotle wanted to find out the length of this part of the wall but did not want to get his sandals muddy. To overcome the problem, Aristotle made the measurements shown on the diagram.



i Find the distance AC in metres, correct to two decimal places.

j Find the length of the wall to be constructed across the marshland. Give your answer to the nearest metre.

2 From a point C, by looking due north, a girl can see a beacon at point B. She can also see a tower at point T, which is 5 km away on a bearing of 056°. The tower at point T is due east of the beacon at B.

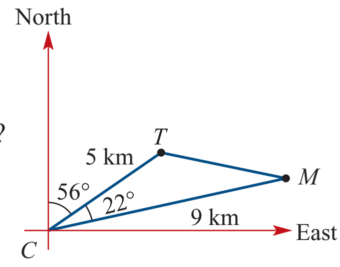


a Calculate the length of BT, the distance of the tower from the beacon. Give your answer correct to three decimal places.

b If she looks a further 22° from the tower at T the girl can see a radio mast at point M, which is 9 km away.

i What is the bearing of the mast at point M from C?

ii What is the distance between the tower and the mast correct to three decimal places?



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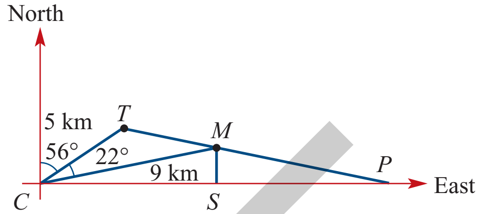
c The girl now walks due east from C.

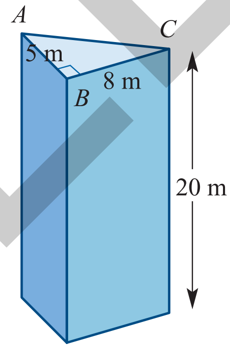


i How far must she walk until she is at point S due south of the mast? Give your answer correct to three decimal places.

ii Given that $\angle CMT = 23.2°$ (correct to one decimal place), how far must she walk from C until she is at point P so that T, M and P lie on a straight line? Give your answer correct to one decimal place.

iii What is the bearing of the mast at M from the tower at T?

d The tower is 20 m high and in the shape of a right triangular prism with dimensions as shown in the diagram.

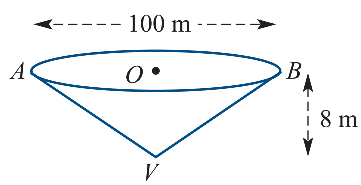


i What is the volume of the tower in cubic metres?

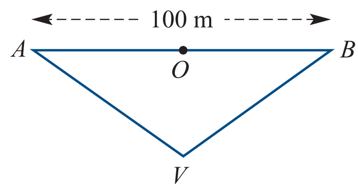
ii What is the length, in metres, of edge AC of the tower? Give your answer correct to two decimal places.

iii A scale model of the tower is to be constructed out of cardboard. The model is to be on a scale of 1 : 110 and must include the roof of the tower but not the floor. How many square metres of cardboard will be needed? Give your answer correct to two decimal places.

3 A water reservoir is shaped like a right circular cone, as shown. AB is a diameter of the circular surface of the cone and O is the centre of this surface. The height of the cone is 8 m.



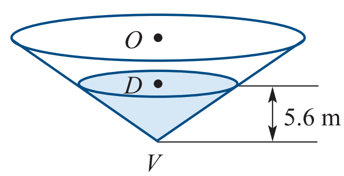
a A cross-section ABV is shown.



i Find the length VB correct to one decimal place.

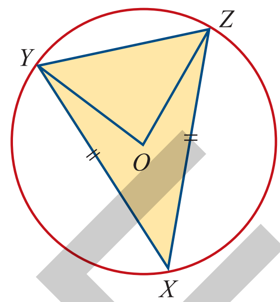
ii If the depth of the water is 4 m, calculate the radius of the circular surface of water in the reservoir.

b If the depth of the water in the reservoir is 5.6 m, calculate the surface area of the water; that is, the area of the circle, centre D (correct to two decimal places).



c If the reservoir is 80% full of water (that is, the volume of water is 80% of capacity) calculate the depth of the water (correct to two decimal places).

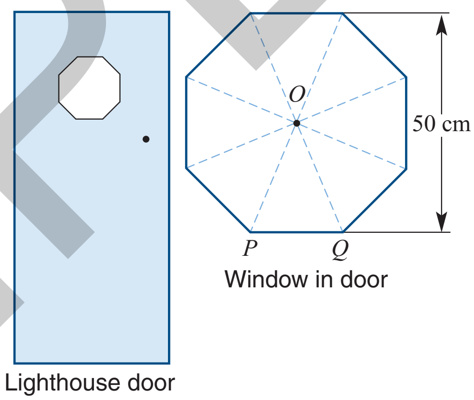
d A mesh is to be placed over the reservoir to partially shade its surface. The first plan is to use a triangular mesh. The triangular mesh, XYZ, is to be supported by three posts around the edge of the reservoir at X, Y and Z respectively as shown. In the diagram, $YX = ZX$ and $\angle YOZ$ is a right angle. O is the centre of the circle and $OZ = OY = 50$ m.



- **i** Find the length YX (correct to two decimal places).
- **ii** Find the area of the triangular mesh (rounded to the nearest whole number).
- **iii** Find the percentage of the area of the circle, centre O, covered by the triangular mesh (correct to one decimal place).

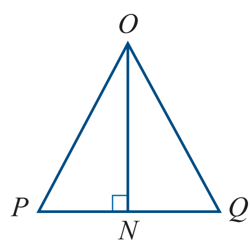
e If the mesh has the form of a regular dodecagon (12-sided regular polygon), with vertices on the circumference of the circle, find the percentage of the area of the circle covered by the mesh (correct to one decimal place).

4 Lee and Nick are staying in the seaside township of Eagle Point, famous for the octagonal window in its lighthouse door. The window is in the shape of a regular octagon. PQ is the bottom side of the window whose diagonals meet at O. The height of the window is 50 cm.



a Show by calculation that the size of angle POQ is 45°.

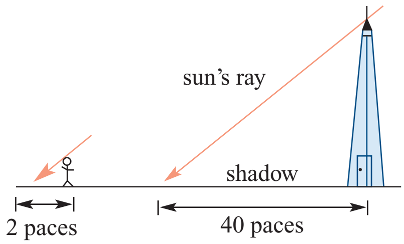
b In triangle POQ, N is the midpoint of PQ.



- **i** Write down the length of ON.
- **ii** Write down the size of angle PON.
- **iii** What is the length of PQ in centimetres correct to two decimal places?

c Find the area of the glass in the octagonal window. Give your answer correct to the nearest square centimetre.

d At midday, the lighthouse casts a shadow directly onto a straight level road leading to the lighthouse. Lee measures the length of the shadow by pacing, and finds that it is 40 paces long when measured from the centre of the base of the lighthouse. When Nick stands on the road, Lee finds that Nick's shadow is two paces long, as shown in the diagram. Nick is 172 cm tall. What is the height of the lighthouse in metres correct to one decimal place?



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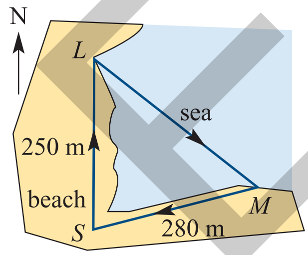
e The Eagle Point Surf Club has set up a training course which requires participants to run 250 metres along the beach from the starting point S to a point L on the shore. They then swim across an inlet to a point M on the opposite shore before running 280 metres directly back to the starting point S as shown. L is due north of S and the bearing of M from S is 078°.

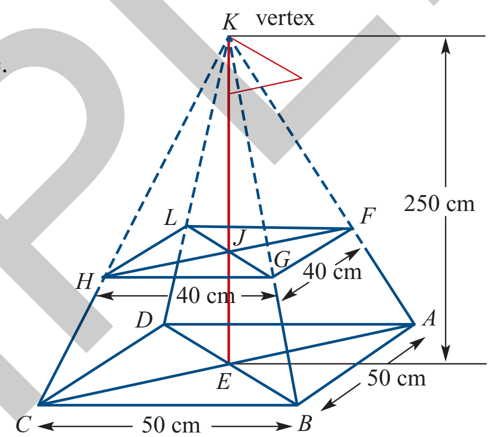
i Write down the size of angle LSM.

ii Find the total length of the training course. Give your answer correct to the nearest metre.

iii What is the bearing of M from L? Give your answer correct to the nearest degree.



f The club places flags on the beach to mark points on the training course. The flagpoles sit in wooden boxes which are in the shape of truncated right pyramids. One such box is shown in the diagram. The base $ABCD$ of the box is a 50 cm by 50 cm square. The top $FGHL$ is a 40 cm by 40 cm square. The flagpole KE sits vertically in the box and is 250 centimetres long. If the pyramid could be completed, its vertex would be at K, the top of the flagpole, as shown.



i Find the angle KCE. Give your answer correct to the nearest degree.

ii Find JE, the depth of the block, in centimetres.

[VCAA pre 2000]