CHAPTER 8
Revision of Chapters 1–7

8.1 Multiple-choice questions

1. The domain of the function whose graph is shown is:
   A. \((-3, 1]\)
   B. \((-1, 3]\)
   C. \([1, 3]\)
   D. \([-1, 3)
   E. \((-1, 3)

2. Which of the following sets of ordered pairs does not represent a function where \(y\) is the value of the function?
   A. \(\{(x, y) : x = 2y^2, x \geq 0\}\)
   B. \(\{(x, y) : y = \frac{1}{x}, x \in R\setminus\{0\}\}\)
   C. \(\{(x, y) : y = 2x^3 + 3, x \in R\}\)
   D. \(\{(x, y) : y = 3x^2 + 7, x \in R\}\)
   E. \(\{(x, y) : y = e^x - 1, x \in R\}\)

3. The implied (largest possible) domain for the function with the rule \(y = \frac{1}{\sqrt{2-x}}\) is:
   A. \(R\setminus\{2\}\)
   B. \((-\infty, 2)\)
   C. \((2, \infty)\)
   D. \((-\infty, 2]\)
   E. \(R^+\)

4. If \(f(x) = \frac{x}{x-1}\) then \(f\left(-\frac{1}{a}\right)\), in simplified form, is equal to:
   A. \(\frac{1}{-1-a}\)
   B. \(-1\)
   C. \(0\)
   D. \(\frac{a^2}{1-a}\)
   E. \(\frac{1}{a+1}\)

5. The graph shown has the equation:
   A. \(y = \begin{cases} x - 2, & x > 0 \\ -2x - 2, & x \leq 0 \end{cases}\)
   B. \(y = \begin{cases} 2x - 2, & x \geq 0 \\ -2x - 2, & x < 0 \end{cases}\)
C \[ y = \begin{cases} x - 2, & x > 0 \\ -2x - 1, & x \leq 0 \end{cases} \]

D \[ y = \begin{cases} x + 2, & x > 0 \\ -2x - 2, & x \leq 0 \end{cases} \]

E \[ y = \begin{cases} x - 2, & x > 0 \\ -x - 2, & x \leq 0 \end{cases} \]

6 If \( f: [0, 2\pi] \rightarrow R \) where \( f(x) = \sin 2x \) and \( g: [0, 2\pi] \rightarrow R \) where \( g(x) = 2\sin x \), then the value of \( (f + g)\left(\frac{3\pi}{2}\right) \) is:

A 2  B 0  C -1  D 1  E -2

7 If \( f(x) = 3x + 2 \) and \( g(x) = 2x^2 \), then \( f(g(3)) \) equals:

A 36  B 20  C 56  D 144  E 29

8 If \( f(x) = 3x^2 \), \( 0 \leq x \leq 6 \) and \( g(x) = \sqrt{2 - x} \), \( x \leq 2 \), the domain of \( f + g \) is:

A \([0, 2]\)  B \([0, 6]\)  C \((-\infty, 2]\)  D \(R^+ \cup \{0\}\)  E \([2, 6]\)

9 If \( g(x) = 2x^2 + 1 \) and \( f(x) = 3x + 2 \), then the rule of the product function \( fg(x) \) equals:

A \(2x^2 + 3x + 3\)  B \(6x^3 + 4x^2 + 3x + 2\)  C \(6x^3 + 3\)  D \(6x^3 + 2x^2 + 3\)  E \(6x^3 + 2\)

10 The domain of the function whose graph is shown is:

A \([1, 5]\)  B \((1, 5]\)  C \((-2, 5]\)  D \((1, 5]\)  E \((-2, 5]\)

11 The implied domain for the function with equation \( y = \sqrt{4 - x^2} \) is:

A \([2, \infty)\)  B \(\{x: -2 < x < 2\}\)  C \([-2, 2]\)  D \((-\infty, 2]\)  E \(R^+\)

12 The graph of the function with rule \( y = f(x) \) is shown.
Which one of the following graphs is the graph of the inverse of \( f \)?

\[ A \quad y = \begin{cases} (x - 2)^2, & x \geq 2 \\ x - 3, & x < 2 \end{cases} \]

\[ B \quad y = \begin{cases} x - 2, & x \geq 2 \\ x - 3, & x < 2 \end{cases} \]

\[ C \quad y = \begin{cases} (2 - x)^2, & x \geq 2 \\ 2x - 3, & x < 2 \end{cases} \]

\[ D \quad y = \begin{cases} (x - 2)^2, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases} \]

\[ E \quad y = \begin{cases} (x - 2)^2, & x \geq 2 \\ 2x - 3, & x < 2 \end{cases} \]

13. The graph shown has the rule:

\[ A \quad y = \begin{cases} (x - 2)^2, & x \geq 2 \\ x - 3, & x < 2 \end{cases} \]

\[ B \quad y = \begin{cases} x - 2, & x \geq 2 \\ x - 3, & x < 2 \end{cases} \]

\[ C \quad y = \begin{cases} (2 - x)^2, & x \geq 2 \\ 2x - 3, & x < 2 \end{cases} \]

\[ D \quad y = \begin{cases} (x - 2)^2, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases} \]

\[ E \quad y = \begin{cases} (x - 2)^2, & x \geq 2 \\ 2x - 3, & x < 2 \end{cases} \]

14. The inverse, \( f^{-1} \), of the function \( f: [2, 3] \rightarrow R \), \( f(x) = 2x - 4 \) is:

\[ A \quad f^{-1}: [0, 2] \rightarrow R, \quad f^{-1}(x) = \frac{x}{2} + 4 \]

\[ B \quad f^{-1}: [3, 2] \rightarrow R, \quad f^{-1}(x) = \frac{x + 4}{2} \]

\[ C \quad f^{-1}: [2, 3] \rightarrow R, \quad f^{-1}(x) = \frac{1}{2x - 4} \]

\[ D \quad f^{-1}: [0, 2] \rightarrow R, \quad f^{-1}(x) = \frac{x}{2x - 4} \]

15. \( f \) is the function defined by \( f(x) = \frac{1}{x^2 + 2}, x \in R \). A suitable restriction for \( f \), \( f^* \) such that \( f^* \) exists, would be:

\[ A \quad f^*: [-1, 1] \rightarrow R, \quad f^*(x) = \frac{1}{x^2 + 2} \]

\[ B \quad f^*: R \rightarrow R, \quad f^*(x) = \frac{1}{x^2 + 2} \]
C \( f^*: [-2, 2] \to R, \quad f^*(x) = \frac{1}{x^2 + 2} \)

D \( f^*: [0, \infty) \to R, \quad f^*(x) = \frac{1}{x^2 + 2} \)

E \( f^*: [-1, \infty) \to R, \quad f^*(x) = \frac{1}{x^2 + 2} \)

16 Let \( h: [a, 2] \to R \) where \( h(x) = 2x - x^2 \). If \( a \) is the smallest real value such that \( h \) has an inverse function, \( h^{-1} \), then \( a \) equals:

A \(-1\)  B \(0\)  C \(1\)  D \(-2\)  E \(\frac{1}{2}\)

17 If \( f(x) = 3x - 2, \ x \in R \), then \( f^{-1}(x) \) equals:

A \(\frac{1}{3x - 2}\)  B \(3x + 2\)  C \(\frac{1}{3}(x - 2)\)  D \(3x + 6\)  E \(\frac{1}{3}(x + 2)\)

18 The solution of the equation \( 2x = \frac{3x}{2} - 4 \) is:

A \(4\)  B \(-2\)  C \(-8\)  D \(1\)  E \(2\)

19 The graph shows:

A \(y + 2 = x\)  B \(y = 2x - 2\)  C \(y + 2x + 2 = 0\)  D \(y = -2x + 2\)  E \(y - 2 = x\)

20 If \( \frac{2(x - 1)}{3} - \frac{x + 4}{2} = \frac{5}{6} \), then \( x \) equals:

A \(-5\)  B \(\frac{7}{5}\)  C \(\frac{21}{5}\)  D \(21\)  E \(3\)

21 The equation of the line that passes through the points \((-2, 3)\) and \((4, 0)\) is:

A \(2y = x + 4\)  B \(y = -\frac{1}{2}x - 2\)  C \(2y + x = 4\)  D \(y = \frac{1}{2}x - 2\)  E \(2y - x = 4\)

22 If the angle between the lines \(2y = 8x + 10\) and \(3x - 6y = 22\) is \(\theta\), then \(\tan \theta\) is best approximated by:

A \(1.17\)  B \(1.40\)  C \(2\)  D \(0.86\)  E \(1\)

23 The line with equation \(y = \frac{4}{5}x - 4\) meets the \(x\)-axis at \(A\) and the \(y\)-axis at \(B\). If \(O\) is the origin, the area of the triangle \(OAB\) is:

A \(\frac{1}{5}\) square units  B \(\frac{2}{5}\) square units  C 10 square units  D 15 square units  E 20 square units
24 If the equations $2x - 3y = 12$ and $3x - 2y = 13$ are simultaneously true, then $x + y$ equals:
A $-5$  B $-1$  C $0$  D $1$  E $5$

25 If the graphs of the relations $7x - 6y = 20$ and $3x + 4y = 2$ are drawn on the same pair of axes, the $x$-coordinate of the point of intersection is:
A $-2$  B $-1$  C $1$  D $2$  E $3$

26 A possible equation for the graph shown is:
A $y - 3 = \frac{1}{x - 1}$
B $y + 3 = \frac{1}{x + 1}$
C $y - 3 = \frac{1}{x + 1}$
D $y - 4 = \frac{1}{x + 1}$
E $y = \frac{1}{x - 1} - 3$

27 The function given by $f(x) = \frac{1}{x + 3} - 2$ has the range given by:
A $R \setminus \{-2\}$  B $R$  C $R \setminus \{3\}$  D $R \setminus \{2\}$  E $R \setminus \{-3\}$

28 A parabola has its vertex at $(2, 3)$. A possible equation for this parabola is:
A $y = (x + 2)^2 + 3$  B $y = (x - 2)^2 - 3$  C $y = (x + 2)^2 - 3$
D $y = (x - 2)^2 + 3$  E $y = 3 - (x + 2)^2$

29 Which one of the following is an even function of $x$?
A $f(x) = 3x + 1$  B $f(x) = x^3 - x$  C $f(x) = (1 - x)^2$
D $f(x) = -x^2$  E $f(x) = x^3 + x^2$

30 The graph of $y = 3\sqrt{x + 2}$ can be obtained from the graph of $y = \sqrt{x}$ by:
A a translation $(x, y) \rightarrow (x - 2, y)$ followed by a dilation of factor 3 from the $x$-axis
B a translation $(x, y) \rightarrow (x + 2, y)$ followed by a dilation of factor $\frac{1}{3}$ from the $x$-axis
C a translation $(x, y) \rightarrow (x + 3, y)$ followed by a dilation of factor 3 from the $y$-axis
D a translation $(x, y) \rightarrow (x - 2, y)$ followed by a dilation of factor 3 from the $y$-axis
E a translation $(x, y) \rightarrow (x + 2, y)$ followed by a dilation of factor 3 from the $y$-axis

31 A function with rule $f(x) = 3\sqrt{x - 2} + 1$ has maximal domain:
A $(-\infty, 2)$  B $[1, \infty)$  C $(2, \infty)$  D $[-2, \infty)$  E $[2, \infty)$
32 A possible equation of the graph shown is:
   A. \( y = 2\sqrt{x - 3} + 1 \)
   B. \( y = -2\sqrt{x - 3} + 1 \)
   C. \( y = \sqrt{x - 3} + 1 \)
   D. \( y = -\sqrt{x - 3} + 1 \)
   E. \( y = -2\sqrt{x - 3} + 2 \)

33 The range of the function \( f: \mathbb{R} \setminus \{2\} \to \mathbb{R} \), \( f(x) = \frac{3}{(x - 2)^2} + 4 \) is:
   A. \((3, 4]\)
   B. \((-\infty, 4)\)
   C. \([3, 4)\)
   D. \([4, \infty)\)
   E. \((4, \infty)\)

34 If \( 3x^2 + kx + 1 = 0 \) when \( x = 1 \), then \( k \) equals:
   A. \(-4\)
   B. \(-1\)
   C. \(1\)
   D. \(4\)
   E. \(0\)

35 The quadratic equation whose roots are 5 and \(-7\) is:
   A. \( x^2 + 2x - 35 = 0 \)
   B. \( x^2 - 2x - 35 = 0 \)
   C. \( x^2 + 12x - 35 = 0 \)
   D. \( x^2 - 12x - 35 = 0 \)
   E. \(-x^2 + 12x + 35 = 0 \)

36 If \( x^3 - 5x^2 + x + k \) is divisible by \( x + 1 \), then \( k \) equals:
   A. \(-7\)
   B. \(-5\)
   C. \(-2\)
   D. \(5\)
   E. \(7\)

37 Which one of the following could be the equation of the graph shown?
   A. \( y = x(x - 2)(x + 2) \)
   B. \( y = -x(x + 2)(x - 2) \)
   C. \( y = -(x + 2)^2(x - 2) \)
   D. \( y = (x - 2)^2(x + 2) \)
   E. \( y = x(x - 2)^2 \)

38 The graph shown is:
   A. \( y + 2 = -2(x + 1)^3 \)
   B. \( y - 2 = 2(x - 1)^3 \)
   C. \( y = x^3 + 2 \)
   D. \( y = -\frac{1}{2}(x + 1)^3 + 2 \)
   E. \( y = 2(x - 1)^3 - 2 \)

39 \( P(x) = x^3 + 2x^2 - 5x - 6 \) has the factors:
   A. \((x - 1)(x - 2)(x + 3)\)
   B. \((x + 1)(x + 2)(x + 3)\)
   C. \((x + 1)(x - 2)(x + 3)\)
   D. \((x + 1)(x - 2)(x - 3)\)
   E. \((x - 1)(x - 2)(-x - 3)\)

40 If \( P(x) = 2x^3 - 2x^2 + 3x + 1 \), when \( P(x) \) is divided by \((x - 2)\) the remainder is:
   A. \(31\)
   B. \(15\)
   C. \(1\)
   D. \(-2\)
   E. \(-29\)
The graph shown is that of the function 
\[ f(x) = mx + 3, \]  
where \( m \) is a constant. 
The inverse \( f^{-1} \) is defined as 
\[ f^{-1}: R \to R, \quad f^{-1}(x) = ax + b, \]  
where \( a \) and \( b \) are constants. Which one of the following statements is true?

A \( a = \frac{-3}{m}, \quad b = \frac{1}{m} \)  
B \( a < 0 \) and \( b < 0 \)  
C \( a = -m, \quad b = 3 \)  
D \( a > 0 \) and \( b > 0 \)  
E \( a = \frac{1}{m}, \quad b = \frac{-3}{m} \)

If \( x^3 + 2x^2 + ax - 4 \) has a remainder 1 when divided by \( x + 1 \), then \( a \) equals:

A \( -8 \)  
B \( -4 \)  
C \( -2 \)  
D \( 0 \)  
E \( 2 \)

Which of these equations is represented by the graph shown?

A \( y = (x + 2)^2(x - 2) \)  
B \( y = 16 - x^4 \)  
C \( y = (x^2 - 4)^2 \)  
D \( y = (x + 2)^2(2 - x) \)  
E \( y = x^4 - 16 \)

The function \( f: R \to R \) where \( f(x) = e^{-x} + 1 \) has an inverse function \( f^{-1} \). The domain of \( f^{-1} \) is:

A \( (0, \infty) \)  
B \( R \)  
C \( [1, \infty) \)  
D \( (1, \infty) \)  
E \( [0, \infty) \)

The function \( f: R^+ \to R \) where \( f(x) = 2 \log_e x + 1 \) has an inverse function \( f^{-1} \). The rule for \( f^{-1} \) is given by:

A \( f^{-1}(x) = 2e^{x-1} \)  
B \( f^{-1}(x) = e^{\frac{1}{2}(x-1)} \)  
C \( f^{-1}(x) = e^{\frac{x}{2} - 1} \)  
D \( f^{-1}(x) = 2e^{x+1} \)  
E \( f^{-1}(x) = \frac{1}{2}e^{x-1} \)

Let \( f: R \to R \) where \( f(x) = e^{-x} \) and \( g: (-1, \infty) \to R \) where \( g(x) = \log_e(x + 2) \). The function with the rule \( y = f(g(x)) \) has the range:

A \( (1, \infty) \)  
B \( (0, 1) \)  
C \( (0, 1] \)  
D \( [1, \infty) \)  
E \( [0, 1] \)

The function \( g: R \to R \) where \( g(x) = e^x - 1 \) has an inverse whose rule is given by:

A \( f^{-1}(x) = \frac{1}{e^x - 1} \)  
B \( f^{-1}(x) = -\log_e(x + 1) \)  
C \( f^{-1}(x) = \log_e(x - 1) \)  
D \( f^{-1}(x) = \log_e(1 - x) \)  
E \( f^{-1}(x) = \log_e(x + 1) \)

The function \( f: [4, \infty) \to R \) where \( f(x) = \log_e(x - 3) \) has an inverse. The domain of this inverse is:

A \( [0, \infty) \)  
B \( (0, \infty) \)  
C \( [4, \infty) \)  
D \( (3, \infty) \)  
E \( R \)
49. The function \( f: \mathbb{R} \to \mathbb{R} \) where \( f(x) = e^{x-1} \) has an inverse whose rule is given by
\[ f^{-1}(x) = \]
A. \( e^{-x-1} \) 
B. \(-\log e x\) 
C. \(1 + \log e x\) 
D. \(\log e (x + 1)\) 
E. \(\log e (x - 1)\)

50. The function \( f: \mathbb{R}^+ \to \mathbb{R} \) where \( f(x) = \log e \frac{x}{2} \) has an inverse function \( f^{-1} \). The rule for \( f^{-1} \) is given by
\[ f^{-1} (x) = \]
A. \(e^{\frac{1}{2}x}\) 
B. \(\log e \left( \frac{2}{x} \right) \) 
C. \(\frac{1}{2} e^{\frac{x}{2}}\) 
D. \(2e^x\) 
E. \(\frac{1}{\log e^2}\)

51. For what values of \( x \) is the function \( f \) with the rule \( f(x) = -2 + \log e (3x - 2) \) defined?
A. \((-2, \infty)\) 
B. \(\left(\frac{2}{3}, \infty\right)\) 
C. \([-2, \infty)\) 
D. \(\left[\frac{2}{3}, \infty\right)\) 
E. \((2, \infty)\)

52. The graphs of the function \( f: (-2, \infty) \to \mathbb{R} \) where \( f(x) = 2 + \log e (x + 2) \) and its inverse \( f^{-1} \) are best shown by which one of the following?
A. 
\[ y = x \] 
B. 
\[ y = x \] 
C. 
\[ y = x \] 
D. 
\[ y = x \] 
E. 
\[ y = x \]

53. For \( \log_2 8x + \log_2 2x = 6 \), \( x = \)
A. 1.5 
B. \(\pm 1.5\) 
C. 2 
D. \(\pm 2\) 
E. 6.4

54. The equation \( \log_{10} x = y(\log_{10} 3) + 1 \) is equivalent to the equation:
A. \( x = 10(3^y) \) 
B. \( x = 30^y \) 
C. \( x = 3^y + 10 \) 
D. \( x = y^3 + 10 \) 
E. \( x = 10y^3 \)

55. The graph indicates that the relationship between \( N \) and \( t \) is:
A. \( N = 2 - e^{-2t} \) 
B. \( N = e^{2-2t} \) 
C. \( N = e^{2t} + 2 \) 
D. \( N = \frac{100}{e^{2t}} \) 
E. \( N = -2e^{2t} \)
56 A possible equation for the graph is:
A \( y = 1 - e^x \)
B \( y = 1 - e^{-x} \)
C \( y = 1 + e^x \)
D \( y = 1 + e^{-x} \)
E \( y = e^{-x} - 1 \)

57 A possible equation for the graph is:
A \( y = \log_e (x - 2) \)
B \( y = \log_e \frac{1}{2}(x + 2) \)
C \( y = \log_e 2(x + 1) \)
D \( y = 2 \log_e (x + 1) \)
E \( y = \frac{1}{2} \log_e (x + 2) \)

58 A possible equation for the graph shown is:
A \( y = 2 \cos 3 \left( \theta + \frac{\pi}{4} \right) - 4 \)
B \( y = 2 \cos 2 \left( \theta + \frac{\pi}{4} \right) - 2 \)
C \( y = 2 \sin 3 \left( \theta + \frac{\pi}{4} \right) - 2 \)
D \( y = 2 \cos 3 \left( \theta + \frac{\pi}{4} \right) - 2 \)
E \( y = 2 \cos 3 \left( \theta - \frac{\pi}{4} \right) - 2 \)

59 The function \( f: R \rightarrow R \) where \( f(x) = 2 - 3 \cos \left( \theta + \frac{\pi}{2} \right) \) has range:
A \([-3, 5]\)
B \([2, 5]\)
C \(R\)
D \([-1, 5]\)
E \([-3, 2]\)

60 Two values between 0 and \( 2\pi \) for which \( 2 \sin \theta + \sqrt{3} = 0 \) are:
A \( \frac{\pi}{3}, \frac{2\pi}{3} \)
B \( 60^\circ, 240^\circ \)
C \( \frac{2\pi}{3}, \frac{5\pi}{3} \)
D \( \frac{4\pi}{3}, \frac{5\pi}{3} \)
E \( \frac{7\pi}{6}, \frac{11\pi}{6} \)

61 A possible equation for the graph shown is:
A \( y = \sin \left( x - \frac{\pi}{6} \right) \)
B \( y = \sin \left( x + \frac{\pi}{6} \right) \)
C \( y = -\sin \left( x - \frac{\pi}{6} \right) \)
D \( y = \cos \left( x - \frac{\pi}{6} \right) \)
E \( y = \cos \left( x + \frac{\pi}{6} \right) \)
The function \( f: \mathbb{R} \rightarrow \mathbb{R} \), \( f(x) = 3 \sin 2x \) has:

A. amplitude 3 and period \( \pi \) 
B. amplitude 2 and period \( \frac{\pi}{2} \) 
C. amplitude 1 and period \( \frac{\pi}{2} \) 
D. amplitude \( \frac{3}{2} \) and period \( 2\pi \) 
E. amplitude \( 1\frac{1}{2} \) and period \( 2\pi \)

The function \( f: \mathbb{R} \rightarrow \mathbb{R} \), where \( f(x) = 3 \sin 2x \) has range:

A. \([0, 3]\) 
B. \([-2, 2]\) 
C. \([2, 3]\) 
D. \([-3, 3]\) 
E. \([-1, 5]\)

Consider the polynomial \( p(x) = (x - 2a)^2(x + a)(x^2 + a) \) where \( a > 0 \). The equation \( p(x) = 0 \) has exactly:

A. 1 distinct real solution 
B. 2 distinct real solutions 
C. 3 distinct real solutions 
D. 4 distinct real solutions 
E. 5 distinct real solutions

The gradient of a straight line perpendicular to the line shown is:

A. 2 
B. -2 
C. \(-\frac{1}{2}\) 
D. \(\frac{1}{2}\) 
E. 3

The graph of a function \( f \) whose rule is \( y = f(x) \) has exactly one asymptote for which the equation is \( y = 6 \). The inverse function \( f^{-1} \) exists. The inverse function will have:

A. a horizontal asymptote with equation \( y = 6 \) 
B. a vertical asymptote with equation \( x = 6 \) 
C. a vertical asymptote with equation \( x = \frac{-1}{6} \) 
D. a horizontal asymptote with equation \( y = -6 \) 
E. no asymptote

The function \( f: \mathbb{R} \rightarrow \mathbb{R} \), \( f(x) = a \sin(bx) + c \) where \( a, b \) and \( c \) are positive constants has period:

A. \( a \) 
B. \( b \) 
C. \( c \) 
D. \( \frac{2\pi}{a} \) 
E. \( \frac{2\pi}{b} \)

The functions \( f: [18, 34] \rightarrow \mathbb{R} \), \( f(x) = 2x - 4 \) and \( g: \mathbb{R}^+ \rightarrow \mathbb{R} \), \( g(x) = \log_2 x \) are used to define the composite function \( g \circ f \). The range of \( g \circ f \) is:

A. \([2, \infty)\) 
B. \(\left[\frac{3}{2}, \infty)\) 
C. \([5, 6]\) 
D. \(\mathbb{R}^+\) 
E. \(\mathbb{R}\)

The rule for the inverse relation of the function with rule \( y = x^2 - 4x + 5 \) and domain \( R \) is:

A. \( y = 2 \pm \sqrt{x + 1} \) 
B. \( y^2 = 2x + 5 \) 
C. \( y = 2 \pm \sqrt{x - 1} \) 
D. \( y = \sqrt{4x - 5} \) 
E. \( y = 4x - 5 \)

The range of the function with rule \( y = -3| \sin 2x | + 3 \) is:

A. \([0, 3]\) 
B. \([0, 6]\) 
C. \([-3, 3]\) 
D. \([0, 6]\) 
E. \([-3, 6]\)
8.2 Extended-response questions

1 An arch is constructed as shown.

The height of the arch is 9 metres \((OZ = 9 \text{ m})\). The width of the arch is 20 metres \((AB = 20 \text{ m})\). The equation of the curve is of the form \(y = ax^2 + b\), taking axes as shown.

a Find values of \(a\) and \(b\).

b A man of height 1.8 m stands at \(C\) \((OC = 7 \text{ m})\). How far above his head is the point \(E\) on the arch? (That is, find the distance \(DE\).)

c A horizontal bar \(FG\) is placed across the arch as shown. The height, \(OH\), of the bar above the ground is 6.3 m. Find the length of the bar.

2 a The expression \(2x^3 + ax^2 - 72x - 18\) leaves a remainder of 17 when divided by \(x + 5\). Determine the value of \(a\).

b Solve the equation: \(2x^3 = x^2 + 5x + 2\)

c \(\text{i} \) Given that the expression \(x^2 - 5x + 7\) leaves the same remainder whether divided by \(x - b\) or \(x - c\), where \(b \neq c\), show that \(b + c = 5\)

\(\text{ii} \) Given further that \(4bc = 21\) and \(b > c\), find the values of \(b\) and \(c\).

3 As a pendulum swings, its horizontal position \(x\), measured from the central position, varies from -4 cm (at \(A\)) to 4 cm (at \(B\)). \(x\) is given by the rule:

\[x = -4 \sin \pi t\]

a Sketch the graph of \(x\) against \(t\) for \(t \in [0, 2]\).

b Find the horizontal position of the pendulum for:

\(\text{i} \) \(t = 0\) \hspace{1cm} \(\text{ii} \) \(t = \frac{1}{2}\) \hspace{1cm} \(\text{iii} \) \(t = 1\)

c Find the first time that the pendulum has horizontal position \(x = 2\).

d Find the period of the pendulum, i.e. the time it takes to go from \(A\) to \(B\) and back to \(A\).
4 Two people are rotating a skipping rope. The rope is held 1.25 m above the ground. It reaches a height of 2.5 m above the ground, and just touches the ground.

The vertical position, $y$ m, of the point $P$ on the rope at time $t$ seconds is given by the rule:

$$y = -1.25 \cos (2\pi t) + 1.25$$

a Find $y$ when:

i $t = 0$

ii $t = \frac{1}{2}$

iii $t = 1$

b How long does it take for one revolution of the rope?

c Sketch the graph of $y$ against $t$.

d Find the first time that the point $P$ on the rope is 2.00 metres above the ground.

5 The population of a country is found to be growing continuously at an annual rate of 2.96% after 1 January 1950. The population $p$ years after 1 January 1950 is given by the formula:

$$p(t) = 150 \times 10^6 e^{kt}$$

a Find the value of $k$.

b Find the population on 1 January 1950.

c Find the expected population on 1 January 2000.

d After how many years would the population be $300 \times 10^6$?

6 A football is kicked so that it leaves the player’s foot with a velocity of $V$ m/s. The horizontal distance travelled by the football after being kicked is given by the formula:

$$x = \frac{V^2 \sin 2\alpha}{10}$$

where $\alpha$ is the angle of projection.

a Find the distance the ball is kicked if $V = 25$ m/s and $\alpha = 45^\circ$.

b For $V = 20$, sketch the graph of $x$ against $\alpha$ for $0 \leq \alpha \leq 90^\circ$.

c If the ball goes 30 m and the initial velocity is 20 m/s, find the angle of projection.

7 A large urn was filled with water. It was turned on, and the water was heated until its temperature reached 95°C. This occurred at exactly 2:00 p.m., at which time the urn was turned off and the water began to cool. The temperature of the room where the urn was located remained constant at 15°C. Commencing at 2:00 p.m. and finishing at midnight, Jenny measured the temperature of the water every hour on the hour for the next 10 hours and recorded the results.

At 4:00 p.m. Jenny recorded the temperature of the water to be 55°C. She found that the temperature ($T$ degrees Celsius) of the water could be described by the equation:

$$T = Ae^{-kt} + 15, \text{ where } 0 \leq t \leq 10$$

where $t$ is the number of hours after 2:00 p.m.
a Find the values of $A$ and $k$.

b Find the temperature at midnight.

c At what time did Jenny first record a temperature less than $24^\circ C$?

d Sketch the graph of $T$ against $t$.

8 On an overnight interstate train an electrical fault meant that the illumination in two carriages, $A$ and $B$, was affected. Before the fault occurred the illumination in carriage $A$ was $I$ units and $0.66I$ units in carriage $B$. Every time the train stopped the illumination in carriage $A$ reduced by 17% and by 11% in carriage $B$.

a Write down exponential expressions for the expected illumination in each carriage after the train had stopped for the $n$th time.

b At some time after the fault occurred the illumination in both carriages was approximately the same. At how many stations did the train stop before this occurred?

9 The diagram shows a conical glass fibre. The circular cross-sectional area at end $B$ is 0.02 mm$^2$. The cross-sectional area diminishes by a factor of $(0.92)^{1/10}$ per metre length of the fibre. The total length is 5 m.

a Write down a rule for the cross-sectional area of the fibre at a distance $x$ m from $B$.

b What is the cross-sectional area of the fibre at a point one-third of its length from $B$?

c The fibre is constructed in such a way that the strength increases in the direction $B$ to $A$. At a distance $x$ m from $B$ the strength is given by the rule $S = (0.92)^{10-3x}$.

If the load the fibre will take at each point before breaking is given by load $= \text{strength} \times \text{cross-sectional area}$, write down an expression, in terms of $x$, for the load the fibre will stand at a distance $x$ m from $B$.

d A piece of glass fibre that will have to carry loads of up to $0.02 \times (0.92)^{2.5}$ units is needed. How much of the 5 m fibre could be used with confidence for this purpose?

10 A pizza is divided by a number of straight cuts as shown. The table shows the largest number of pieces $f(n)$ into which it is divided by $n$ cuts.

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(n)$</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a Find a quadratic model for this data.

b Use your model to find the greatest number of pizza pieces produced by:

i 4 straight cuts

ii 5 straight cuts

c Check your answers to b by drawing diagrams.
11 a The graph is of one complete cycle of:
\[ y = h - k \cos \left( \frac{\pi t}{6} \right) \]
- How many units long is OP?
- Express OQ, OR in terms of h and k.

b The number of hours of daylight on the 21st of each month in a city in the northern hemisphere is given by the table:

<table>
<thead>
<tr>
<th>x</th>
<th>Dec</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>7.5</td>
<td>8.2</td>
<td>9.9</td>
<td>12.0</td>
<td>14.2</td>
<td>15.8</td>
<td>16.5</td>
<td>15.9</td>
<td>14.3</td>
<td>12.0</td>
<td>9.8</td>
<td>8.1</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Using suitable scales, plot these points and draw a curve through them. Call December month 0, January month 1, etc., and treat all months as of equal length.

c Find the values of h and k so that your graph is approximately that of:
\[ y = h - k \cos \left( \frac{\pi t}{6} \right) \]

12 a In the figure \( y = 1 - a(x - 3)^2 \) intersects the x-axis at A and B. Point C is the vertex of the curve and a is a positive constant.
- Find the coordinates of A and B in terms of a.
- Find the area of triangle ABC in terms of a.

b The graph shown has rule:
\[ y = (x - a)^2(x - 2a) + a \text{ where } a > 0 \]
- Use a calculator to sketch the graph for \( a = 1, 2, 3 \)
- Find the values of a for which \( \frac{-4}{27} a^3 + a = 0 \)
- Find the values of a for which \( \frac{-4}{27} a^3 + a < 0 \)
- Find the value of a for which \( \frac{-4}{27} a^3 + a = -1 \)
- Find the value of a for which \( \frac{-4}{27} a^3 + a = 1 \)
- Plot the graphs \( y = (x - a)^2(x - 2a) + a \) for the values of a obtained in iv and v.

c Triangle PSQ is a right-angled triangle.
- Give the coordinates of S.
- Find the length of PS and SQ in terms of a.
- Give the area of triangle PSQ in terms of a.
- Find the value of a for which the area of the triangle is 4.
- Find the value of a for which the area of the triangle is 1500.