19.1 Multiple-choice questions

1. A box contains 12 red balls and 4 green balls. A ball is selected at random from the box and not replaced, and then a second ball is drawn. The probability that the two balls are both green is equal to:
   A $\frac{1}{4}$  B $\frac{1}{16}$  C $\frac{3}{64}$  D $\frac{1}{8}$  E $\frac{1}{20}$

2. In a six-item true/false test the probability that a student who guesses will obtain six correct answers is:
   A 0.9844  B 0.0278  C 0.5  D 0.0156  E 0.17

Questions 3, 4, 5 and 6 refer to the following probability distribution:

<table>
<thead>
<tr>
<th>$x$</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr($X = x$)</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.4</td>
</tr>
</tbody>
</table>

3. For this probability distribution the mean, $E(X)$, is equal to:
   A 6.7  B 0.275  C 6.5  D 2.75  E 2.59

4. For this probability distribution the variance, $Var(X)$, is equal to:
   A 19.45  B 4.41  C 6.7  D 2.1  E 0.61

5. Let $Y = 2X - 1$, where $X$ has the probability distribution given in the table. The probability distribution of $Y$ is:
   A $ y $ | 4 | 6 | 7 | 9 |
   | Pr($Y = y$) | 0.3 | 0.2 | 0.1 | 0.4 |

   B $ y $ | 8 | 12 | 14 | 18 |
   | Pr($Y = y$) | 0.3 | 0.2 | 0.1 | 0.4 |
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6. Let $Z = 4 - X$, where $X$ has the probability distribution given in the table. The variance of $Z$ is:
   A. 11.59  B. 2.1  C. 0.41  D. 4.41  E. -0.41

7. $X$ has the probability distribution given by:
   ![Table with columns $x$ and $Pr(X=x)$ with values 1: $4c^2$, 2: $5c^2$, 3: $4c^2$, 4: $3c^2$]

   The value of $c$ is:
   A. 0.5  B. 0.0263  C. 0.1622  D. 0.25  E. 0.0625

8. Suppose a spinner numbered 1, 2, 3, 4, 5, 6 is spun until a ‘3’ appears. The number of spins is noted. The sample space for this random experiment is:
   A. {1, 2, 3, 4, 5, 6}  B. {0, 1, 2, 3, 4, 5, 6}  C. {1, 2, 3, 4, ...}  D. {3}  E. {1, 2, 3}

9. Suppose that in Melbourne the probability of the temperature exceeding 30°C on a particular day is 0.6 if the temperature exceeded 30°C on the previous day, and 0.25 if it did not. If the temperature exceeds 30°C on Monday, then the probability that it exceeds 30°C on Wednesday is:
   A. 0.36  B. 0.10  C. 0.60  D. 0.30  E. 0.46

10. For a random variable $X$, $E(X) = 11$ and $E(X^2) = 202$. The standard deviation of $X$ is equal to:
    A. 191  B. 13.82  C. 9  D. 3.72  E. 81

11. A set of test scores have a probability distribution with mean $\mu = 50$ and the standard deviation $\sigma = 10$. Which of the following intervals contains about 95% of the test scores?
    A. (40, 60)  B. (30, 70)  C. (20, 80)  D. (46.84, 53.16)  E. (43.68, 56.32)

12. If three fair coins are tossed what is the probability that there are at least two heads?
    A. $\frac{1}{3}$  B. $\frac{6}{7}$  C. $\frac{1}{4}$  D. $\frac{1}{2}$  E. $\frac{1}{8}$

13. Let $X$ be a binomial random variable with parameters $n = 400$ and $p = 0.1$. The mean of $X$, $E(X)$, is equal to:
    A. 36  B. 6  C. 40  D. 6.32  E. 360
14 Which of the following does not define a binomial variable?

A A die is tossed 10 times and the number of sixes observed.
B A die is rolled until a six is obtained and the number of rolls counted.
C A die is rolled five times and the number of even numbers showing observed.
D A sample of 20 people is chosen from a population and the number of females counted.
E The number of questions answered correctly by a student who is guessing every question is noted.

15 Let \( X \) be a binomial random variable with parameters \( n = 900 \) and \( p = 0.2 \). The standard deviation of \( X \) is equal to:

A 18  B 144  C 180  D 13.42  E 12

16 Let \( X \) be a binomial random variable with a variance of 9.4248. If \( n = 42 \) then the probability of success \( p \) is equal to:

A 0.45  B 0.22  C 0.34  D 0.68  E 0.34 or 0.66

17 \( \binom{7}{5} p^5 (1 - p)^2 \) is the probability of:

A exactly two failures  B exactly two successes  C at least two failures  D exactly five failures  E more failures than successes

18 The proportion of female students in a particular university is 0.2. A sample of 10 students is chosen at random from the entire student population. What is the probability that the sample contains exactly four female students?

A 0.0881  B 0.5000  C 0.0328  D 0.0016  E 0.9672

19 Jane decides to call five friends to invite each of them to a party. The probability of a friend not being home when Jane calls is 0.4. What is the probability that Jane finds at least one of her friends at home?

A 0.0778  B 0.9222  C 0.0102  D 0.9898  E 0.0768

20 Consider the matrices

\[
U = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad V = \begin{bmatrix} 0.4 & 0.5 \\ 0.6 & 0.5 \end{bmatrix} \quad W = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.4 \end{bmatrix} \quad X = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.2 \end{bmatrix} \quad Y = \begin{bmatrix} 0.1 & 0 & 0.2 \\ 0.3 & 0.2 & 0 \\ 0.1 & 0.1 & 0.2 \end{bmatrix}
\]

The matrix that could be a transition matrix for a Markov chain is:

A U  B V  C W  D X  E Y

21 Suppose that a four-state Markov chain is defined by a transition matrix \( T \) and an initial state matrix \( S_0 \). Which of the following is a true statement?

A The dimension of \( T \) is 4 \( \times \) 4 and the dimension of \( S_0 \) is 4 \( \times \) 1.
B The dimension of \( T \) is 2 \( \times \) 2 and the dimension of \( S_0 \) is 2 \( \times \) 1.
C The dimension of \( T \) is 4 \( \times \) 1 and the dimension of \( S_0 \) is 4 \( \times \) 4.
The dimension of $T$ is $4 \times 4$ and the dimension of $S_0$ is $1 \times 1$.

The dimension of $T$ is $4 \times 4$ and the dimension of $S_0$ is $4 \times 4$.

A Markov chain is defined by a transition matrix $T = \begin{bmatrix} 0.59 & 0.44 \\ 0.41 & 0.56 \end{bmatrix}$ and an initial state matrix $S_0 = \begin{bmatrix} 100 \\ 500 \end{bmatrix}$. For this Markov chain, $S_2 =$

$A \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$  $B \begin{bmatrix} 279 \\ 321 \end{bmatrix}$  $C \begin{bmatrix} 558 \\ 642 \end{bmatrix}$  $D \begin{bmatrix} 305.85 \\ 294.15 \end{bmatrix}$  $E \begin{bmatrix} 309.8775 \\ 290.1225 \end{bmatrix}$

The following information is needed for Questions 23 and 24:
Suppose that the probability of a badminton player winning a point is 0.77 if he has won the preceding point and 0.44 if he has lost the preceding point, and that $X_n = 1$ if the player wins the $n$th point and $X_n = 0$ if the player loses the $n$th point.

23. The steady state probability, $\Pr(X_n = 1)$, is equal to:

A 0.66  B 0.34  C 44/67  D 23/67  E 0.77

24. The probability that the player wins four points in a row after losing the first point, and then loses the next point, is closest to:

A 0.0462  B 0.0600  C 0.2009  D 0.1364  E 0.1050

25. If a random variable $X$ has probability density function

$$f(x) = \begin{cases} kx^3 + \frac{3}{4}x & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

then $k$ is equal to:

A $\frac{3}{16}$  B $\frac{6}{25}$  C $\frac{9}{16}$  D $\frac{1}{8}$  E $\frac{3}{8}$

26. If a random variable $X$ has probability density function

$$f(x) = \begin{cases} \frac{1}{9}(4x - x^2) & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

then $\Pr(X \leq 2)$ is closest to:

A 0.6667  B 0.4074  C 0.5926  D 0.4444  E 0.5556

27. If a random variable $X$ has probability density function

$$f(x) = \begin{cases} \frac{8}{3}(1 - x) & 0 < x < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

then the median of $X$ is equal to:

A 0.222  B 0.667  C 0.250  D 1.791  E 0.209
28 If a random variable $X$ has probability density function

$$f(x) = \begin{cases} \frac{24}{x^3} & 3 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

then the mode of $X$ is closest to:

A 4.5  
B 3  
C 6  
D 4  
E 3.33

29 If a random variable $X$ has a probability density function

$$f(x) = \begin{cases} 2 \left(1 - \frac{1}{x^2}\right) & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

then the mean of $X$ is closest to:

A 1  
B 1.614  
C 2  
D 1.5  
E 0.609

30 The probability of obtaining a $z$-value which falls between $z = -1.0$ and $z = 0$ for a standard normal distribution is approximately:

A 0.05  
B 0.20  
C 0.34  
D 0.68  
E 0.16

31 For a normal probability distribution which of the following is true?

A The mean is always positive.
B No value can be more than four standard deviations away from the mean.
C The area under the normal curve is approximately equal to 1.
D The standard deviation is always positive.
E The standard deviation is less than the mean.

32 If $X$ is a normally distributed random variable with mean $\mu = 2$ and standard deviation $\sigma = 0.5$, then the probability that $X$ is greater than 2.6 is closest to:

A 0.8849  
B 0.9918  
C 0.1151  
D 0.0082  
E 0.0302

33 If $X$ is a normally distributed random variable with mean $\mu = 2$ and standard deviation $\sigma = 2$, then the probability that $X$ is less than $-2$ is given by:

A 0.1587  
B 0.8413  
C 0.9772  
D 0.1228  
E 0.0228

34 If $X$ is a normally distributed random variable with mean $\mu = 3$ and variance $\sigma^2 = 0.4$, then the probability that $X$ is greater than $-2.73$ is given by:

A 0.1  
B 0  
C 0.9115  
D 0.0885  
E 0.5537

35 If $X$ is a normally distributed random variable with mean $\mu = 2$ and variance $\sigma^2 = 4$, then $\Pr(1 < X < 2.5)$ is given by:

A 0.5987  
B 0.2902  
C 0.6915  
D 0.4013  
E 0.3085
36 A machine fills cups with cordial when money is inserted in an automatic dispensing machine. If the amount of cordial in the cup is a normally distributed random variable with a mean of 50 mL and a standard deviation of 2 mL, then 90% of the cups contain more than:

A 44.87 mL  B 53.29 mL  C 46.71 mL  D 52.56 mL  E 47.44 mL

37 Lengths of blocks of cheese are found to be normally distributed with a mean of 10 cm and a variance of 0.5 cm. Then 95% of the blocks of cheese are shorter than:

A 11.39 cm  B 8.84 cm  C 11.16 cm  D 8.61 cm  E 9.18 cm

38 If $X$ is a normally distributed random variable with mean $\mu = 1$ and variance $\sigma^2 = 2.25$ and $\Pr(\mu - k < X < \mu + k) = 0.7$, then $k =$

A 1.555  B 1.037  C 0.787  D 0.524  E 2.332

39 The weight of a packet of biscuits is known to be normally distributed with a mean of 1 kg. If a packet is more than 0.05 kg underweight, it is unacceptable. If it is found that 3% of packets are unacceptable, then the standard deviation of the weight is:

A 1.881  B 0.027  C 10.488  D 0.030  E 37.616

40 Consider the diagram, which shows the probability density functions of two normally distributed random variables, one with a mean $\mu_1$ and a standard deviation $\sigma_1$, and the other with a mean $\mu_2$ and a standard deviation $\sigma_2$. Which of the following statements is true?

A $\mu_1 = \mu_2$, $\sigma_1 < \sigma_2$
B $\mu_1 = \mu_2$, $\sigma_1 > \sigma_2$
C $\mu_1 > \mu_2$, $\sigma_1 = \sigma_2$
D $\mu_1 < \mu_2$, $\sigma_1 = \sigma_2$
E $\mu_1 = \mu_2$, $\sigma_1 = \sigma_2$

41 If the heights of a certain population of men are normally distributed with a mean of 173 cm and a variance of 25 cm, then about 68% of men in the population have heights in the interval (in cm):


19.2 Extended-response questions

1 At each of a series of trials, the probability of the occurrence of a certain event is $\frac{1}{2}$, except that it cannot occur in two consecutive trials.

a Show that the probability of it occurring just twice in three trials is $\frac{1}{4}$, and ii in four trials is $\frac{1}{2}$.

b What is the probability of it occurring just twice in five trials?
2 Katia and Mikki play a game in which a fair six-sided die is thrown five times. Katia will receive $1 from Mikki if there is an odd number of sixes, and Mikki will receive $x from Katia if there is an even number of sixes. Find $x$ for the game to be fair. (Count zero as even.)

3 Lleyton Hewitt and Roger Federer are playing the final of a tennis tournament. The winner of the match will be the first player to win two sets. The result of each set follows a Markov sequence, as described below:

\[
\begin{align*}
\Pr(L_1) &= 0.45 \\
\Pr(R_1) &= 0.55 \\
\Pr(L_{i+1} | L_i) &= 0.6 & \Pr(L_{i+1} | R_i) &= 0.45 \\
\Pr(R_{i+1} | L_i) &= 0.4 & \Pr(R_{i+1} | R_i) &= 0.55
\end{align*}
\]

where $L$ represents the event that Lleyton Hewitt wins, and $R$ represents the event that Roger Federer wins.

a What is the probability that Lleyton Hewitt wins the first two sets?

b What is the probability that Roger Federer wins the first two sets?

c What is the probability that Lleyton Hewitt will win the match:

i given that he wins the first set?

ii given that Roger Federer wins the first set?

d What is the probability that Lleyton Hewitt will win the match?

4 A newspaper seller buys papers for 50 cents and sells them for 75 cents, and cannot return unsold papers. Daily demand has the following distribution, and each day’s demand is independent of the previous day’s demand:

<table>
<thead>
<tr>
<th>Number of customers</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.05</td>
<td>0.10</td>
<td>0.10</td>
<td>0.25</td>
<td>0.25</td>
<td>0.15</td>
<td>0.10</td>
</tr>
</tbody>
</table>

If the newspaper seller stocks too many papers a loss is incurred. If too few papers are stocked potential profit is lost because of the excess demand. Let $s$ represent the number of newspapers stocked, and $X$ the daily demand.

a If $P$ is the newspaper seller’s profit for a particular stock level $s$, find and expression for $P$ in terms of $s$ and $X$.

b Find the expected value of the profit, $E(P)$, when $s = 26$.

c Hence find an expression for the expected profit when $s$ is unknown.

d By evaluating the expression for expected profit for different values of $s$ determine how many papers the newspaper seller should stock.

5 Show that if $X$ is a random variable with mean $\mu$ and variance $\sigma^2$, and $Z = \frac{X - \mu}{\sigma}$, then $E(Z) = 0$ and $\text{Var}(Z) = 1$.

6 Anne and Jane play a game against each other, which starts with Anne aiming to throw a bean bag into a circle marked on the ground.

a The probability that the bean bag lands entirely inside the circle is $\frac{1}{2}$, and the probability that it lands on the rim of the circle is $\frac{1}{3}$.
i Show that the probability that the bag lands entirely outside the circle is $\frac{1}{6}$.

ii What is the probability that two successive throws land outside the circle?

iii What is the probability that for two successive throws, the first lands on the rim of the circle and the second inside the circle?

b Jane then shoots at a target on which she can score 10, 5 or 0. With any one shot the probability that she scores 10 is $\frac{2}{5}$, the probability that she scores 5 is $\frac{1}{10}$, and the probability that she scores 0 is $\frac{1}{2}$. With exactly two shots, what are the probabilities that she scores:

i 20?

ii 10?

c When the bean bag thrown by Anne lands outside the circle, Jane is allowed two shots at her target. If however, the bean bag lands on the rim of the circle Jane has one shot, and if it lands inside the circle Jane is not allowed any shots. Find the probability that Jane scores 10 as a result of any one throw from Anne.

7 Suppose that the outcome of a rugby test series between Australia and England follows a Markov sequence. If $X_i = 0$ if England wins game $i$, and $X_i = 1$ if Australia wins game $i$ then:

$$
\begin{align*}
\Pr(X_0 = 0) &= \frac{5}{8} \\
\Pr(X_{i+1} = 0 | X_i = 0) &= \frac{3}{4} \\
\Pr(X_{i+1} = 0 | X_i = 1) &= \frac{1}{2} \\
\Pr(X_0 = 1) &= \frac{3}{8} \\
\Pr(X_{i+1} = 1 | X_i = 0) &= \frac{1}{4} \\
\Pr(X_{i+1} = 1 | X_i = 1) &= \frac{1}{2}
\end{align*}
$$

(Assume that the game is played until one team wins and that there are no draws.)

a Write down the transition matrix that describes this situation, and use it to determine the probability that:

i England wins game 3 if they win the first game

ii England wins game 5 if Australia wins the first game

b Find the steady state probability of Australia winning a game, $\Pr(X_n = 1)$.

8 Suppose that there are two dentists in a country town. Records show that, from year to year:

- Of Dr Laslett’s patients, 85% of patients will stay with Dr Laslett and 15% will move to Dr Kildare.
- Of Dr Kildare’s patients, 94% of patients will stay with Dr Kildare and 6% will move to Dr Laslett.

a Find the transition matrix that can be used to represent this information.

b At the beginning of the year 65% of the patients go to Dr Laslett and 35% to Dr Kildare. Find:

i the percentage of patients estimated to be at each dentist at the end of year 2

ii the percentage of patients estimated to be at each dentist after 10 years

(cont’d.)
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c  Assume that 12 000 patients are treated by dentists in the town each year. Neither of
the practices can afford to operate if the number of patients seen falls below 4000
annually. Will either of the dentists go out of business and, if so, who and during
which year?

9  The lifetime of a certain brand of light globe is normally distributed with mean \( \mu = 400 \) hours and standard deviation \( \sigma = 50 \) hours.
   a  Find the probability that a randomly chosen light globe will last more than 375 hours.
   b  The light globes are sold in boxes of 10. Find the probability that at least nine of the
globes in a randomly selected box will last more than 375 hours.

10  A large taxi company determined that the distance travelled annually per taxi is normally
distributed with a mean of 80 000 km and a standard deviation of 20 000 km.
   a  What is the probability that a randomly selected taxi will travel between 56 000 and
60 000 kilometres in a year?
   b  What percentage of taxis can be expected to travel either below 48 000 or above
96 000 kilometres in a year?
   c  How many of the 250 taxis in the fleet are expected to travel between 48 000 and
96 000 kilometres in a year?
   d  At least how many kilometres would be travelled by 85% of the taxis?

11  The amount of cereal in boxes, packed by a particular machine, is normally distributed
with mean \( \mu \) grams and standard deviation \( \sigma = 5 \) grams.
   a  Find:
      i  the proportion of boxes that will be underweight (that is, weigh less than
500 grams) when \( \mu = 505 \) grams
      ii  the value of \( \mu \) required to ensure that only 1% of boxes are underweight
   b  As a check on the setting of the machine a random sample of five boxes is chosen and
the setting changed if more than one of them is underweight. Find the probability that
the setting of the machine is changed when \( \mu = 505 \).

12  A random variable \( X \) is normally distributed with a mean of \( \mu = 2 \) and a variance
\( \sigma^2 = 13 \). A random variable \( Y \) is also normally distributed with a mean of \( \mu = -17 \) and a
variance \( \sigma^2 = \frac{1}{8} \). \( X \) and \( Y \) are independent.
   a  Find \( E(8X + Y) \).
   b  If \( X \) and \( Y \) are independent random variables, then:

\[
\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y), \quad a \text{ and } b \text{ are constants}
\]

Find \( \text{sd}(8X + Y) \).
   c  Given that a linear combination of two normally distributed random variables is also
normally distributed, find \( \Pr(8X + Y > -3) \).
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13 A factory has two machines that produce widgets. The time taken to produce a widget using machine 1, $X$, is normally distributed with a mean of 10 seconds and a standard deviation of 2 seconds. The time taken to produce a widget using machine 2, $Y$, has the following probability density function:

$$f(y) = \begin{cases} 
k(y - 8) & 8 < y < 12 \\
0 & \text{otherwise}
\end{cases}$$

a  i Find the value of $k$.
   ii Show that machine 1 has a greater probability of producing a widget in less than 11 seconds than machine 2.
   iii Find which machine, on average, is quicker in producing widgets.

b Suppose that 60% of the widgets manufactured at the factory are produced by machine 1, and 40% by machine 2. If a widget selected at random is known to have been produced in less than 10 seconds, what is the probability that it was produced by machine 1?

14 The queuing time, $X$ minutes, at the box office at a movie theatre has probability density function:

$$f(x) = \begin{cases} 
kx(100 - x^2) & 0 \leq x \leq 10 \\
0 & \text{otherwise}
\end{cases}$$

a Find:
   i the value of $k$
   ii the mean of $X$
   iii the probability that a theatregoer will have to queue for more than 3 minutes
   iv the probability that a theatregoer will have to queue for more than 3 minutes given that she queues for less than 7 minutes

b If 10 moviegoers go independently to the theatre, find the probability that at least five of them will be required to queue for more than 3 minutes.

15 Electronic sensors of a certain type fail when they become too hot. The temperature at which a randomly chosen sensor fails is $T \degree C$, where $T$ is modelled as a normal random variable with mean $\mu$ and standard deviation $\sigma$.

a In a laboratory test, 98% of a random sample of sensors continued working at a temperature of 80$\degree$ but only 4% continued working at 104$\degree$.
   i Show the given information on a sketch of the distribution of $T$.
   ii Determine estimates of the values of $\mu$ and $\sigma$.

b More extensive tests confirm that $T$ is normally distributed, but with $\mu = 94.5$ and $\sigma = 5.7$. Use these values in the rest of the question.
   i Determine what proportion of sensors will operate in boiling water (i.e. at 100$\degree$).
   ii The manufacturers wish to quote a safe operating temperature at which 99% of the sensors will work. What temperature should they quote?

16 Jam is packed in tins of nominal net weight 1 kg. The actual weight of jam delivered to a tin by the filling machine is normally distributed about the mean weight set on the machine, with a standard deviation of 12 g.
If the machine is set to 1 kg find the probability that a tin chosen at random contains less than 985 g.

b It is a legal requirement that no more than 1% of tins contains less than the nominal weight. Find the minimum setting of the filling machine which will meet this requirement.

17 The random variable $X$ has the probability function

$$
Pr(X = x) = \begin{cases} 
\frac{c}{x} & x = 1, 2, \ldots, 6 \\
0 & \text{otherwise}
\end{cases}
$$

where $c$ is a constant.

Find the value of:

a $c$

b $E(X)$

c $\text{Var}(X)$

18 A flight into an airport is declared to be ‘on time’ if it touches down within 3 minutes either side of the advertised arrival time; otherwise it is declared early or late. On any one occasion, the probability that a flight is on time is 0.5 and the probability that it is late is 0.3. The time of arrival on each day of a particular flight is independent of the time of arrival on any other day.

a Calculate the probabilities that:

i on any one day the flight arrives early

ii on any one day it does not arrive late

iii it arrives on time on three consecutive days

iv it arrives late on Monday one week, but is on time for all of the remaining 4 weekdays

b In a given week of 5 days find the probability that it is:

i late exactly once

ii early exactly twice

c The airline is reported to the authority if the flight is late on more than two occasions in a 5-day week. Find the probability that this happens.

19 In a factory machines $A$, $B$, and $C$ are all producing springs of the same length. Of the total production of springs in the factory, machine $A$ produces 35% and machines $B$ and $C$ produce 25% and 40% respectively. Of their production machines $A$, $B$ and $C$ produce 3%, 6% and 5% defective springs respectively.

a Find the probability that:

i a randomly selected spring is produced by machine $A$ and is defective

ii a randomly selected spring is defective

b Given that a randomly selected spring is defective find the probability that it was produced by machine $C$.

c Given that a randomly selected spring is not defective find the probability that it was produced by either machine $A$ or machine $B$. 

20 An electronic game comes with five batteries. The game only needs four batteries to work, but because the batteries are sometimes faulty the manufacturer includes five of them with the game. Suppose that \( X \) is the number of good batteries included with the game. The probability distribution of \( X \) is given in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Pr(X = x) )</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>0.45</td>
<td>0.45</td>
</tr>
</tbody>
</table>

a Use the information in the table to:

i find \( \mu \), the expected value of \( X \)

ii find \( \sigma \), the standard deviation of \( X \) correct to one decimal place

iii find, exactly, the proportion of the distribution that lies within two standard deviations of the mean

iv find the probability that a randomly selected game works, i.e. \( \Pr(X \geq 4) \)

b The electronic games are packed in boxes of 20. Whether or not an electronic game in the box will work is independent of any other game in the box working. Let \( Y \) be the number of working games in the box.

i Name the distribution of \( Y \).

ii Find the expected number of working games in the box.

iii Find the standard deviation of the number of working games in the box.

iv Find the probability that a randomly chosen box will contain at least 19 working games.