CAMBRIDGE TECHNOLOGY IN MATHS

Year 12

Arithmetic and geometric sequences for the TI-83/84

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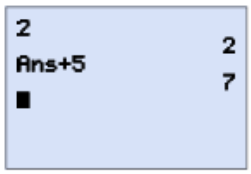
How to generate the terms of an arithmetic sequence using the TI-83/84's Home screen

Generate the first five terms of the arithmetic sequence: 2, 7, 12, 17, 22, . . .

Steps

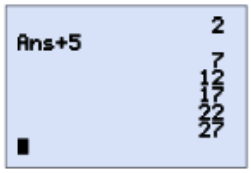
1 Start on the **Home** screen. **Clear**. Enter the value of the first term 2. Press ENTER.

2 The common difference for this sequence is 5. So, type in $+5$. Press ENTER. The second term in the sequence, 7, is generated.



3 Pressing ENTER again generates the next term, 12.

4 Pressing ENTER again generates the next term, 17. Keep pressing ENTER until the required number of terms is generated.



How to generate the terms of a sequence using the TI-83/84

Generate the terms in an arithmetic sequence with $a = 10$ and $d = 4$.

Steps

Strategy: Find an expression for the nth term of the sequence as for Example 5. A graphics calculator can then be used to display the sequence in a table.

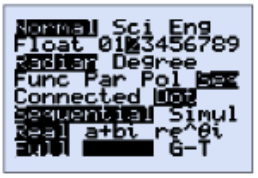
1 For this sequence, $a = 10$ and $d = 4$.

$a = 10,\ d = 4$

$t_n = 10 + (n - 1) \times 4$

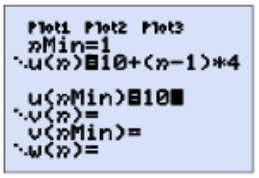
2 Use $t_n = a + (n - 1)d$ to write down an expression for the nth term, t_n. Don't simplify.

3 Set the calculator to **Seq**uence mode. Press [MODE] then use the arrow keys ([▼] and [▶]) to highlight **Seq**. Press [ENTER] to select. The calculator is now in **Seq**uence mode.



4 Enter the expression for the nth term. Press [Y=] and, as opposite,

- ***n*Min** = enter **1**, the minimum value of n.
- ***u*(n)** = enter **10 + (n - 1)*4**, the expression for t_n.
- ***u*(*n*Min)** = enter **10**, the value of a.

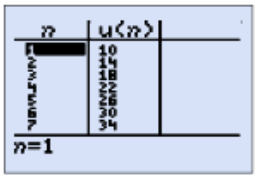


Notes:

1 The calculator uses ***u*(*n*)** to represent the nth term of a sequence.

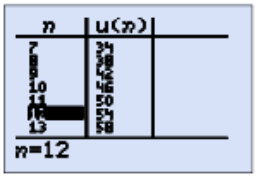
2 When you are in sequence mode, the [X,T,Θ,n] key changes from **X** to ***n***.

5 Press [2nd] [TABLE] to display the sequence in a table. The first column ***n*** displays the term numbers, 1, 2, 3, . . . and the second column ***u*(*n*)** displays the values of the terms in the sequence: 10, 14, 18, 22, 26, 30, 34, . . .



Note: If the values of ***n*** do not increase in steps of 1, press [2nd] [TBLSET] and set △ **Tbl** = **1**. Pressing [2nd] [TABLE] returns you to the table.

6 Finally, press the down arrow key ([▼]) to display further terms.



Example: Finding when a term in a sequence first exceeds a given value

How many terms would we have to write down in the arithmetic sequence 10, 14, 18, 22, ... before we found a term greater than 51?

Solution

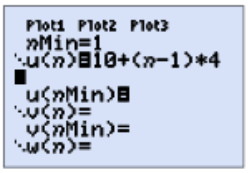
Strategy: Find an expression for the nth term of the sequence as for Example 5. A graphics calculator can then be used to display the sequence in a table. The first term that exceeds 51 can then be found.

1 For this sequence, $a = 10$ and $d = 4$

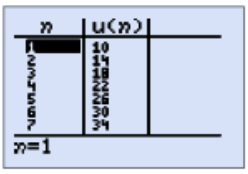
$a = 10, d = 4$

$t_n = 10 + (n - 1) \times 4$

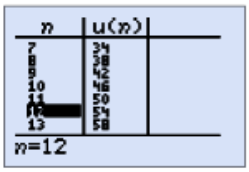
2 Using **Seq**uence mode, enter the expression for u_n into your graphics calculator and tabulate the sequence.



3 Press [2nd] [TABLE] to display the sequence in a table.



4 Finally, use the down arrow key ([▼]) to step down through the table until you find the first term in the sequence that exceeds 51. This is clearly the 12th term.



5 Write down key values in the sequence (to show how you solved the problem) and your answer.

n	1	2	...	10	11	12	...
t_n	10	14	...	46	50	54	...

The first term to exceed 51 is t_{12}.

Example: Application of the *n*th term of an arithmetic sequence

Before starting on a weight-loss program a man weighs 124 kg. He plans to lose weight at a rate of 1.5 kg a week until he reaches his recommended weight of 94 kg.

a Write down a rule for the man's weight, W_n, at the start of week n.

b If he keeps to his plan, how many weeks will it take the man to reach his target weight of 94 kg?

Solution

Strategy: You need to recognise that by losing a constant amount of weight each week, the man's weekly weight follows an arithmetic sequence. Using this information, you can write down an expression for his weight in the nth week. You can then use this expression to display the sequence of weights in a table and hence determine when the target weight is reached.

a 1 Arithmetic sequence with $a = 124$ and $d = -1.5$

2 Use the rule $W_n = a + (n-1)d$ to write down an expression for W_n.

Arithmetic sequence

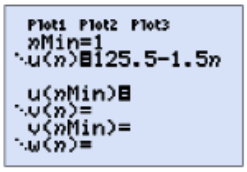
$a = 124,\ d = -1.5$

$W_n = 124 + (n-1) \times (-1.5)$

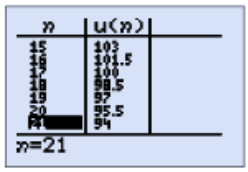
$= 124 - 1.5n + 1.5$

$\therefore W_n = 125.5 - 1.5n$

b 1 Using **Seq**uence mode, enter the expression for W_n into your graphics calculator and tabulate the sequence.



2 Press 2nd [TABLE] to display the sequence in a table. Use the down arrow key (▼) to find the first term in the sequence that is 94 or less.



3 Write down key values in the sequence (to show how you solved the problem) and your answer.

n	1	2	...	19	20	21	...
W_n	124	122.5	...	97	95.5	94	...

If the man keeps to his plan, he will reach his target weight by the start of week 21, or after 20 weeks of being on the program.

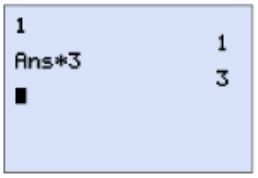
How to generate the terms of a geometric sequence using the TI-83/84's Home screen

Generate the first five terms of the arithmetic sequence 1, 3, 9, 27, . . .

Steps

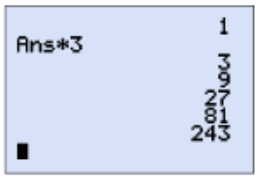
1 Start on the **Home** screen. **Clear**. Enter the value of the first term **1**. Press ENTER.

2 The common ratio for this sequence is 3. So, type in ***3**. Press ENTER. The second term in the sequence, 3, is generated.



3 Pressing ENTER again generates the next term, 9.

4 Pressing ENTER again generates the next term, 27. Keep pressing ENTER until the required number of terms is generated.



How to generate the terms of a sequence using the TI-83/84

Plot the terms of the following sequences on the same graph:

- sequence 1: arithmetic with $a = 2$ and $d = 2$
- sequence 2: geometric with $a = 2$ and $r = 2$

for $n = 1, 2, \ldots 6$. These are the sequences plotted previously.

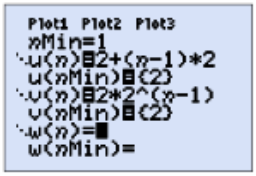
Steps

1 Write down an expression for the nth term of the two sequences using the rules.
Set the calculator to **Seq**uence mode.

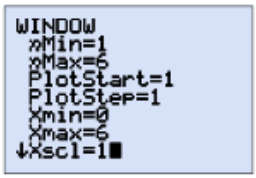
arithmetic: $t_n = 2 + (n - 1) \times 2$
geometric: $t_n = 2 \times 2^{(n-1)}$

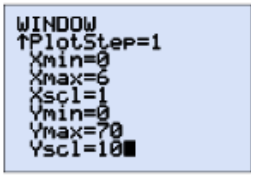
2 Enter the expression for the nth terms.
Press [Y=] and, opposite:

- ***n*Min** = enter **1**, the minimum value of n
- ***u*(*n*)** = enter **2 + (n - 1)*2**, the expression for t_n for the arithmetic sequence
- ***u*(*n*Min)** = enter **2**, the value of a (the curly brackets will appear as you move on)
- ***v*(*n*)** = enter **2*2^(n - 1)**, the expression for t_n for the geometric sequence.
- ***v*(*n*Min)** = enter **2**, the value of a (the curly brackets will appear as you move on)

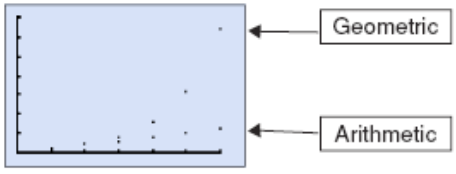


3 Set up the viewing window as shown opposite (2 screens).





4 Press [GRAPH] to plot.
Note: The arithmetic sequence increases in a linear manner, while the geometric sequence increases in an exponential manner.



Questions on generating the terms of a sequence using the TI-83/84

Using a graphics calculator, on the same axes plot the first five terms of the following pairs of sequences. In each case, comment on the differences that you notice in the two plots.

Note: You will need to readjust the window settings as you move through the exercises.

1 a arithmetic sequence: $a = 32$ and $d = 0.5$ geometric sequence: $a = 32$ and $r = 0.5$

b arithmetic sequence: $a = 1$ and $d = 2$ geometric sequence: $a = 1$ and $r = 2$

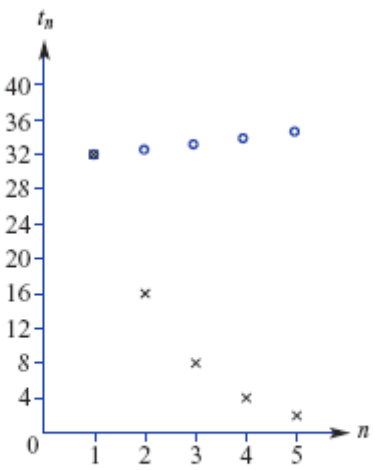
c arithmetic sequence: $a = 100$ and $d = -5$ geometric sequence: $a = 100$ and $r = 0.5$

d arithmetic sequence: $a = 100$ and $d = 5$ geometric sequence: $a = 1$ and $r = 5$

Answers

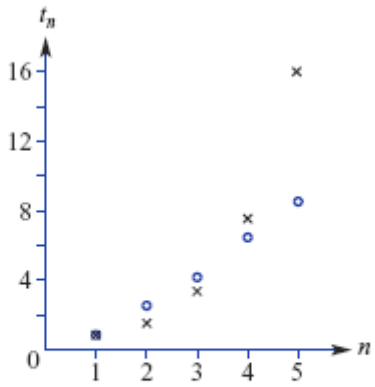
Graphing questions

1 a



The arithmetic sequence is linear and increasing.
The geometric sequence is exponential and decreasing rapidly.

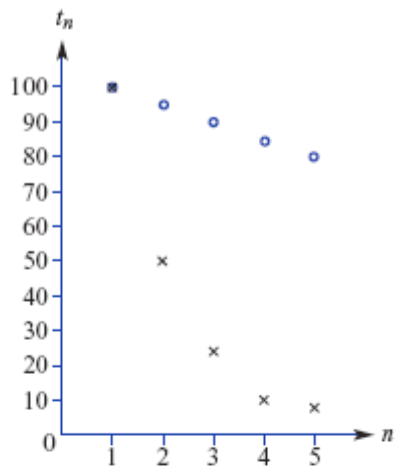
b



The arithmetic sequence is linear and increasing.
The geometric sequence is exponential and increasing.

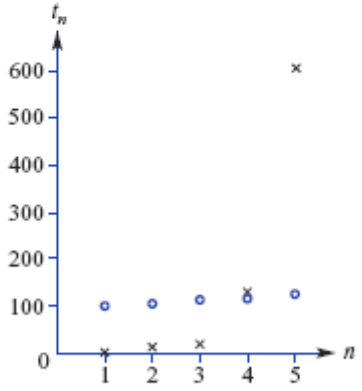
The geometric sequence falls behind the arithmetic sequence but quickly passes it.

c



The arithmetic sequence is linear and decreasing.
The geometric sequence is exponential and decreasing rapidly.

d



The arithmetic sequence is linear and increasing.
The geometric sequence is exponential and increasing.
The terms in the geometric sequence are initially smaller than those of the arithmetic sequence, but soon exceed them in size.