CAMBRIDGE TECHNOLOGY IN MATHS

Year 11

Rates of change

CONTENTS

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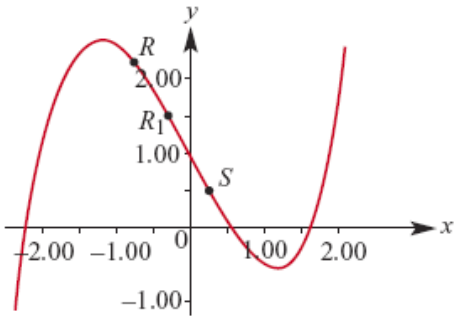
Approximating rate of change using a CAS calculator

The gradient of $y = 0.5x^3 - 2x + 1$ at the point (0, 1) will be investigated.

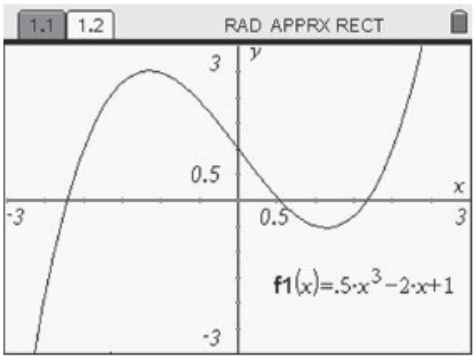
The graph of $y = 0.5x^3 - 2x + 1$ is shown.



Method A

Using the TI-Nspire:

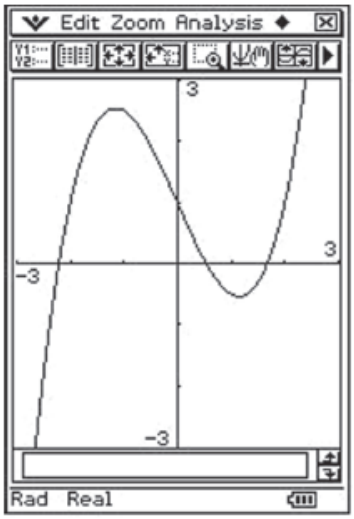
1 Enter the equation into $f1(x)$, press (enter) and choose a suitable Window.



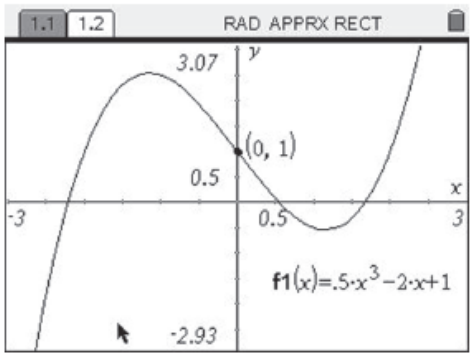
Method A

Using the ClassPad:

1 Enter the equation into $y1$, press (EXE), tap [A] and choose a suitable Window.



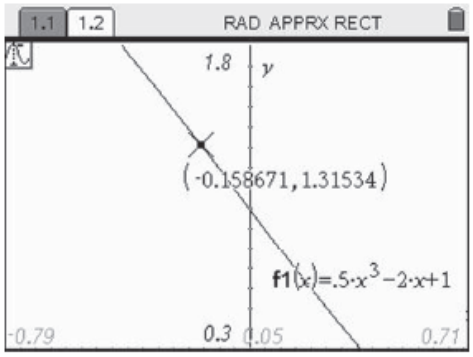
2 Use *Graph Trace* to take the cursor to the point (0, 1).

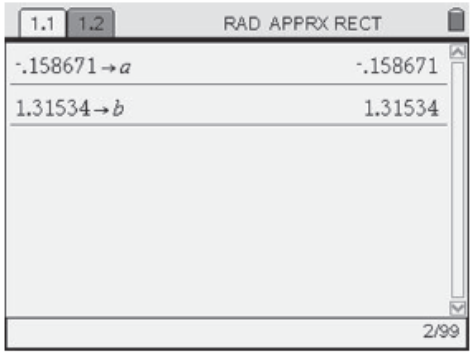


3 Choose **3:Zoom In** from the Window menu and press enter.

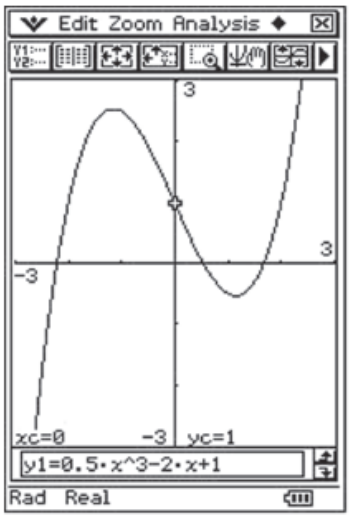
4 Take the cursor to the point (0, 1) and again choose **3:Zoom In** from the Window menu.

5 Choose two points on this section of the curve through (0, 1) and save the values as (A, B) and (C, D).





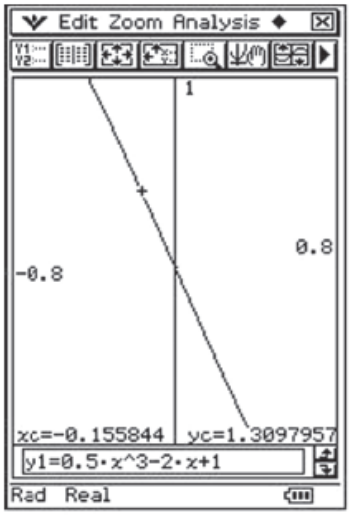
2 Use Trace to take the cursor to the point (0, 1).

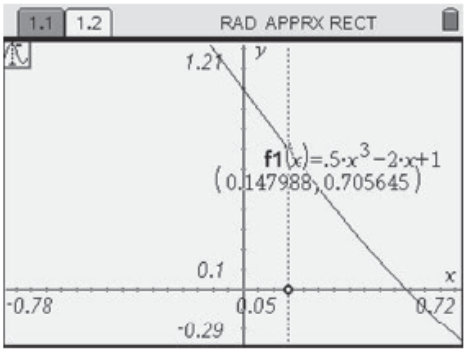


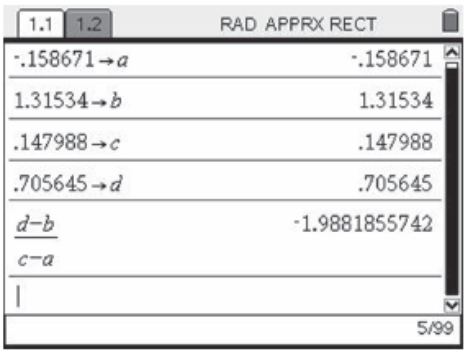
3 Tap **Zoom In** from the Zoom menu.

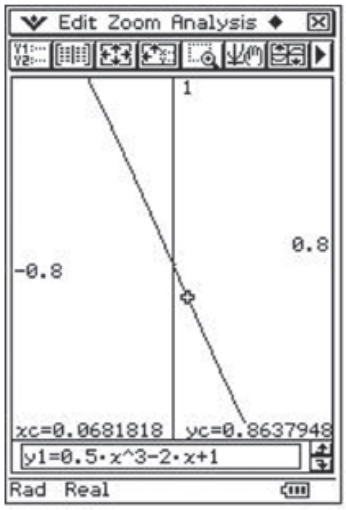
4 Take the cursor to the point (0, 1) and again tap **Zoom In** from the Zoom menu.

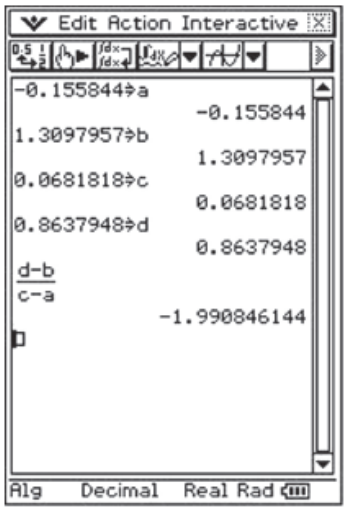
5 Choose two points on this section of the curve through (0, 1) and save the values as (A, B) and (C, D).









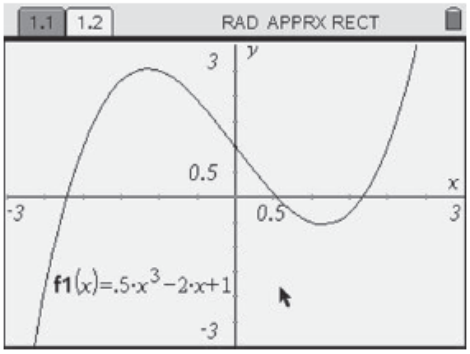


Hence, the gradient is close to –2.

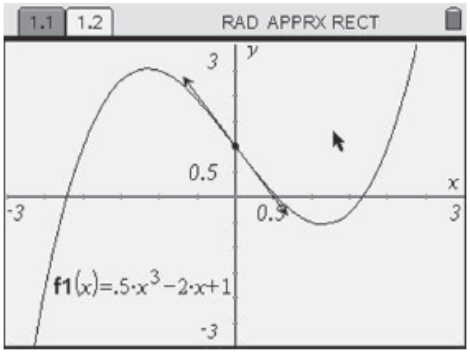
Method B

Using the TI-Nspire:

1 Display the plot $y = 0.5x^3 - 2x + 1$.

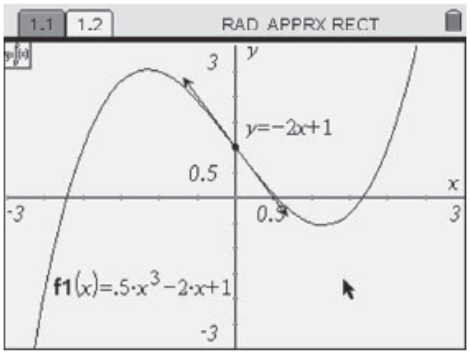


2 To draw a tangent line, select *Tangent* from the Points & Lines submenu. Move the cursor to the point (0, 1) and press enter to see the tangent.



3 To view the equation of the tangent line, select **6: Coordinates and Equations** from the Actions submenu.

4 Move the cursor to the tangent line and press enter to display its equation.

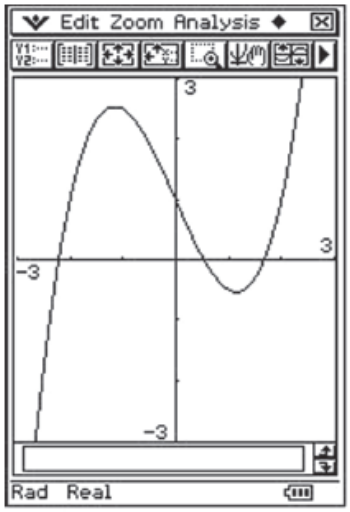


The gradient of the tangent is –2.

Method B

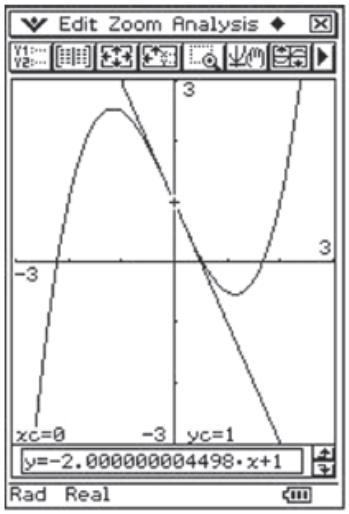
Using the ClassPad:

1 Display the plot $y = 0.5x^3 - 2x + 1$.



2 To draw a tangent line, tap Analysis, then select *Tangent* from the Sketch submenu.

3 Move the cursor to the point (0, 1) and then press EXE.

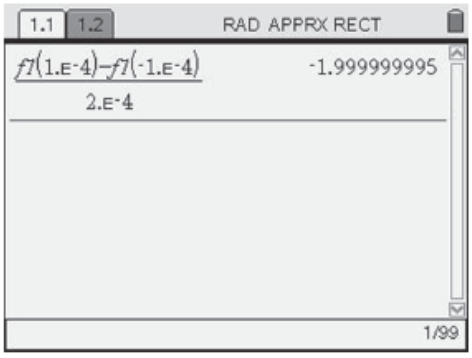


4 The equation of the tangent is displayed on the screen.

Alternatively, the rate of change may be approximated directly using the Calculator/Main application.

Method C

With the equation stored in $f1(x)$, type the following:



Method C

With the equation stored in $y1$, type the following:

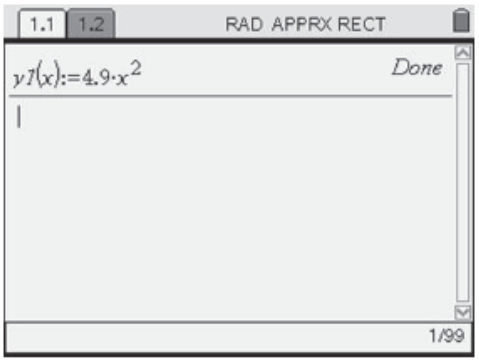
Gradient $= -1.999999\ldots \approx -2$

Calculating the gradient at a point using a CAS calculator

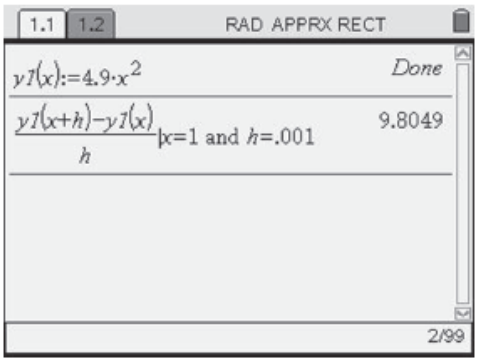
Using the TI-Nspire:

In the Calculator application do the following:

1 Define the function $y = 4.9x^2$ by typing $y1(x) := 4.9x\text{^}2$ and then press (enter).



2 Now type $(y1(x+h) - y1(x))/h|x = 1$ and $h = 0.001$ and press (enter).



Using the ClassPad:

In the Main application do the following:

1 Define the function $y = 4.9x^2$ by typing **Define** $y(x) = 4.9x\text{^}2$ and then press (EXE).

2 Now type $(y(x+h) - y(x))/h|x = 1$ **and** $h = 0.001$ and press (EXE).

Example: Gradient of a curve

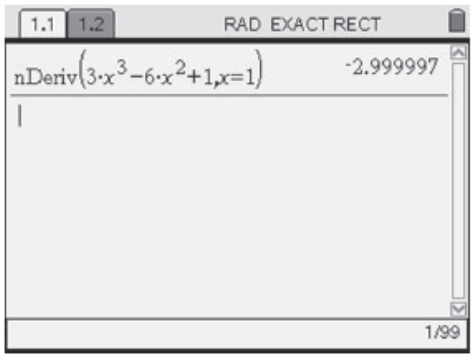
Find the gradient of the curve determined by the rule $f(x) = 3x^3 - 6x^2 + 1$ at the point $(1, -2)$.

Solution

The graphics calculator function **nDeriv** evaluates the numerical derivative at an **X** value.

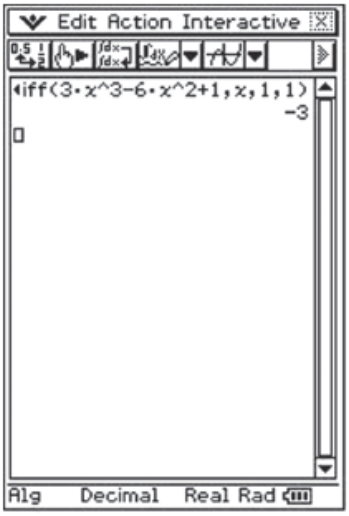
Using the TI-Nspire:

1 In the Calculator application, type **nDeriv** $(3x^3 - 6x^2 + 1, x = 1)$ and press enter.

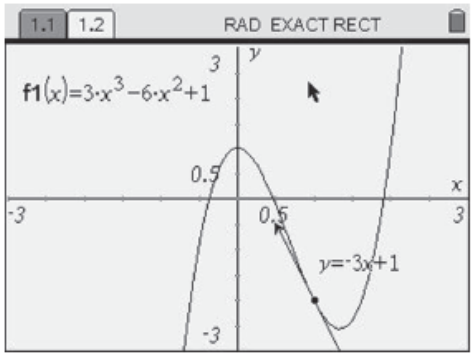


Using the ClassPad:

1 In the Main application, type **diff** $(3x^3 - 6x^2 + 1, x, 1, 1)$ and press EXE.



As described earlier, draw a tangent line and display its equation.



In the Main application, type **tanLine** $(3x^3 - 6x^2 + 1, x, 1)$ and then press EXE.

The tangent gradient is -3.

Example: Coordinates of points on curves

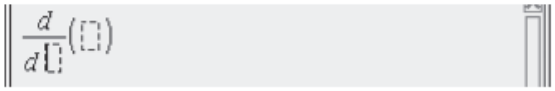
Find the coordinates of the points on curves determined by each of the following equations at which the gradient has the given values:

a $y = x^3$; gradient $= 8$ **b** $y = x^2 - 4x + 2$ gradient $= 0$

c $y = 4 - x^3$; gradient $= -6$

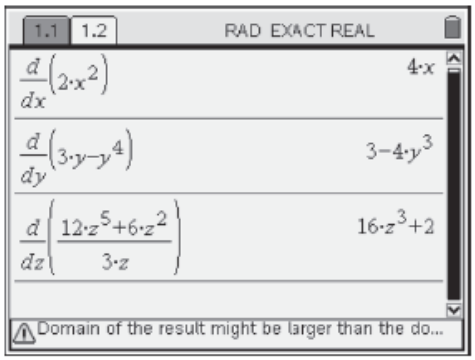
Using the TI-Nspire:

1 In the calculator application, press (menu) and select *Derivative* from the Calculus submenu.



2 In the denominator enter the variable with which you wish to differentiate.

3 In the brackets enter the expression you wish to differentiate, then press (enter).



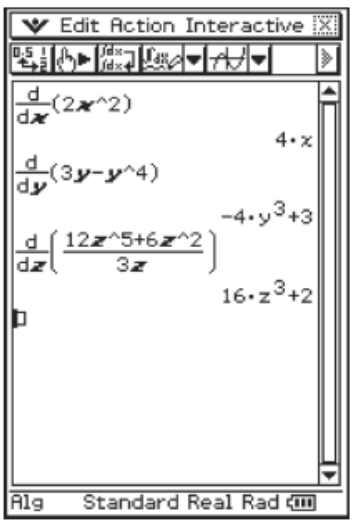
Using the ClassPad:

1 In the main application, press (Keyboard) and then tap the 2D tab.

2 Now tap the CALC tab to access the derivative operator shown below.

3 In the denominator enter the variable with which you wish to differentiate.

4 In the brackets enter the expression you wish to differentiate and then press (EXE).



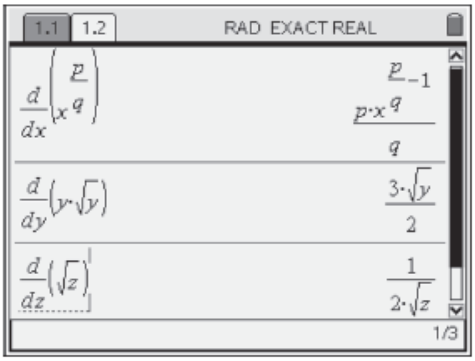
Example: Finding the derivative

Find the derivative of each of the following with respect to x:

a $4\sqrt[3]{x^2}$ **b** $\sqrt[5]{x} - \frac{2}{x^3}$ **c** $\frac{(1+\sqrt{x})^2}{\sqrt{x}}$

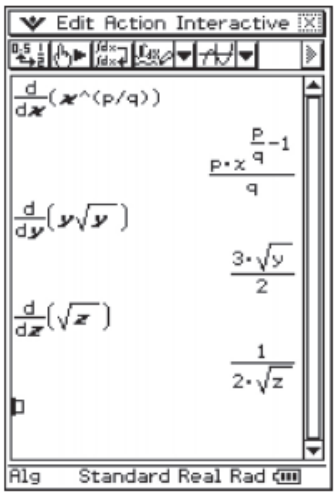
Using the TI-Nspire:

1. In the calculator application, access the derivative operator, as discussed earlier, for each of the steps below.
2. Place an x in the denominator, type $x\text{^}(p/q)$ in the brackets and then press enter.
3. Place a y in the denominator, type $y\sqrt{y}$ in the brackets and then press enter.
4. Place a z in the denominator, type $\sqrt{z}$ in the brackets and then press enter.



Using the ClassPad:

1. In the main application, access the derivative operator, as discussed earlier, for each of the steps below.
2. Place an x in the denominator, type $x\text{^}(p/q)$ in the brackets and then press EXE.
3. Place a y in the denominator, type $y\sqrt{y}$ in the brackets and then press EXE.
4. Place a z in the denominator, type $\sqrt{z}$ in the brackets and then press EXE.



Original location: Ch 9 Example 34 (p.542-543)