CAMBRIDGE TECHNOLOGY IN MATHS

Year 11

Sequence and series for the TI-83/84

CONTENTS

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How to use recursion to generate an arithmetic sequence using the TI-83/84

Use recursion on a graphics calculator to generate the arithmetic sequence: 2, 9, 16, . . .

a Write the first five terms. **b** Find the 10th term, t_{10}.

Steps

1 Find the common difference.

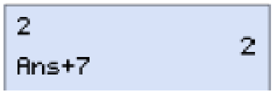
Common difference: $d = t_2 - t_1$
$= 9 - 2 = 7$

2 Check that this difference makes all the given terms.

+7 +7
2, 9, 16, . . .

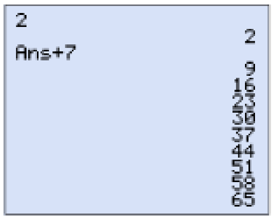
3 Press [2] [ENTER] so that the first term, 2, becomes the value of **Ans**, the most recent answer.

4 Press [+] [7], as 7 is the common difference. The last **Ans** variable is automatically inserted before **+7**.



Each time [ENTER] is pressed, 7 will be added to the previous answer.

5 To find the tenth term, t_{10}, we need to add 7 nine times. So press [ENTER] nine times.



6 **a** Write the first five terms.

(a) The first five terms are:
2, 9, 16, 23, 30.

b State the value of the 10th term.

(b) The tenth term is 65.

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How to generate a position counter beside the terms of an arithmetic sequence using the TI-83/84

Generate the arithmetic sequence: 26, 17, 8, −1, . . . with a position number in front of each term.

Steps

1 Find the common difference.

Common difference: $d = t_2 - t_1$

$= 17 - 26 = -9$

2 Check that this difference makes all the given terms.

$$\underset{26,}{} \overset{-9}{\quad} \underset{17,}{} \overset{-9}{\quad} \underset{8,}{} \overset{-9}{\quad} \underset{-1,}{} \quad \ldots$$

3 Enter {**1, 26**} using the curly brackets. Here **1, 26** says the 1st term is 26. The **1** starts the position counter and the **26** is the first term. This pair become **Ans(1), Ans(2)**, two previous answers for the counter and the term, which we can use to make the next counter number and the next term.

```
{1,26}
                 {1 26}
```

4 Enter {**Ans(1) +1, Ans(2)–9**} to add 1 to the counter and subtract 9 from the last term each time [ENTER] is pressed.

Tip: Press [2nd] [ANS] to type **Ans**.

Note: Ans is the second function of the [(-)] key.

```
{Ans(1)+1,Ans(2)
-9}
```

5 Press [ENTER] to generate each new term with its counter in front of it.

```
                {2 17}
                 {3 8}
                {4 -1}
               {5 -10}
               {6 -19}
               {7 -28}
               {8 -37}
```

6 Write the results in a table.

Position, n	1	2	3	4	. . .
Term, t_n	26	17	8	−1	. . .

Original location: Chapter 8 (p.305)

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Example: Application of an arithmetic sequence

The hire of a car costs \$180 for the first day and \$150 for each following day.

a How much would it cost to hire the car for 7 days?

b Find a rule for the cost of hiring the car for n days.

c For how many days can the car be hired using \$1530?

Solution

a

1 Identify values for a, n and d.

$a = 180, n = 7, d = 150$

2 Substitute the values for a, n and d into $t_n = a + (n - 1)d$.

$t_n = a + (n - 1)d$

$t_7 = 180 + (7 - 1)(150)$

3 Evaluate.

$= 180 + (6)(150)$

$= 1080$

4 Write your answer.

It would cost \$1080 to hire the car for 7 days.

b Substitute $a = 180$ and $d = 150$ into $t_n = a + (n - 1)d$.

$t_n = a + (n - 1)d$

$t_n = 180 + (n - 1)(150)$

Note: This rule is saying that it costs \$180 for the first day and the $(n - 1)$ days left each cost \$150.

c

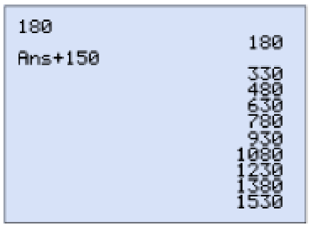
Method 1: Repeated use of the addition command on your graphics calculator

Strategy: Use your graphics calculator to enter \$180 as the first entry, then continue counting the number of times the **Ans + 150** command is entered until \$1530 is reached.

1 Press [1] [8] [0] [ENTER] so that 180 becomes the value of **Ans**, the most recent answer.

2 Press [+] [1] [5] [0], to use 150 as the common difference.

3 Count 180 as the first entry and count 1 more each time [ENTER] is pressed to make another term.

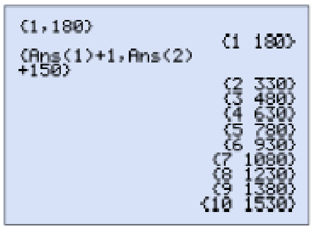


4 We find that 1530 occurs as the 10th term. Write your answer.

\$1530 can be used to hire the car for 10 days.

Method 2: Generate a position counter beside each term

Strategy: Using a counter is very helpful when a long list of terms needs to be generated to find the position of a particular term.



1 Enter {**1, 180**} to start the counter at 1 and put the first term as 180.
2 Enter {**Ans(1) + 1, Ans(2) + 150**} to add 1 to the counter and 150 to the previous term each time [ENTER] is pressed.

Tip: Press [2nd] [ANS] to type **Ans**.
Note: **Ans** is the second function of the [(-)] key.

3 Press [ENTER] until the term 1530 occurs.
4 We see that the 10th term is 1530. Write your answer.

$1530 can be used to hire the car for 10 days.

Example: Application of an arithmetic sequence

Consider the arithmetic series: $7 + 11 + 15 + 19 + \ldots + 87$

a Find the number of terms, n. **b** Calculate the sum of the terms.

Solution

a

Method 1: Using the formula

1 The first and last terms are given. $a = 7, l = 87$

2 Find the common difference. $d = 11 - 7 = 4$

3 Substitute the values for a, l and d into $n = \dfrac{l - a}{d} + 1$.

$$n = \frac{l - a}{d} + 1 = \frac{87 - 7}{4} + 1$$

4 Evaluate. $= 21$

5 Write your answer. The number of terms is 21.

Method 2: Counting the terms as they are generated on a graphics calculator

Strategy: Enter the first term, then count on each time [ENTER] is pressed to make new terms until the last term is reached.

Note: Unless we are told otherwise, we should assume that there could be many terms, not just one term, in the gap shown by '. . .' in the sequence.

1 Press [7] [ENTER] so that the first term is 7.

```
7
                7
```

2 Press [+] [4], as 4 is the common difference.

3 Each time [ENTER] is pressed, 4 will be added and a new term will be made. So as the first term has already been entered, count that as 1, then count on every time [ENTER] is pressed until the required term is made.

Note: Only a few of the early and later terms are shown.

```
Ans+4
               11
               15
               19
```

```
               75
               79
               83
               87
```

4 The value 87 occurs as the 21st term. Write your answer. $n = 21$

Method 3: Using a counter to keep track of the number of terms generated

Strategy: When a large number of terms needs to be generated to find the position of the last term, a counter in front of each term is helpful.

1 Enter {**1, 7**} to say the 1st term is 7.

2 Enter {**Ans(1) + 1, Ans(2) + 4**} to add 1 to the counter and add 4 (the common difference) to the last term each time [ENTER] is pressed.

```
{1,7}
            {1 7}
{Ans(1)+1,Ans(2)
+4}
           {2 11}
           {3 15}
           {4 19}
```

3 Repeatedly press [ENTER] until the required term is made and note the position number in front of it.

Note: Not all terms generated are shown.

```
          {19 79}
          {20 83}
          {21 87}
```

4 The value 87 occurs as the 21st term. Write your anwer. $n = 21$

b

1 Substitute the values for a, n and l into $S_n = \frac{n}{2}(a + l)$.

$S_n = \frac{n}{2}(a + l)$

$S_{21} = \frac{21}{2}(7 + 87)$

2 Evaluate.

$= 987$

3 Write your answer.

The sum of the terms is 987.

Note: As we also know that $d = 4$, we could have used $S_n = \frac{n}{2}[2a + (n - 1)d]$.

How to use recursion to generate a geometric sequence using the TI-83/84

Use recursion on a graphics calculator to generate the geometric sequence:
4, −12, 36, −108, . . .

a Write the first five terms. **b** Find the 10th term, t_{10}

Steps

1 Find the common ratio.

Ratio: $r = \frac{t_2}{t_1} = \frac{-12}{4} = -3$

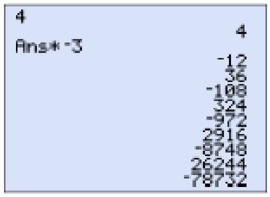
2 Check that this ratio makes all the given terms.

$4, \xrightarrow{\times -3} -12, \xrightarrow{\times -3} 36, \xrightarrow{\times -3} -108, \ldots$

3 Press [4] [ENTER] so that 4 becomes the value of **Ans**, the most recent answer.

4 Press [X] [(-)] [3] to multiply the most recent (last) answer by −3 each time [ENTER] is pressed. Notice that the last answer variable **Ans** is automatically inserted before ∗ − **3**.

5 To find the 10th term, t_{10}, we need to multiply by −3 nine times. So press [ENTER] nine times.



6 **a** Write the first five terms.

(a) The first five terms are:
4, −12, 36, −108, 324.

b State the value of the 10th term.

(b) The 10th term is −78 732.

How to generate a position counter beside the terms of a geometric sequence using the TI-83/84

Generate the geometric sequence: 80, 40, 20, 10, . . . with a position number in front of each term.

Steps

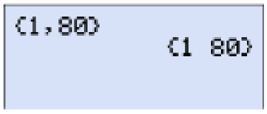
1 Find the common ratio.

Common ratio: $r = \frac{t_2}{t_1} = \frac{40}{80} = \frac{1}{2}$

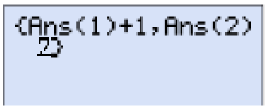
2 Check that this ratio makes all the given terms.

$80 \xrightarrow{\times \frac{1}{2}} 40 \xrightarrow{\times \frac{1}{2}} 20 \xrightarrow{\times \frac{1}{2}} 10, \ldots$

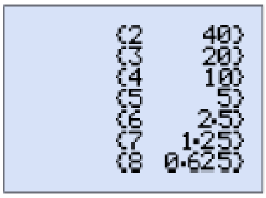
3 Enter {**1, 80**} using the curly brackets. Here {**1, 80**} says the 1st term is 80. The **1** starts the position counter and the **80** is the first term. This pair become {**Ans(1), Ans(2)**}, two previous answers for the counter and the term, which we can use to make the next counter number and the next term.



4 Multiplying by $\frac{1}{2}$ is the same as dividing by 2. Enter {**Ans(1) + 1, Ans(2)/2**} to add 1 to the counter and divide the last term by 2 each time [ENTER] is pressed.
Tip: Press [2nd] [ANS] to type **Ans**.
Note: Ans is the second function of the [(-)] key.



5 Press [ENTER] to generate each new term with its counter in front of it.



6 Write the results in a table.

Position, n	1	2	3	4	. .
Term, t_n	80	40	20	10	. . .

Example: Application of a geometric sequence

As a park ranger, Ayleisha has been working on a project to increase the number of rare native orchids in Wilsons Promontory National Park.

At the start of the project, a survey found 200 of the orchids in the park. It is assumed from similar projects that the number of orchids will increase by about 18% each year.

a State the first term a, and the common ratio r, for the geometric sequence for the number of orchids each year.

b Find a rule for the number of orchids at the start of the nth year.

c How many orchids are predicted in 10 years time?

d It is believed that a viable population of orchids will be established when the numbers reach 1000. When will this happen?

Solution

a

1 The number of orchids starts at 200. $a = 200$

2 Use $r = 1 + \frac{R}{100}$ with $R = 18$. $r = 1 + \frac{R}{100}$ Put $R = 18$.

or

An 18% increase means each year there are 118% of the previous year. So $r = 1.18$.

$$= 1 + \frac{18}{100}$$
$$= 1.18$$

b Substitute $a = 200$ and $r = 1.18$ into $t_n = ar^{n-1}$.

$$t_n = ar^{n-1}$$
$$t_n = 200 \times 1.18^{n-1}$$

c

1 Substitute $n = 10$ into the rule for t_n.

$$t_n = 200 \times 1.18^{n-1}$$
$$t_{10} = 200 \times 1.18^{10-1}$$

2 Evaluate.

$$= 200 \times 1.18^9$$
$$\approx 887$$

3 Write your answer. There will be about 887 orchids in 10 years time.

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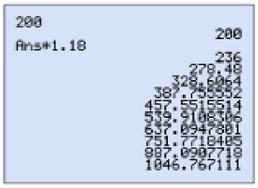
d

Method 1: Repeated use of the multiplication command on your graphics calculator

Strategy: Use your graphics calculator to enter 200 as the first entry, then apply the **Ans x 1.18** command repeatedly until a term of at least 1000 is reached.

1 Press [2] [0] [0] [ENTER] so that 200 becomes the value of **Ans**, the most recent answer.
2 Press [×] [1] [.] [1] [8], to use 1.18 as the common ratio.
3 Count 200 as the first term and count 1 more each time [ENTER] is pressed to make another term.
4 We find that 1046.8 occurs as the 11th term. Write your answer.

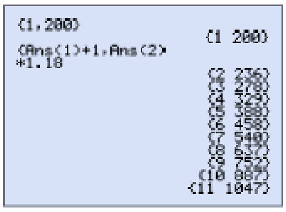


Over 1000 orchids are predicted in the eleventh year.

Method 2: Generate a position counter beside each term.

Strategy: Using a counter is very helpful when a long list of terms needs to be generated to find the position of a particular term.

1 Enter {**1, 200**} to start the counter at 1 and put the first term as 200.
2 Enter {**Ans**{**1**} **+ 1, Ans(2) x 1.18**} to add 1 to the counter and to multiply the previous term by 1.18 each time [ENTER] is pressed.
   Tip: Press [2nd] [ANS] to type **Ans**.
   Note: **Ans** is the second function of the [(-)] key.
   Tip: To get answers to the nearest whole number, press [MODE] [▼] [▶] [ENTER]. Press [2nd] [QUIT] to return to the home screen.
3 Press [ENTER] until a term exceeding 1000 occurs.
4 We see that the 11th term is 1047. Write your answer.



Over 1000 orchids are predicted in the eleventh year.

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How to use difference equations to generate recursive sequences using the TI-83/84

Generate the first four terms for each of these difference equations.

a $t_{n+1} = 2t_n + 3, \quad t_1 = 5$ **b** $t_{n+1} = t_n^2, \quad t_1 = 2$ **c** $t_{n+1} = t_n(t_n + 2), \quad t_1 = 1$

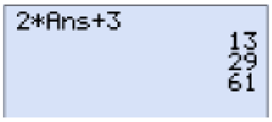
a $t_{n+1} = 2t_n + 3, \quad t_1 = 5$

Strategy: Multiply the previous term by 2, then add 3. Start at 5.

Steps

1 Press [5] [ENTER] to enter the first term.

2 Type **2* Ans+3**.
Note: Press [×] for the multiplication sign *.
Press [2nd] [ANS] for the previous answer symbol **Ans**.



3 Press [ENTER] repeatedly to generate each new term.
Write the first four terms.

The first four terms are:
5, 13, 29, 61.

b $t_{n+1} = t_n^2, t_1 = 2$

Strategy: Square the previous term to make the next term. Start at 2.

1 Press [2] [ENTER] to enter the first term.

2 Press [x^2].
Notice that the previous answer symbol **Ans** has automatically been inserted before the squaring operation.

3 Press [ENTER] repeatedly to generate each new term.

4 Write the first four terms.

The first four terms are:
2, 4, 16, 256

c $t_{n+1} = t_n(t_n + 2), t_1 = 1$

Strategy: The previous answer multiplies the number 2 greater than the previous answer. Start at 1.

1 Press [1] [ENTER] to enter the first term.

2 Type **Ans*(Ans+2)**.
Note: Press [2nd] [ANS] for the previous answer symbol **Ans**.

3 Press [ENTER] repeatedly to generate each new term.

4 Write the first four terms.

The first four terms are:
1, 3, 15, 255.

Questions on generating recursive sequences using the TI-83/84

6 Use your graphics calculator to find the required term for each sequence with the difference equation given. Give answers correct to 2 decimal places where necessary.
Hint: The symbol **Ans**, for the previous answer, can be inserted into the required place in the repeated command by pressing [2nd] [ANS]. See 'How to use difference equations to generate recursive sequences on a graphics calculator' (page 340).

- a $t_{n+1} = t_n + 5, t_1 = 2$ Find t_{12}.
- b $t_{n+1} = t_n + 7, t_1 = 4,$ Find t_{10}.
- c $t_{n+1} = 4t_n, t_1 = 3$ Find t_9.
- d $t_{n+1} = 3t_n, t_1 = 1$ Find t_{11}.
- e $t_{n+1} = 2t_n - 5, t_1 = 1$ Find t_{10}.
- f $t_{n+1} = 5t_n + 4, t_1 = 2$ Find t_8.
- g $t_{n+1} = 1.06t_n + 200, t_1 = 1000$ Find t_{12}.
- h $t_{n+1} = 1.08t_n - 150, t_1 = 5000$ Find t_{10}.
- i $t_{n+1} = t_n(t_n + 1), t_1 = 2$ Find t_5.
- j $t_{n+1} = 2t_n(t_n - 10), t_1 = 11$ Find t_4.

8 Steven invested \$10 000 at 5% interest per year. At the start of each following year the previous amount in the account was multiplied by 1.05, to allow for the 5% increase because of the interest paid, and he deposited an extra \$2000.

- a Give the difference equation for the sequence showing the amount in his account each year.
- b List the amounts for the first 4 years.
- c Use your graphics calculator to find the amount in his account at the end of the tenth year.

Answers

Recursive sequence questions

6 a 57 **b** 67 **c** 196 608 **d** 59 049
e −2043 **f** 234 374 **g** 4892.63 **h** 8121.89
i 3 263 442 **j** 547 008

8 a $t_{n+1} = 1.05t_n + 2000, \quad t_1 = 10\,000$
b 10 000, 12 500, 15 125, 17 881.25 dollars
c $37 566.41

Original location: Answers (p.501)

Cambridge University Press