CAMBRIDGE TECHNOLOGY IN MATHS

Year 11
Quadratics for the ClassPad

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Example: Expanding and collecting like terms

Expand $(2x - 1)(3x^2 + 2x + 4)$.

Solution

$$\begin{aligned}(2x-1)(3x^2+2x+4) &= 2x(3x^2+2x+4) - 1(3x^2+2x+4)\\ &= 6x^3+4x^2+8x-3x^2-2x-4\\ &= 6x^3+x^2+6x-4\end{aligned}$$

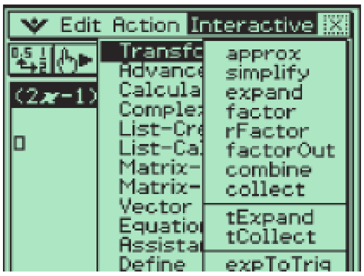
Using the Casio ClassPad

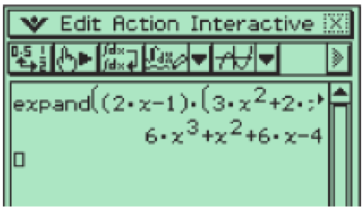
Enter the expression

$(2x - 1)(3x^2 + 2x + 4)$

into Main, then highlight and select

Interactive—Transformation—expand.





Original location: Chapter 4 Example 4 (p.94-95)
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Example: Factorising quadratics expressions

Factorise $6x^2 - 13x - 15$.

Solution

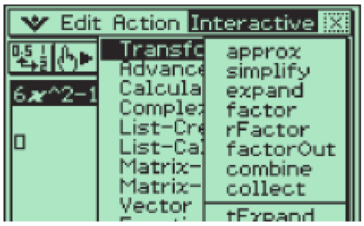
There are several combinations of factors of $6x^2$ and -15 to consider. Only one combination is correct.

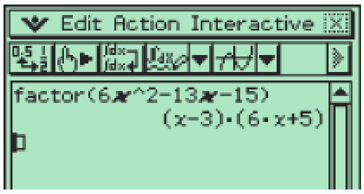
Factors of $6x^2$	Factors of -15	'Cross-products' add to give $-13x$
$6x$	$+5$	$+5x$
x	-3	$-18x$
		$-13x$

$\therefore$ Factors of $6x^2 - 13x - 15$
$= (6x + 5)(x - 3)$

Using the Casio ClassPad

Enter the expression $6x^2 - 13x - 15$ into Main, then highlight and select **Interactive—Transformation—factor**.





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Example: Sketching quadratics in polynomial form

Find the x- and y-axis intercepts and the turning point, and hence sketch the graph of $y = x^2 + x - 12$.

Solution

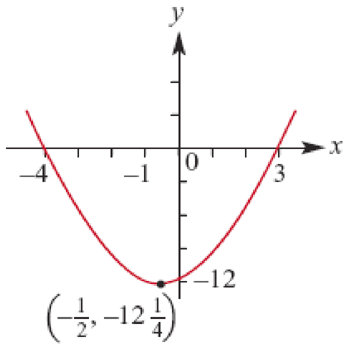
Step 1 $c = -12$, $\therefore$ y-axis intercept is $(0, -12)$

Step 2 Set $y = 0$ and factorise the right-hand side:

$$0 = x^2 + x - 12$$
$$0 = (x + 4)(x - 3)$$

$\therefore x = -4$ or $x = 3$

x-axis intercepts are $(-4, 0)$ and $(3, 0)$



Step 3 Due to the symmetry of the parabola, the axis of symmetry will be the line bisecting the two x-axis intercepts.

$\therefore$ the axis of symmetry is the line with equation $x = \frac{-4 + 3}{2} = -\frac{1}{2}$.

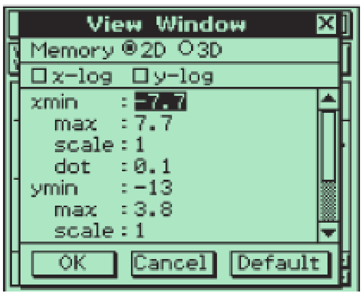
When $x = -\frac{1}{2}$, $y = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 12$

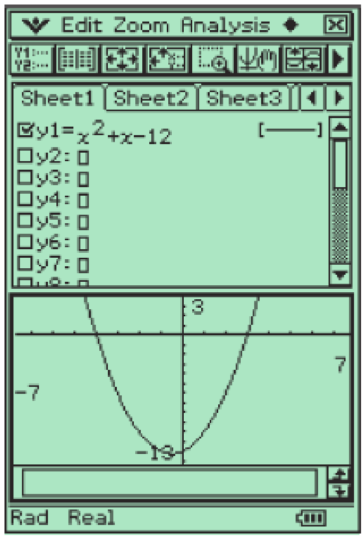
$$= -12\frac{1}{4}$$

$\therefore$ the turning point has coordinates $\left(-\frac{1}{2}, -12\frac{1}{4}\right)$.

Using the Casio ClassPad

To graph the quadratic relation with rule $y = x^2 + x - 12$, enter the rule in the Graph&Tab screen, tick the box and click . It may be necessary to change the view window using .





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Example: The general quadratic formula

Solve each of the following equations for x by using the quadratic formula:

a $x^2 - x - 1 = 0$ **b** $x^2 - 2kx - 3 = 0$

Solution

a $x^2 - x - 1 = 0$

$a = 1, b = -1$ and $c = -1$

The formula gives

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

b $x^2 - 2kx - 3 = 0$

$a = 1, b = -2k$ and $c = -3$

The formula gives

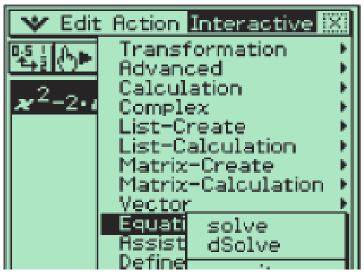
$$x = \frac{-(-2k) \pm \sqrt{(-2k)^2 - 4 \times 1 \times (-3)}}{2 \times 2}$$

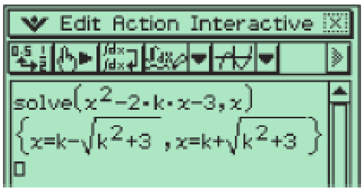
$$= \frac{2k \pm \sqrt{4k^2 + 12}}{4}$$

$$= \frac{k \pm \sqrt{k^2 + 3}}{2}$$

Using the Casio ClassPad

To solve the equation $x^2 - 2kx - 3 = 0$ for x, enter and highlight the equation (use VAR on the keyboard to enter the variables). Select **Interactive**—**Equation/ Inequality**—**solve** and set the variable to x.





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Calculating iteration using the ClassPad

Using the Casio ClassPad

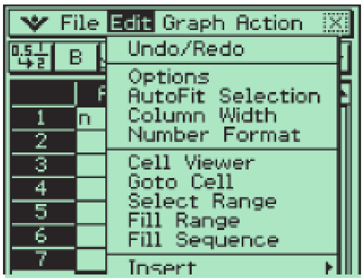
Method 1

Choose a starting value near the positive solution of $x^2 + 3x - 5 = 0$. Let $x_1 = 2$.

In Spreadsheet enter column headings n, x(n), x(n + 1). Enter the numbers 1 to 10 in the n column. Enter the value for x_1 into cell B2. In cell C2 enter the formula $= 5/(B2 + 3)$ for x_{n+1}. The formula will appear in the formula bar towards the bottom of the screen as you enter it and the answer will appear in cell C2.

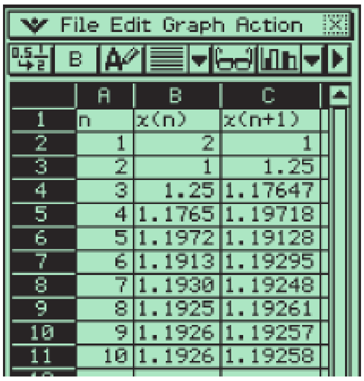
Type the formula =C2 into cell B3 to set the answer to the first iteration as the x_n value for the next iteration.

Click on cell B3 and drag to select cells B3 to B11. Select **Edit—Fill Range** and OK. The formula is copied and the column fills with zeroes as there are no values in cells C3 downwards.



Click on cell C2 and drag to select cells C2 to C11. Select **Edit—Fill Range** and OK to copy the formula.

You can try some other starting points by replacing the x_1 value in cell B2; for example, set $x_1 = -400$ or $x_1 = 200$. Convergence is always towards the solution $x = \dfrac{-3 + \sqrt{29}}{2}$.



	A	B	C
1	n	x(n)	x(n+1)
2	1	2	1
3	2	1	1.25
4	3	1.25	1.17647
5	4	1.1765	1.19718
6	5	1.1972	1.19128
7	6	1.1913	1.19295
8	7	1.1930	1.19248
9	8	1.1925	1.19261
10	9	1.1926	1.19257
11	10	1.1926	1.19258

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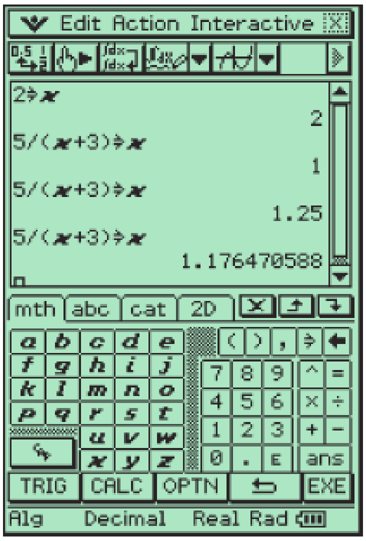
Method 2

Another method which can be used to generate the sequence is to enter the first value as variable x. Turn on the keyboard and remember to use x from the VAR menu or from the keyboard.

Enter the formula for x_{n+1} in the next entry line and send its value to x as shown. Press EXE repeatedly, enter the previous value and calculate the next value in the iterative sequence.



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Example: Solving simultaneous linear and quadratic equations

Find the points of intersection of the line with the equation $y = -2x + 4$ and the parabola with the equation $y = x^2 - 8x + 12$.

Solution

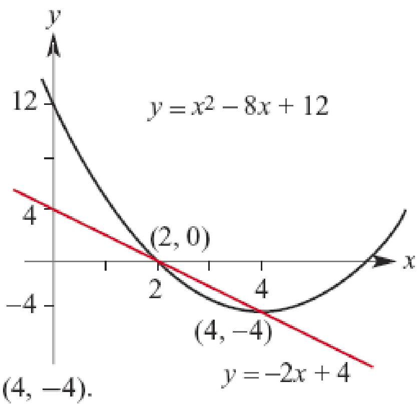
At the point of intersection

$$x^2 - 8x + 12 = -2x + 4$$
$$x^2 - 6x - 8 = 0$$
$$(x - 2)(x - 4) = 0$$

Hence $x = 2$ or $x = 4$

When $x = 2, y = -2(2) + 4 = 0$

$x = 4, y = -2(4) + 4 = -4$



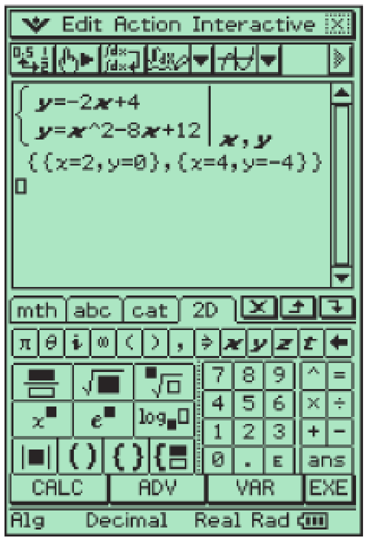
Therefore the points of intersection are (2, 0) and (4, −4).

The result can be shown graphically.

Using the Casio ClassPad

In Main turn on the keyboard, select 2D (and at bottom left if necessary), then click the simultaneous equations entry button.

Enter the simultaneous equations $y = -2x + 4$ and $y = x^2 - 8x + 12$ into the two lines and the variables x, y as the variables to be solved.



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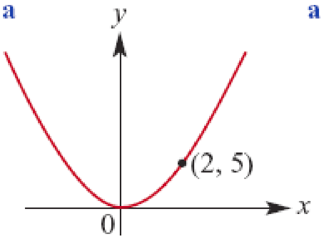
Example: Determining quadratic rules

Determine the quadratic rule for each of the following graphs, assuming each is a parabola.

Solution

a



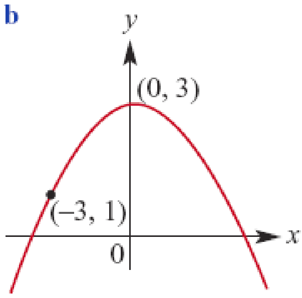
a This is of the form $y = ax^2$

When $x = 2, y = 5$, thus $5 = 4a$

Therefore $a = \frac{5}{4}$

and the rule is $y = \frac{5}{4}x^2$

b



b This is of the form $y = ax^2 + c$

For $(0, 3)$ $3 = a(0) + c$

Therefore $c = 3$

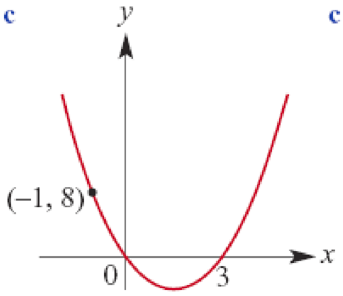
For $(-3, 1)$ $1 = a(-3)^2 + 3$

$1 = 9a + 3$

Therefore $a = -\frac{2}{9}$

and the rule is $y = -\frac{2}{9}x^2 + 3$

c



c This is of the form $y = ax(x - 3)$

For $(-1, 8)$ $8 = -a(-1 - 3)$

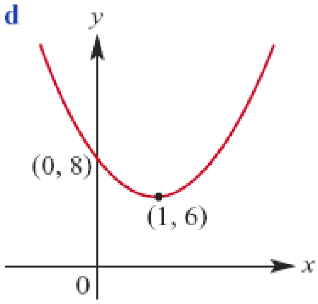
$8 = 4a$

Therefore $a = 2$

and the rule is $y = 2x(x - 3)$

$y = 2x^2 - 6x$

d



d This is of the form $y = k(x - 1)^2 + 6$

When $x = 0$, $y = 8$

$\therefore$ $8 = k + 6$

and $k = 2$

$\therefore$ $y = 2(x - 1)^2 + 6$

and the rule is $y = 2(x^2 - 2x + 1) + 6$

$y = 2x^2 - 4x + 8$

Original location: Chapter 4 Example 37 (p.130-132)

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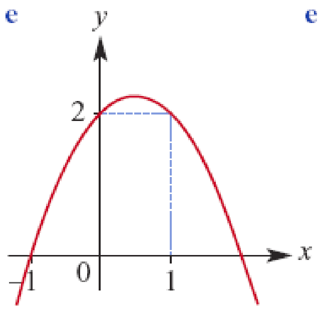
e



e This is of the form $y = ax^2 + bx + c$

For $(-1, 0)$ $\quad 0 = a - b + c \quad (1)$

For $(0, 2)$ $\quad 2 = c \quad (2)$

For $(1, 2)$ $\quad 2 = a + b + c \quad (3)$

Substitute $c = 2$ in (1) and (3)

$$0 = a - b + 2$$

$$-2 = a - b \quad (1a)$$

$$0 = a + b \quad (3a)$$

Substract (3a) from (1a)

$$-2 = -2b$$

Therefore $\quad b = 1$

Substitute $b = 1$ and $c = 2$ in (1)

Therefore $\quad 0 = a - 1 + 2$

$$0 = a + 1$$

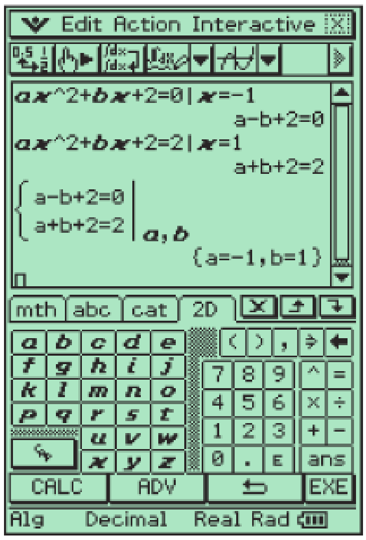
and hence $\quad a = -1$

Thus the quadratic rule is $\quad y = -x^2 + x + 2$

Using the Casio ClassPad

The equation $y = ax^2 + bx + c$ is used to generate equations in a and b. These equations are then solved simultaneously to find a and b.

Note: To generate the equation in a and b when $x = -1$, the | symbol is found by clicking **mth** and **OPTN** on the keyboard. Remember to use VAR to enter the variables a and b.



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