Back to Menu >>>

CHAPTER

2

Linear relations and equations

- How do we use a formula?
- How do we create a table of values?
- How do we use a graphics calculator to create a table of values?
- How do we solve linear equations?
- What is a literal equation?
- How do we solve literal equations?
- How do we develop a formula?
- How do we transpose a formula?
- What is recursion?
- How do we use recursion to solve problems?
- How do we find the intersection of two linear graphs?
- What are simultaneous equations?
- How do we solve simultaneous equations?
- How can we use simultaneous equations to solve practical problems?

2.1 Substitution of values into a formula

A **formula** is a mathematical relationship connecting two or more variables.

For example:

- $C = 45t + 150$ is a formula for relating the cost, C dollars, of hiring a plumber for t hours. C and t are the variables.
- $P = 4L$ is a formula for finding the perimeter of a square, where P is the perimeter and L is the side length of the square. P and L are the variables.

By substituting all known variables into a formula, we are able to find the value of an unknown variable.

Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4

Example 1 **Using a formula**

The cost of hiring a windsurfer is given by the rule

$$C = 5t + 8$$

where C is the cost in dollars and t is the time in hours.
Annie wants to sail for 2 hours. How much will it cost her?

Solution

1 Write the formula. $C = 5t + 8$

2 To determine the cost of hiring a windsurfer for 2 hours, substitute $t = 2$ into the formula. $C = 5(2) + 8$

Remember: 5(2) means 5×2.

3 Evaluate. $C = 18$

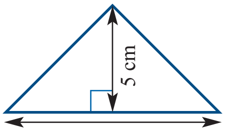
4 Write your answer. It will cost Annie $18 to hire a windsurfer for 2 hours.

Example 2 **Using a formula**

The area of a triangle with base b and height h is given by the formula

$$A = \frac{1}{2}bh$$

Find the area of a triangle with base 12 cm and height 5 cm.



Solution

1 Write the formula. $A = \frac{1}{2}bh$

2 Substitute values for b and h into the formula. $A = \frac{1}{2} \times 12 \times 5$

3 Evaluate. $A = 30$

4 Give your answer with correct units. Since we are finding area, units are cm^2. The area of the triangle is 30 cm^2.

Exercise 2A

1 The cost of hiring a dance hall is given by the rule

$$C = 50t + 1200$$

where C is the total cost in dollars and t is the number of hours for which the hall is hired.

Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4

Find the cost of hiring the hall for:

a 4 hours **b** 6 hours **c** 4.5 hours.

2 The distance, d km, travelled by a car in t hours at an average speed of v km/h is given by the formula

$$d = v \times t$$

Find the distance travelled by a car travelling at a speed of 95 km/hour for 4 hours.

3 Taxi fares are calculated using the formula

$$F = 1.3K + 4$$

where K is the distance travelled in kilometres and F is the cost of the fare in dollars. Find the costs of the following trips.

a 5 km **b** 8 km **c** 20 km

4 The circumference, C, of a circle with radius, r, is given by

$$C = 2\pi r$$

Find, correct to 2 decimal places, the circumferences of the circles with the following radii.

a $r = 25$ cm **b** $r = 3$ mm **c** $r = 5.4$ cm **d** $r = 7.2$ m

5 If $P = 2(L + W)$, find the value of P if:

a $L = 3$ and $W = 4$ **b** $L = 15$ and $W = 8$ **c** $L = 2.5$ and $W = 9$.

6 If $A = \frac{1}{2}h\,(x + y)$, find A if:

a $h = 1, x = 3, y = 5$ **b** $h = 5, x = -2, y = 7$ **c** $h = 2, x = -3, y = -4$.

7 The formula used to convert temperature from degrees Fahrenheit to degrees Centigrade is

$$C = \frac{5}{9}(F - 32)$$

Use this formula to convert the following temperatures to degrees Centigrade.
Give your answers correct to 1 decimal place.

a 50°F **b** 0°F **c** 212°F **d** 92°F

8 The formula for calculating simple interest is

$$I = \frac{PRT}{100}$$

where P is the principal (amount invested or borrowed), R is the interest rate per annum and T is the time (in years). In the following questions, give your answers to the nearest cent (correct to 2 decimal places).

a Frank borrows \$5000 at 12% for 4 years. How much interest will he pay?
b Chris borrows \$1500 at 6% for 2 years. How much interest will he pay?
c Jane invests \$2500 at 5% for 3 years. How much interest will she earn?
d Henry invests \$8500 for 3 years with an interest rate of 7.9%. How much interest will he earn?

Back to Menu >>>

9 In Australian football, a goal, G, is worth 6 points and a behind, B, is worth 1 point. The rule showing the total number of points, P, is given by

$$P = 6G + B$$

Find the number of points if:

a 2 goals and 3 behinds are kicked **b** 5 goals and 7 behinds are kicked
c 8 goals and 20 behinds are kicked

10 The rule for finding the nth term (t_n) of the sequence 3, 5, 7, . . . is given by

$$t_n = a + (n - 1)d$$

where a is the value of the first term and d is the common difference.
If $a = 3$ and $d = 2$, find the value of the:

a 6th term **b** 11th term **c** 50th term

2.2 Constructing a table of values

We can use a formula to construct a **table of values**. This can be done by substitution (by hand) or using a graphics calculator.

Example 3 Constructing a table of values

The formula for converting degrees Centigrade to degrees Fahrenheit is given by

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

Solution

Draw up a table of values for $F = \frac{9}{5}C + 32$, then substitute values of $C = 0, 10, 20, 30, \ldots,$ 100 into the formula to find F.

If $C = 0, F = \frac{9}{5}(0) + 32$
$= 32$

If $C = 10, F = \frac{9}{5}(10) + 32$
$= 50$

and so on.

The table would then look as follows:

C	0	10	20	30	40	50	60	70	80	90	100
F	32	50	68	86	104	122	140	158	176	194	212

How to construct a table of values using the TI-Nspire CAS

The formula for converting degrees Centigrade to degrees Fahrenheit is given by

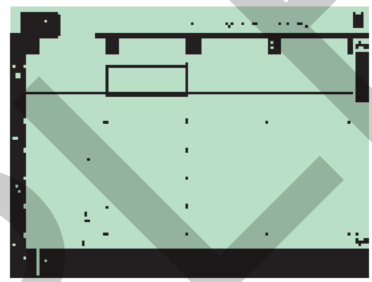
$$F = \tfrac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

Steps

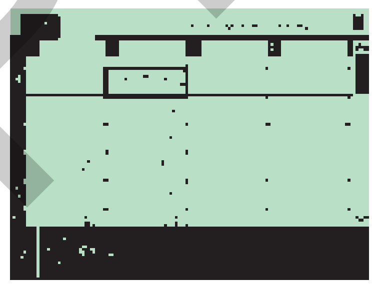
1 Start a new document: press ctrl + N.

2 Select **3:Add Lists & Spreadsheet**.
Name the lists **c** (for Centigrade) and **f** (for Fahrenheit).
Enter the data 0–100 in intervals of 10 into the list named 'c', as shown.



3 Place the cursor in the grey formula cell in column B (i.e. list 'f') and type in $= 9 \div 5 \times c + 32$

Hint: If you typed in c you will need to select **Variable Reference** when prompted. This prompt occurs because c can also be a column name. Alternatively, press sto→var key and select c from the variable list.



Press enter to display the values given. Use the ▼ arrow to move down through the table.

How to construct a table of values using the ClassPad

The formula for converting degrees Centigrade to degrees Fahrenheit is given by

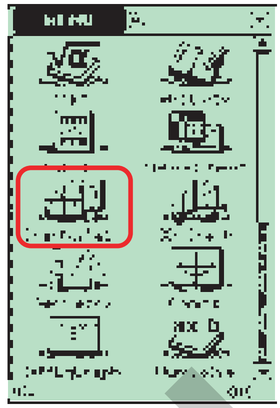
$$F = \tfrac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

Back to Menu >>>

Steps

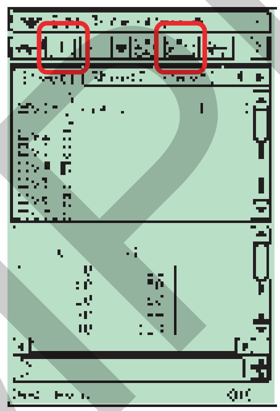
1 Enter the data into your calculator using the **Graph & Table** application. From the application menu screen, locate the built-in **Graph & Table** application, Graph&Tab... . Tap to open. Tapping Menu from the icon panel (just below the touch screen) will display the application menu if it is not already visible.



2 a Adjacent to **y**1 = type in the formula $\mathbf{9/5x + 32}$. Then press EXE.

b Tap the **Table Input** ( ) icon from the toolbar to set the table entries as shown.

c Tap the icon to display the required table of values. Scrolling down will show more values in the table.



Exercise 2B

1 A football club wishes to purchase pies at a cost of \$2.15 each. Use the formula

$$C = 2.15x$$

where C is the cost (\$) and x is the number of pies, to complete the table showing the amount of money needed to purchase from 40 to 50 pies.

x	40	41	42	43	44	45	46	47	48	49	50
C (\$)	86	88.15	90.30								

2 The area of a circle is given by

$$A = \pi r^2$$

where r is the radius. Complete the table of values to show the areas of circles with radii from 0 to 1 cm in intervals of 0.1 cm. Give your answers correct to 3 decimal places.

r (cm)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
A (cm^2)	0	0.031	0.126	0.283							

Back to Menu >>>

3 A phone bill is calculated using the formula

$$C = 40 + 0.18n$$

where C is the total cost and n represents the number of calls made.
Complete the table of values to show the cost for 50, 60, 70, . . . , 200 calls.

n	50	60	70	80	90	100	110	...	...
C (\$)	49	50.80	52.60						

4 The amount of energy (E) in kilojoules expended by an adult male of mass (M) at rest, can be estimated using the formula

$$E = 110 + 9M$$

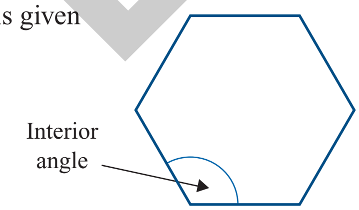
Complete the table of values in intervals of 5 kg for males of mass 60–120 kg to show the corresponding values of E.

M (kg)	60	65	70	75	80	85	90	95	100				
E (kJ)	650	695											

5 The sum, S, of the interior angles of a polygon with n sides is given by the formula

$$S = 90(2n - 4)$$

Construct a table of values showing the sum of the interior angles of polygons with 3 to 10 sides.



n	3	4	5					
S	180°	360°						

6 A car salesman's weekly wage, E dollars, is given by the formula

$$E = 60n + 680$$

where n is the number of cars sold.

a Construct a table of values to show how much his weekly wage will be if he sells from 0 to 10 cars.

b Using your table of values, if the salesman earns \$1040 in a week, how many cars did he sell?

7 Anita has \$10 000 that she wishes to invest at a rate of 7.5% per annum. She wants to know how much interest she will earn after 1, 2, 3 . . . , 10 years. Using the formula

$$I = \frac{PRT}{100}$$

where P is the principal and R is the interest rate (%), construct a table of values with a calculator to show how much interest, I, she will have after $T = 1, 2, \ldots, 10$ years.

8 The formula for finding the amount, A, accumulated at compound interest is given by

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

where P is the principal, r is the annual interest rate (%) and t is the time in years.
Construct a table of values showing the amount accumulated when \$5000 is invested at a rate of 5.5% over 5, 10, 15, 20 and 25 years. Give your answers to the nearest dollar.

Back to Menu >>>

2.3 Solving linear equations with one unknown

Practical applications of mathematics often involve the need to be able to solve linear equations. An **equation** is a mathematical statement that says that two things are equal. For example, these are all equations:

$$x - 3 = 5 \qquad 2w - 5 = 17 \qquad 3m = 24$$

Linear equations come in many different forms in mathematics but are easy to recognise because the powers on the unknown values are always 1. For example:

- $m - 4 = 8$ is a linear equation. The unknown value is m.
- $3x = 18$ is a linear equation. The unknown value is x.
- $4y - 3 = 17$ is a linear equation. The unknown value is y.
- $a + b = 0$ is a linear equation. The unknown values are a and b.
- $x^2 + 3 = 9$ is *not* a linear equation (the power of x is 2 not 1). The unknown value is x.
- $c = 16 - d^2$ is *not* a linear equation (the power of d is 2). The unknowns are c and d.

The process of finding the unknown value is called **solving the equation.** When solving an equation, **opposite** (or **inverse**) **operations** are used so that the unknown value to be solved is the only term remaining on one side of the equation. Opposite operations are indicated in the table below.

Operation	$+$	$-$	$\times$	$\div$	x^2 (power of 2, square)	$\sqrt{x}$ square root
Opposite operation	$-$	$+$	$\div$	$\times$	$\sqrt{x}$ (square root)	x^2 (power of 2, square)

Remember: The equation must remain **balanced**. This can be done by adding or subtracting the *same* number on *both* sides of the equation, or by multiplying or dividing *both* sides of the equation by the *same* number.

Example 4 Solving a linear equation

Solve the equation $x + 6 = 10$.

Solution

Method 1: By inspection

Write the equation. $x + 6 = 10$

What needs to be added to 6 to make 10?

The answer is 4. $\therefore \quad x = 4$

Method 2: Inverse operations

This method requires the equation to be 'undone', leaving the unknown value by itself on one side of the equation.

Back to Menu >>>

1 Write the equation.

$$x + 6 = 10$$

2 Subtract 6 from both sides of the equation. This is the opposite process to adding 6.

$$x + 6 - 6 = 10 - 6$$
$$\therefore x = 4$$

3 Check your answer by substituting the found value for x into the original equation. If each side gives the same value, the solution is correct.

$$\begin{aligned} \text{LHS} &= x + 6 \\ &= 4 + 6 \\ &= 10 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

Example 5 Solving a linear equation

Solve the equation $3y = 18$.

Solution

1 Write the equation.

$$3y = 18$$

2 The opposite process of multiplying by 3 is dividing by 3. Divide both sides of the equation by 3.

$$\frac{3y}{3} = \frac{18}{3}$$
$$\therefore y = 6$$

3 Check that the solution is correct by substituting $y = 6$ into the original equation.

$$\begin{aligned} \text{LHS} &= 3y \\ &= 3 \times 6 \\ &= 18 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

Example 6 Solving a linear equation

Solve the equation $4x + 5 = 17$.

Solution

1 Write the equation.

$$4x + 5 = 17$$

2 Subtract 5 from both sides of the equation.

$$4x + 5 - 5 = 17 - 5$$
$$4x = 12$$

3 Divide both sides of the equation by 4.

$$\frac{4x}{4} = \frac{12}{4}$$
$$\therefore x = 3$$

4 Check that the solution is correct by substituting $x = 3$ into the original equation.

$$\begin{aligned} \text{LHS} &= 4x + 5 \\ &= 4 \times 3 + 5 \\ &= 12 + 5 \\ &= 17 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4

Back to Menu >>>

Example 7 Solving a linear equation

Solve the equation $-4 - 3b = 8$.

Solution

1 Write the equation.

$$-4 - 3b = 8$$

2 Add 4 to both sides of the equation.

$$-4 - 3b + 4 = 8 + 4$$

$$-3b = 12$$

3 Divide both sides of the equation by –3.

$$\frac{-3b}{-3} = \frac{12}{-3}$$

$$\therefore b = -4$$

4 Check that the solution is correct by substituting $x = -4$ into the original equation.

$$\begin{aligned} \text{LHS} &= -4 - 3b \\ &= -4 - 3 \times -4 \\ &= -4 + 12 \\ &= 8 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

Exercise 2C

1 Solve the following linear equations.

a $x + 6 = 15$	**b** $y + 11 = 26$	**c** $t + 5 = 10$	**d** $m - 5 = 1$
e $g - 3 = 3$	**f** $f - 7 = 12$	**g** $f + 5 = 2$	**h** $v + 7 = 2$
i $x + 11 = 10$	**j** $g - 3 = -2$	**k** $b - 10 = -5$	**l** $m - 5 = -7$
m $2 + y = 8$	**n** $6 + e = 9$	**o** $7 + h = 2$	**p** $3 + a = -1$
q $4 + t = -6$	**r** $8 + s = -3$	**s** $9 - k = 2$	**t** $5 - n = 1$
u $15 - y = 7$	**v** $4 - u = 5$	**w** $10 - h = -5$	**x** $-3 - t = 4$

2 Solve the following linear equations.

a $5x = 15$	**b** $3g = 27$	**c** $9n = 36$	**d** $2x = -16$
e $6j = -24$	**f** $4m = 28$	**g** $2f = 11$	**h** $2x = 7$
i $3y = 15$	**j** $3s = -9$	**k** $-5b = 25$	**l** $4d = -18$
m $\frac{r}{3} = 4$	**n** $\frac{q}{5} = 6$	**o** $\frac{x}{8} = 6$	**p** $\frac{t}{-2} = 6$
q $\frac{h}{-8} = -5$	**r** $\frac{m}{-3} = -7$	**s** $\frac{14}{a} = 7$	**t** $\frac{24}{f} = -12$

3 Solve the following linear equations.

a $3a + 5 = 11$	**b** $4b + 3 = 27$	**c** $2w + 5 = 9$	**d** $7c - 2 = 12$
e $3y - 5 = 16$	**f** $4f - 1 = 7$	**g** $3 + 2h = 13$	**h** $2 + 3k = 6$
i $4 + 3g = 19$	**j** $16 - 3c = 10$	**k** $28 - 4e = 16$	**l** $110 - 5g = 65$
m $9 + 2c = 3$	**n** $2y + 5 = -13$	**o** $3a + 15 = 9$	

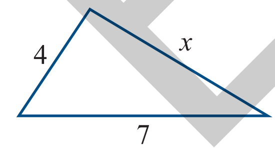
Back to Menu >>>

2.4 Developing a formula: setting up linear equations in one unknown

In many practical problems, mathematicians often need to set up a linear equation before finding the solution to a problem. Some practical examples are given below showing how a linear equation is set up and then solved.

Example 8 Setting up a linear equation

Find an expression for the perimeter of the triangle shown.



Note: Perimeter is the distance around the outside of a shape.

Solution

1 Choose a variable to represent the perimeter. — Let P be the perimeter.

2 Add up all sides of the triangle and let them equal the perimeter, P. — $P = 4 + 7 + x$

$P = 11 + x$

3 Write your answer. — The required expression is $P = 11 + x$.

Example 9 Setting up and solving a linear equation

If 11 is added to a certain number, the result is 25. Find the number.

Solution

1 Choose a variable to represent the number. — Let n be the number.

2 Using the information, write an equation. — $n + 11 = 25$

3 Solve the equation by subtracting 11 from both sides of the equation. — $n + 11 - 11 = 25 - 11$

$\therefore n = 14$

4 Write your answer. — The required number is 14.

Example 10 Setting up and solving a linear equation

At a recent show, Chris spent \$100 on 8 showbags, each costing the same price, \$$x$.

a Using x as the cost of one showbag, write an equation showing the cost of 8 showbags.

b Use the equation to find the cost of one showbag.

Solution

a

1 Write the cost of one showbag using the variable given. — Let x be the cost of one showbag.

2 Use the information to write an equation. — $8x = 100$

Remember: $8 \times x = 8x$

Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4

Back to Menu >>>

b

1 Write the equation.
$$8x = 100$$

2 Solve the equation by dividing both sides of the equation by 8.
$$\frac{8x}{8} = \frac{100}{8}$$
$$\therefore x = 12.5$$

3 Write your answer.

The cost of one showbag is \$12.50.

Example 11 Setting up and solving a linear equation

A car rental company has a fixed charge of \$110 plus \$84 per day for the hire of a car. The Brown family have budgeted \$650 for the hire of a car during their family holiday. For how many days can they hire a car?

Solution

1 Choose a variable (d) for the number of days that the car is hired for.

Let d be the number of days that the car is hired for.

2 Use the information to write an equation.
$$110 + 84d = 650$$

3 Solve the equation.

First, subtract 110 from both sides of the equation.
$$110 + 84d - 110 = 650 - 110$$
$$84d = 540$$

Then divide both sides of the equation by 84.
$$\frac{84d}{84} = \frac{540}{84}$$
$$\therefore d = 6.428\ldots$$

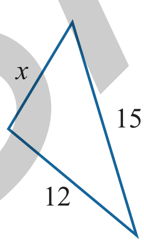
4 Write your answer in terms of complete days.

The Brown family could hire a car for 6 days.

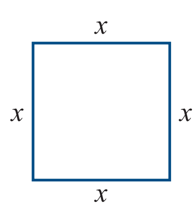
Exercise 2D

1 Find an expression for the perimeter of each of the following shapes.

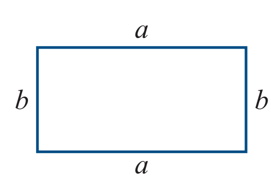
a



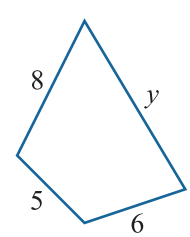
b



c

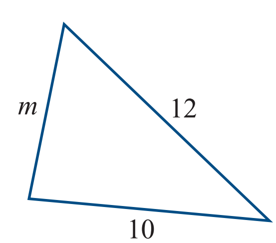


d



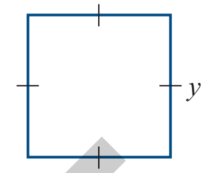
2 a Write an expression for the perimeter of the triangle shown.

b If the perimeter, P, of the triangle is 30 cm, solve the equation to find the value of m.



3 **a** Write an equation for the perimeter of the square shown.

b If the perimeter is 52 cm, what is the length of one side?



4 Seven is added to a number and the result is 15.

a Write an equation using n to represent the number.

b Solve the equation for n.

5 Five is added to twice a number and the result is 17. What is the number?

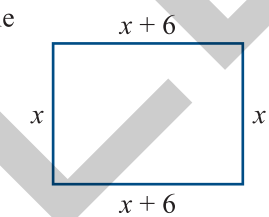
6 When a number is doubled and 15 is subtracted, the result is 103. Find the number.

7 The perimeter of a rectangle is 84 cm. The length of the rectangle is 6 cm longer than the width, as shown in the diagram.

a Write an expression for the perimeter, P, of the rectangle.

b Find the value of x.

c Find the lengths of the sides of the rectangle.



8 Year 11 students want to run a social. The cost of hiring a band is \$820 and they are selling tickets for \$8 per person. The profit, P, is found by subtracting the band hire cost from the money raised from selling tickets. The students want to make a profit of \$300. Use the information to write an equation, then solve the equation to find how many tickets they need to sell.

9 Kate's new mobile phone plan costs her \$10.95 a month. She then pays \$0.20 cents per minute. Kate's phone bill for the month of May was \$39.95. For how many minutes did she use the phone?

10 A raffle prize of \$1000 is divided between Anne and Barry so that Anne receives 3 times as much as Barry. How much does each receive?

2.5 Solving linear equations with two unknowns (literal equations)

A **literal equation** is an equation whose solution will be expressed in terms of another variable rather than a number.

For example:

- $2x + 3 = 13$ is a linear equation in one unknown, whose solution for x is the number 5.
- $x + 2y = 6$ is a *literal* equation in two unknowns, whose solution for x is $6 - 2y$. It is a literal equation because its solution for x is expressed in terms of another variable.

Example 12 **Solving a linear equation with two unknowns**

If $x + 5y = 9$, find an expression for x.

Solution

Strategy: Rearrange the equation so that x becomes the subject.

1 Write the original equation. $x + 5y = 9$

2 Subtract $5y$ from both sides of the equation to give your answer.

$x + 5y - 5y = 9 - 5y$

$\therefore x = 9 - 5y$

Example 13 Solving a linear equation with two unknowns

If $2a - 4b = 8$, find an expression for a.

Solution

Strategy: Rearrange the equation so that a becomes the subject.

1 Write the original equation. $2a - 4b = 8$

2 Add $4b$ to both sides of the equation.

$2a - 4b + 4b = 8 + 4b$

$2a = 8 + 4b$

3 Divide both sides of the equation by 2 to give your answer.

$\therefore a = 4 + 2b$

Note: All terms must be divided by 2.

Example 14 Solving a linear equation with two unknowns

If $3x + 2y = 4$, find:

a an expression for y **b** an expression for x.

Solution

Strategy: Rearrange the equation so that y, and then x, become the subject.

a an expression for y

1 Write the original equation. $3x + 2y = 4$

2 Subtract $3x$ from both sides of the equation.

$3x + 2y - 3x = 4 - 3x$

$2y = 4 - 3x$

3 Divide both sides of the equation by 2 to give your answer.

$\therefore y = 2 - \frac{3}{2}x$

b an expression for x

1 Write the original equation. $3x + 2y = 4$

2 Subtract $2y$ from both sides of the equation.

$3x + 2y - 2y = 4 - 2y$

$3x = 4 - 2y$

3 Divide both sides of the equation by 3 to give your answer.

$\therefore x = \frac{4}{3} - \frac{2}{3}y$

Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4

Example 15 Solving a linear literal equation

If $y = mx + c$, find an expression for m.

Solution

Strategy: Rearrange the equation so that m becomes the subject.

1 Write the original equation.

$$y = mx + c$$

2 Subtract c from both sides of the equation.

$$y - c = mx + c - c$$

$$y - c = mx$$

3 Divide both sides of the equation by x.

$$\frac{y - c}{x} = \frac{mx}{x}$$

$$\frac{y - c}{x} = m$$

4 Rewrite the equation with the required variable, m, on the left-hand side.

$$\therefore m = \frac{y - c}{x}$$

Exercise 2E

1 Find an expression for x.

a $x + 7y = 22$ **b** $x - 4y = 11$ **c** $x + 6y = 5$
d $5y = x - 12$ **e** $2y = x + 5$ **f** $4y = 8 - x$
g $4y = 2x + 6$ **h** $3x - 5 = 2y$ **i** $5y + 2x = 10$
j $4x + 12y = -24$ **k** $2x - 3y = 12$ **l** $7y - 5x = 40$

2 The formula for the circumference, C, of a circle is given by

$$C = 2\pi r$$

Find an expression for r.

3 The sum, S, of the interior angles of a polygon with number of sides n is given by

$$S = 180n - 360$$

a Find an expression for n. **b** Calculate the value of n when $S = 900°$.

4 The velocity, v, of an object is described by the rule

$$v = u + at$$

where u is the initial velocity, a is the acceleration and t is the time in seconds.

a Find an expression for t. **b** Find t if $v = 30$, $u = 12$ and $a = 5$.

5 The formula for the area of a triangle is

$$A = \tfrac{1}{2}bh$$

where b is the base and h is the height. Find an expression for h in terms of b and A.

6 The formula for converting temperature in Centigrade to temperature in Fahrenheit is given by

$$F = \tfrac{9}{5}C + 32.$$

Find an expression for C.

Back to Menu >>>

7 The formula for simple interest is given by

$$I = \frac{PRT}{100}$$

where P is the principal, R is the interest rate per annum (%) and T is the time in years.

a Rearrange the formula to make T the subject.

b For how many years does \$5000 need to be invested at a rate of 4% to obtain interest of at least \$1000?

2.6 Developing a formula: setting up linear equations in two unknowns

It is often necessary to develop formulae so that problems can be solved. Constructing a formula is similar to developing an equation from a description.

Example 16 Setting up and solving a linear equation in two unknowns

Sausage rolls cost \$1.30 each and party pies cost 75 cents each.

a Construct a formula for finding the cost, C dollars, of buying x sausage rolls and y party pies.

b Find the cost of 12 sausage rolls and 24 party pies.

Solution

a

1 Work out a formula using x.
One sausage roll costs \$1.30.
Two sausage rolls cost $2 \times \$1.30 = \$2.60$.
Three sausage rolls cost $3 \times \$1.30 = \$3.90$, etc.
Write a formula using x. — x sausage rolls cost $x \times 1.30 = 1.3x$.

2 Similarly, for y:
One party pie costs \$0.75.
Two party pies cost $2 \times \$0.75 = \$1.50$.
Three party pies cost $3 \times \$0.75 = \$2.25$, etc.
Write a formula using y. — y party pies cost $y \times 0.75 = 0.75y$.

3 Combine to get a formula for total cost, C. — $C = 1.3x + 0.75y$

b

1 Write the formula for C. — $C = 1.3x + 0.75y$

2 Substitute $x = 12$ and $y = 24$ into the formula. — $C = 1.3 \times 12 + 0.75 \times 24$

3 Evaluate. — $C = 33.6$

4 Give your answer in dollars and cents. — The total cost for 12 sausage rolls and 24 party pies is \$33.60.

Skillsheet TEACHER

Exercise 2F

1 Balloons cost 50 cents each and streamers costs 20 cents each.

a Construct a formula for the cost, C, of x balloons and y streamers.

b Find the cost of 25 balloons and 20 streamers.

2 Tickets to a concert cost \$40 for adults and \$25 for children.

a Construct a formula for the total amount, C, paid by x adults and y children.

b How much money altogether was paid by 150 adults and 315 children?

3 At the football canteen, chocolate bars cost \$1.60 and muesli bars cost \$1.40.

a Construct a formula to show the total money, C, made by selling x chocolate bars and y muesli bars.

b How much money would be made if 55 chocolate bars and 38 muesli bars were sold?

4 At the bread shop, custard tarts cost \$1.75 and iced doughnuts \$0.70 cents.

a Construct a formula to show the total cost, C, if x custard tarts and y iced doughnuts are purchased.

b On Monday morning, Mary bought 25 custard tarts and 12 iced doughnuts. How much did it cost her?

5 At the beach café, Marion takes orders for coffee and milkshakes. A cup of coffee costs \$2.50 and a milkshake costs \$4.00.

a Using x (coffee) and y (milkshakes),write a formula showing the cost, C, of coffee and milkshake orders taken.

b Marion took orders for 52 cups of coffee and 26 milkshakes. How much money did this make?

6 Joe sells budgerigars for \$30 and parrots for \$60.

a Write a formula showing the money, C, made by selling x budgerigars and y parrots.

b Joe sold 60 budgerigars and 28 parrots. How much money did he make?

7 James has been saving 50c and 20c pieces.

a If James has x 50c pieces and y 20c pieces, write a formula to show the number, N, of coins that James has.

b Write a formula to show the value, V dollars, of James's collection.

c When James counts his coins, he has forty-five 50c pieces and seventy-seven 20c pieces. How much money does he have in total?

Back to Menu >>>

2.7 Setting up and solving simple non-linear equations

Not all equations that are solved in mathematics are linear equations. Some equations are **non-linear**.

For example:

- $y = x^2 + 2$ is a non-linear equation with two unknowns, x and y.
- $d^2 = 25$ is a non-linear equation with one unknown, d.
- $6m^3 = 48$ is a non-linear equation with one unknown, m.

Example 17 Solving a non-linear equation

Solve the equation $x^2 = 81$.

Solution

1 Write the equation.

$$x^2 = 81$$

2 Take the square root of both sides of the equation. (The opposite process of squaring a number is to take the square root.)

$$\sqrt{x^2} = \sqrt{81}$$
$$\therefore x = \pm 9$$

Note: Both the positive and negative answers should be given, as $-9 \times -9 = 81$ and $9 \times 9 = 81$.

Example 18 Solving a non-linear equation

Solve the equation $a^3 = -512$.

Solution

1 Write the equation.

$$a^3 = -512$$

2 Take the cube root of both sides of the equation. (The opposite process of cubing a number is to take the cube root.)

$$\sqrt[3]{a^3} = \sqrt[3]{-512}$$
$$\therefore a = -8$$

Note: $\sqrt[3]{-512} = (-512)^{\frac{1}{3}}$

Note: $(-8) \times (-8) \times (-8) = -512$

but $\quad 8 \times 8 \times 8 = 512$

Example 19 Solving a non-linear equation

Solve the equation $2r^2 = 10$, correct to 2 decimal places.

Solution

1 Write the equation.

$$2r^2 = 10$$

2 Divide both sides of the equation by 2.

$$\frac{2r^2}{2} = \frac{10}{2}$$
$$r^2 = 5$$

Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4

Back to Menu >>>

3 Take the square root of both sides of the equation.

$\sqrt{r^2} = \sqrt{5}$

$r = \sqrt{5}$

4 Round your answer to 2 decimal places.

$\therefore r = \pm 2.24$

Exercise 2G

1 Evaluate the following.

a 4^2 **b** $(-9)^2$ **c** 7^2
d 3^3 **e** 2^3 **f** 6^3
g $(-5)^3$ **h** 4^4 **i** $(-10)^4$

2 Solve the following non-linear equations.

a $a^2 = 25$ **b** $b^2 = 100$ **c** $c^2 = 64$
d $d^2 = 256$ **e** $e^2 = 676$ **f** $f^3 = 8$
g $g^3 = 64$ **h** $h^3 = -125$ **i** $i^3 = -729$

3 Solve the following non-linear equations correct to 2 decimal places.

a $a^2 = 12$ **b** $b^2 = 72$ **c** $c^2 = 568$
d $d^3 = 76$ **e** $e^3 = 300$ **f** $f^3 = -759$

4 Solve the following non-linear equations correct to 2 decimal places.

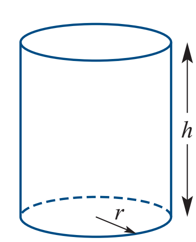
a $3x^2 = 24$ **b** $5y^2 = 25$ **c** $2a^2 = 11$
d $6f^2 = 33$ **e** $4h^2 = 19$ **f** $11c^2 = 75$
g $x^3 = 81$ **h** $r^3 = 18$ **i** $y^3 = 96$
j $2r^3 = 50$ **k** $4m^3 = 76$ **l** $8b^3 = 21$
m $d^4 = 84$ **n** $3e^4 = 159$ **o** $9x^5 = 732$

5 The volume of a cylinder is given by

$$V = \pi r^2 h$$

where h is the height and r is the radius of the base.

a Find an expression for h.
b Find an expression for r.

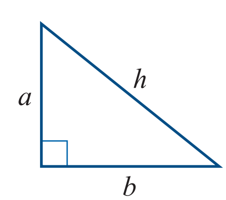


6 Pythagoras' Theorem states that, for any right-angled triangle, the hypotenuse, h, is given by

$$h^2 = a^2 + b^2$$

where a and b are the other two sides of the triangle.

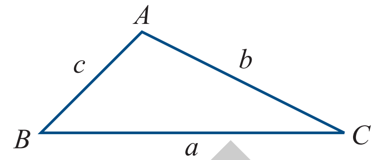
a Find an expression for a.
b Find a, correct to 1 decimal place, when $h = 17.5$ cm and $b = 7.8$ cm.



Back to Menu >>>

7 The cosine rule for a triangle with side lengths a, b and c can be written as

$$a^2 = b^2 + c^2 - 2bc \cos A$$



a Rewrite this rule to make $\cos A$ the subject.

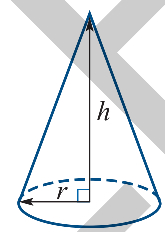
b Find the value of $\cos A$, correct to 4 decimal places, if the sides of the triangle are $a = 30$ cm, $b = 22$ cm and $c = 18$ cm.

8 If $d = \frac{1}{2}at^2$, find an expression for t.

9 The volume, V, of a cone is given by

$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius and h is the height of the cone.



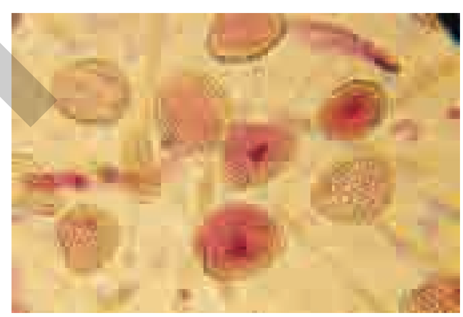
a Rewrite the formula to make r the subject.

b Find, to the nearest cm, the radius if the height of the cone is 15 cm and the volume is 392.7 cm^3.

2.8 Linear recursion

A **recursive relationship** is one where the same thing keeps happening over and over again.

A **recursion** is the process of using a repeated procedure. Recursion can be used to solve problems by repeating a sequence of operations.



For example, a slowly developing bacterial population doubles every day. This situation can be described by a recursive relationship. Assuming that we start with two bacteria, ●●, the population of bacteria can be seen to develop as in the following diagram.

Day	1	2	3	4	....	....	$n - 1$	n	$n + 1$
Number of bacteria	●●	●●●●	●●●●●●●●	●●●●●●●●●●●●●●●●					
Term	$t_1 = 2$	$t_2 = 4$	$t_3 = 8$	$t_4 = 16$	...	...	...	$t_n = 2t_{n-1}$	$t_{n+1} = 2t_n$

On day 1, there are 2 bacteria.	$t_1 = 2$
On day 2, there are $2 \times 2 = 4$ bacteria.	$t_2 = 2 \times t_1$
On day 3, there are $4 \times 2 = 8$ bacteria.	$t_3 = 2 \times t_2$
On day 4, there are $8 \times 2 = 16$ bacteria.	$t_4 = 2 \times t_3$
On day n, there are $t_{n-1} \times 2$ bacteria	$t_n = 2 \times t_{n-1}$
On the $(n + 1)$th day, there are $t_n \times 2$ bacteria.	$t_{n+1} = 2 \times t_n$

Thus a rule for this recursive relationship is $t_{n+1} = 2t_n$, with a starting value $t_1 = 2$.

We can use linear recursion on a graphics calculator to generate a sequence of terms.

How to generate a sequence of terms using linear recursion using the TI-Nspire CAS

A slowly developing bacterial population doubles every day. The rule for this recursive relationship is

$$t_{n+1} = 2t_n.$$

Show the terms of this relationship, if the starting value is 2.

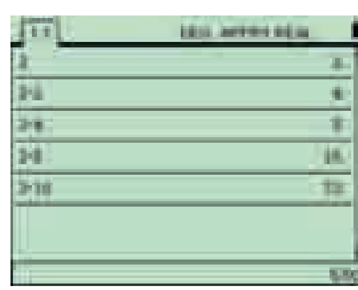
Steps

1 Start a new document: press ctrl + N.

2 Select **1:Add Calculator**.
Type in **2**, the value of the first term. Press enter. The calculator stores the value 2 as **Ans**wer (although you can't see this yet).

3 Now type in $\times$**2** (the screen will show **Ans·2**) and press enter. The second term in the sequence is 4. This value is now stored as **Ans**.
Note: To see the value stored as **Ans** at any time, press ctrl (−) enter.

4 Press enter to generate the next term. Continue pressing enter until the required number of terms is generated.



How to generate a sequence of terms using the ClassPad

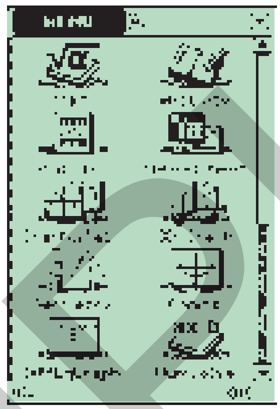
A slowly developing bacterial population doubles every day. The rule for this recursive relationship is

$$t_{n+1} = 2t_n.$$

Show the terms of this relationship, if the starting value is 2.

Steps

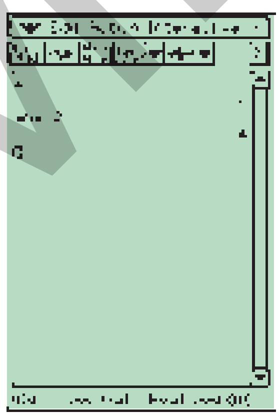
1 Enter the data into your calculator using the built-in **Main** application.
From the application menu screen, locate the built-in **Main** application, Main. Tap to open.
Tapping Menu from the icon panel (just below the touch screen) will display the Application menu if it is not already visible.

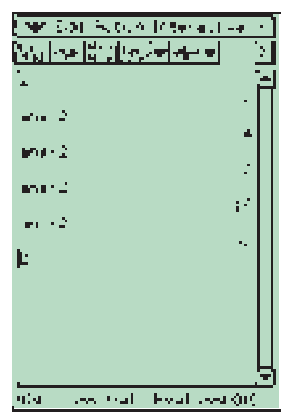


2 **a** Starting with a clean screen, enter the value of the first term, **2**. The calculator stores this value as **ans**.

b Next, press ×**2** followed by (EXE). (The calculator automatically inserts **ans**.)
The second term in the sequence is now displayed; in this case, **4**.

c Pressing (EXE) again generates the next term, **8**, and so on.





Example 20 Using linear recursion

A linear recursion relationship is given by

$$t_{n+1} = t_n + 14$$

Write the first ten terms if the starting value t_1 is 32.

Back to Menu >>>

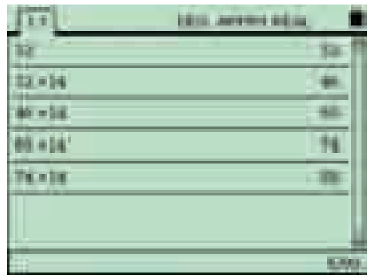
Solution

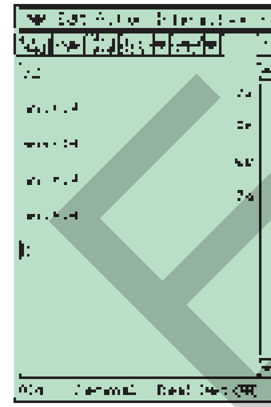
1 On the calculation screen, type in **32** and press (enter) (or (EXE)).

2 Type in + **14** and press (enter) (or (EXE)).

3 Continue pressing (enter) (or (EXE)) until ten terms have been generated.

4 Write your answer.





The first ten terms are 32, 46, 60, 74, 88, 102, 116, 130, 144, 158.

Example 21 Using linear recursion

A linear recursion relationship is given by

$$t_{n+1} = 3t_n - 2$$

Write the first six terms if the starting value is $t_1 = 6$.

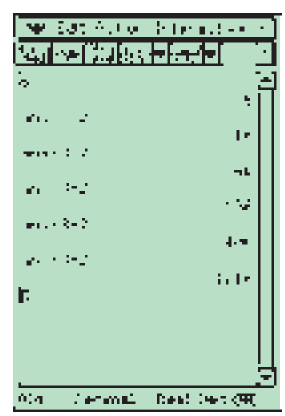
Solution

1 On the calculation screen, type in **6** and press (enter) (or (EXE)).

2 Type in ×**3** − **2** and press (enter) (or (EXE)).

3 Continue pressing (enter) (or (EXE)) until six terms have been generated.

4 Write your answer.



The first six terms are 6, 16, 46, 136, 406, 1216.

Example 22 Using linear recursion to solve practical problems

Maree has \$3000 in her bank account. She adds \$45 to it at the end of each month. How much will she have after 8 months?

Back to Menu >>>

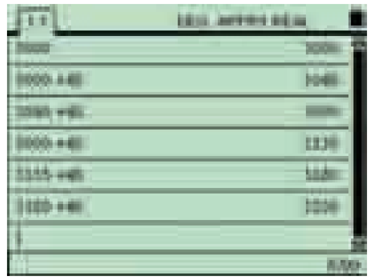
Solution

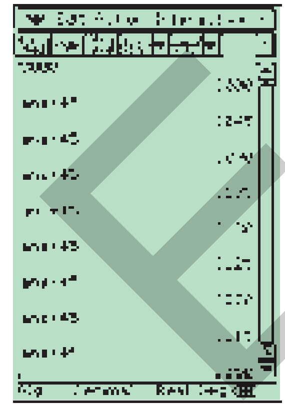
This is a linear recursion relationship for which the starting value is $3000.

1 On the calculation screen, type in **3000** and press enter (or EXE).

2 Each month $45 is added to the account, so type in +**45** and press enter (or EXE). At the end of the first month, Marie has $3045.

3 Continue pressing enter (or EXE) to generate all eight values.





4 Give your answer with the correct units. *After 8 months, Maree will have $3360.*

Example 23 Using linear recursion to solve problems

A person starts a job on an annual salary of $35 000 and receives annual increases of $3500. What will be their salary at the beginning of the fifth year?

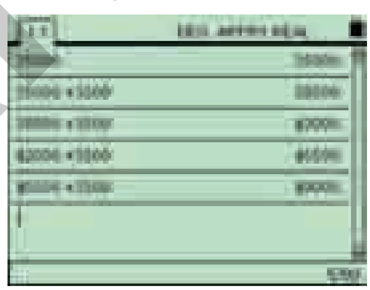
Solution

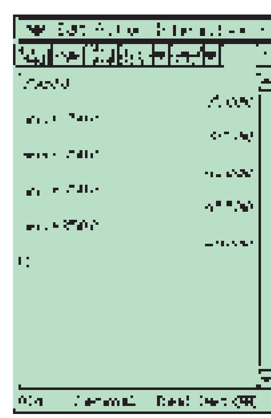
This is a linear recursion relationship with a starting value of $35 000.

1 On the calculation screen, type in **35 000** and press enter (or EXE).

2 Each year there is a salary increase of $3500, so type in + **3500** and press enter (or EXE).

3 Press enter (or EXE) three more times.





At the start of the second year, the salary will be $38 500.
At the start of the third year, the salary will be $42 000.
At the start of the fourth year, the salary will be $45 500.
At the start of the fifth year the salary will be $49 000.

4 Write your answer. *At the start of the fifth year, the salary will be $49 000.*

Back to Menu >>>

Example 24 Using linear recursion to solve problems

A person inherits $25 000 and invests it at 10% per annum. A linear recursion relationship for this is given by

$$t_{n+1} = 1.1t_n$$

Show how the amount of $25 000 increases over 4 years.

Note: An increase of 10% means that the amount is 110% of the original.

$$110\% \text{ means } \frac{110}{100} = 1.1$$

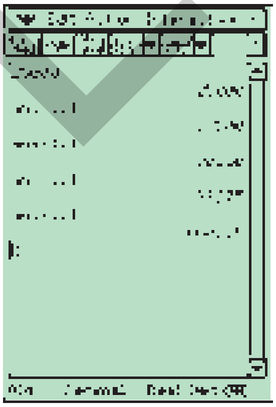
Each year is therefore multiplied by 1.1. The linear recursion relationship is thus defined by $t_{n+1} = 1.1t_n$.

Solution

1 On the calculation screen, type in **25 000** and press enter (or EXE).

2 Each year, there is a 10% increase, so type in $\times$**1.1** and press enter (or EXE).

3 Press enter (or EXE) three more times.



4 Write your answer, showing how the amount increases each year.

At the end of the first year, the amount is $27 500.
At the end of the second year, the amount is $30 250.
At the end of the third year, the amount is $33 275.
At the end of the fourth year, the amount is $36 602.50.

Example 25 Using linear recursion to solve problems

Bruce invests $35 000 at 10% per annum and decides to spend $5000 each year. Show how the balance changes over a 4-year period.

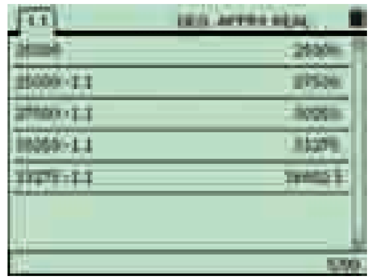
Solution

The linear recursion relationship is $t_{n+1} = 1.1t_n - 5000$

1 On the calculation screen, type in **25 000** and press enter (or EXE).

2 Next type in $\times$**1.1** $-$ **5000** and press enter (or EXE).

3 Press enter (or EXE) three more times.



4 Write your answer, showing how the balance changes each year.

At end of the first year, the balance is \$33 500.
At end of the second year, the balance is \$31 850.
At end of the third year, the balance is \$30 035.
At end of the fourth year, the balance is \$28 038.50.

Exercise 2H

1 A linear recursion relationship is given by $t_{n+1} = t_n - 12$. Write the first five terms if the starting value is $t_1 = 200$.

2 Sarah is saving up for a new car. She already has \$1500 and she is able to save \$400 a month. How much will she have:

a after 6 months? **b** after 12 months?

3 Peter owes \$18 000 to his father. He decides to pay his father \$800 every month.

a How much will he owe:

i after 10 months? **ii** after 1 year? **iii** after 18 months?

b How long will it take Peter to pay the money back to his father?

4 Erica is offered a job with a starting salary of \$29 500 per year and annual increases of \$550.

a What her salary be would:

i at the start of her fifth year on the job?

ii at the start of her eighth year on the job?

b At this rate, how many years would she have to be in the job to receive a salary of \$35 000?

Back to Menu >>>

5 You have \$900 to spend on yourself while on holidays. You plan your budget to spend \$75 per day.

- **a** How much spending money would you have left at the start of the sixth day?
- **b** At this spending rate, for how many days could you afford to stay on holidays?

6 A particular fish population doubles its size every 6 months. Starting with a population of 1000 fish, how many fish would there be after 2 years?

7 Jessica invests \$26 000 with an interest rate of 8% for 6 years. The linear recursion relationship is given by

$$t_{n+1} = 1.08t_n$$

How much will she have after 6 years?

8 Michael inherits \$20 000 and invests it at 10% per annum. He spends \$4000 of the investment each year. The linear recursion relationship is given by

$$t_{n+1} = 1.1t_n - 4000$$

After how many years will the investment all be spent?

9 Jonathon's mother gives him \$5000 to invest at 5% per annum. He is able to save \$500 each year, which he adds to his investment. The linear recursion relationship is given by

$$t_{n+1} = 1.05t_n + 500$$

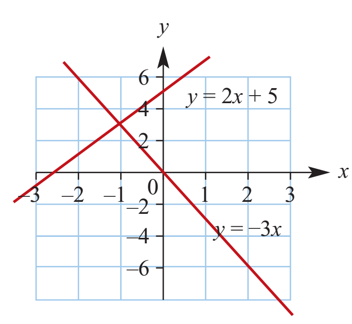
How much will he have after 5 years?

2.9 Finding the point of intersection of two linear graphs

Two straight lines will always intersect unless they are parallel. The point at which two straight lines intersect can be found by sketching the two graphs on the one set of axes and then reading off the coordinates at the point of intersection. When we find the **point of intersection**, we are said to be **solving the equations simultaneously**.

Example 26 **Finding the point of intersection of two linear graphs**

The graphs of $y = 2x + 5$ and $y = -3x$ are shown. Write their point of intersection.



Solution

From the graph it can be seen that the point of intersection is $(-1, 3)$.

Back to Menu >>>

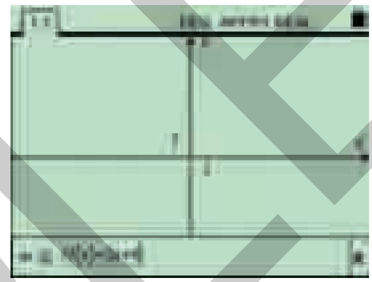
A graphics calculator can also be used to find the point of intersection.

How to find the point of intersection of two linear graphs using the TI-Nspire CAS

Use a graphics calculator to find the point of intersection of the simultaneous equations $y = 2x + 6$ and $y = -2x + 3$.

Steps

1 Start a new document (by pressing ctrl + N) and select **2:Add Graphs & Geometry**.

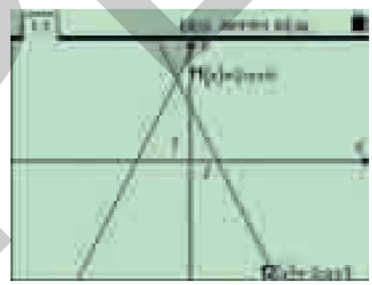


2 Type in the first equation, as shown. Note that $f1(x)$ represents the y term. Press enter to plot the first graph. The edit line will change to $f2(x)$. Type in the second equation and press enter to plot the second graph.

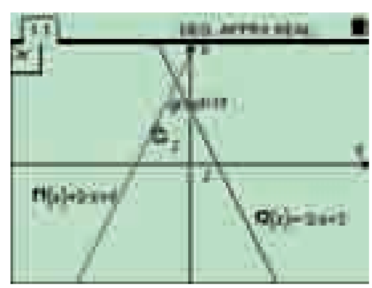
Hint: To see all entered equations move the cursor onto the ▸ and press ⊙.

Hint: ctrl + G will hide the entry line.

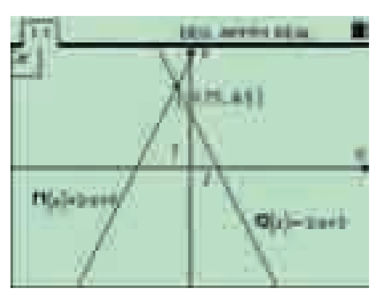
Note: To change window settings, press menu **/4:Window/1:Window Settings** and change to suit. Press enter when finished.



3 To find the point of intersection, press menu **/6:Points & Lines/3:Intersection Point(s)**. Move the cursor to one of the graphs until it flashes, press ⊙, then move to the other graph and press ⊙. The solution will appear.



4 Press enter to display the solution on the screen.
The coordinates of the point of intersections are $x = -0.75$ and $y = 4.5$.



Back to Menu >>>

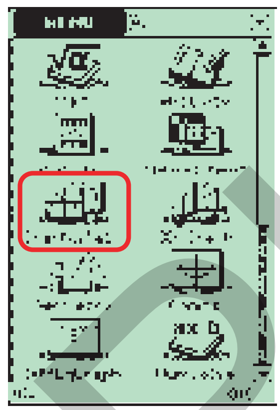
How to find the point of intersection of two linear graphs using the ClassPad

Use a graphics calculator to find the point of intersection of the simultaneous equations $y = 2x + 6$ and $y = -2x + 3$.

Steps

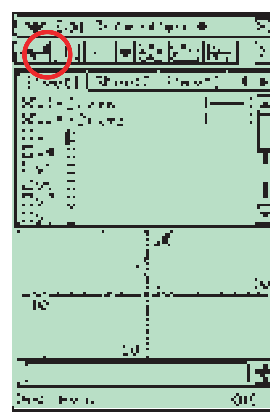
1. Open the built-in **Graphs and Tables** application.

   Tapping Menu from the icon panel (just below the touch screen) will display the Application menu if it is not already visible.



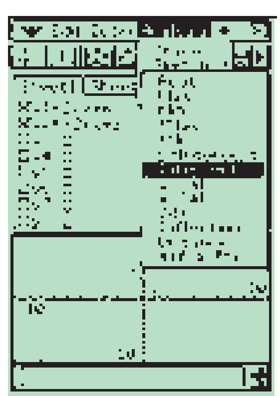
2. Tap the icon and complete the **View Window**.
3. Enter the equations into the graph editor window.

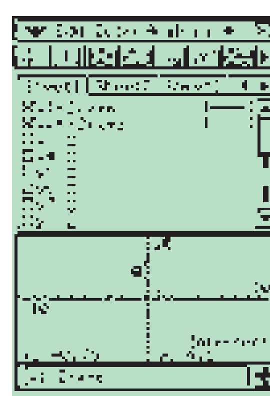
   Tap the icon to plot the graphs.



4. Solve by finding the point of intersection. Select **Analysis** from the menu bar, then **G-solve**, then **Intersect**.

   The required solution is displayed on the screen: $x = -0.75$ and $y = 4.5$.





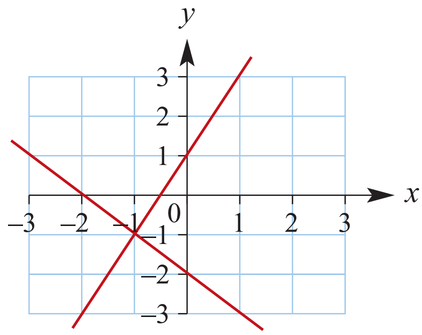
Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4

Back to Menu >>>

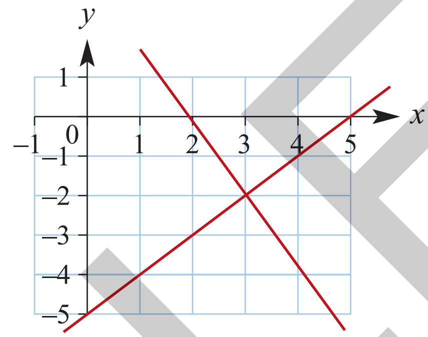
Exercise 2I

1 State the point of intersection for each of these pairs of straight lines.

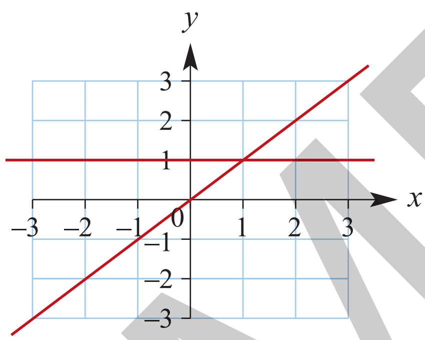
a



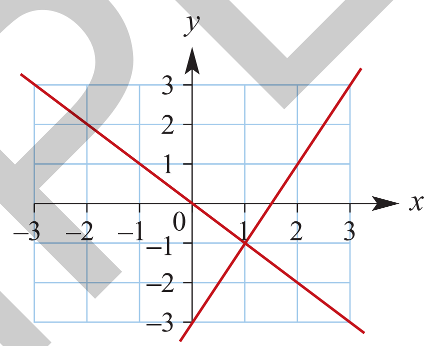
b



c



d



2 Using a graphics calculator, find the point of intersection of each of these pairs of lines.

a $y = x - 6$ and $y = -2x$
b $y = x + 5$ and $y = -x - 1$
c $y = 3x - 2$ and $y = 4 - x$
d $y = x - 1$ and $y = 2x - 3$
e $y = 2x + 6$ and $y = 6 + x$
f $x - y = 5$ and $y = 2$
g $x + 2y = 6$ and $y = 3 - x$
h $2x + y = 7$ and $y - 3x = 2$
i $3x + 2y = -4$ and $y = x - 3$
j $y = 4x - 3$ and $y = 3x + 4$
k $y = x - 12$ and $y = 2x - 4$
l $y + x = 7$ and $2y + 5x = 5$

2.10 Solving simultaneous linear equations algebraically

When solving simultaneous equations algebraically, there are two methods that can be used: **substitution** or **elimination:** Both methods will be demonstrated here.

Method 1: Substitution

When solving simultaneous equations by substitution, the process is to substitute one variable from one equation into the other equation.

Example 27 Solving simultaneous equations by substitution

Solve the pair of simultaneous equations $y = 5 - 2x$ and $3x - 2y = 4$.

Solution

1 Number the two equations as (1) and (2).	$y = 5 - 2x$ (1) $3x - 2y = 4$ (2)
2 Substitute the y-value from equation (1) into equation (2).	Substitute (1) into (2). $3x - 2(5 - 2x) = 4$
3 Expand the brackets and then collect like terms.	$3x - 10 + 4x = 4$ $7x - 10 = 4$
4 Solve for x. Add 10 to both sides of the equation.	$7x - 10 + 10 = 4 + 10$ $7x = 14$
Divide both sides of the equation by 7.	$\frac{7x}{7} = \frac{14}{7}$ $\therefore x = 2$
5 To find y, substitute $x = 2$ into equation (1).	Substitute $x = 2$ into (1). $y = 5 - 2(2)$ $y = 5 - 4$ $\therefore y = 1$
6 Check by substituting $x = 2$ and $y = 1$ into equation (2).	LHS $= 3(2) - 2(1)$ $= 6 - 2 = 4 =$ RHS
7 Write your solution as a set of coordinates.	Solution is (2, 1).

Example 28 Solving simultananeous equations by substitution

Solve the pair of simultaneous equations $y = x + 5$ and $y = -3x + 9$.

Solution

1 Number the two equations as (1) and (2).	$y = x + 5$ (1) $y = -3x + 9$ (2)
2 Both equations are expressions for y, so they can be made equal to each other.	$x + 5 = -3x + 9$
3 Solve for x. Add $3x$ to both sides of the equation.	$x + 5 + 3x = -3x + 9 + 3x$ $4x + 5 = 9$
Subtract 5 from both sides of the equation.	$4x + 5 - 5 = 9 - 5$ $4x = 4$
Divide both sides of the equation by 4.	$\frac{4x}{4} = \frac{4}{4}$ $\therefore x = 1$

Back to Menu >>>

4 Find y by substituting $x = 1$ into either equation (1) or equation (2).

Substitute $x = 1$ into (1).
$y = 1 + 5$
$\therefore y = 6$

5 Check by substituting $x = 1$ and $y = 6$ into equation (2).

LHS $= 6$
RHS $= -3(1) + 9 = -3 + 9 = 6$

6 Write your answer as a pair of coordinates.

Solution is (1, 6).

Method 2: Elimination

When solving simultaneous equations by elimination, one of the unknown variables is eliminated by the process of adding or subtracting multiples of the two equations.

Example 29 Solving simultaneous equations by elimination

Solve the pair of simultaneous equations $x + y = 3$ and $2x - y = -9$.

Solution

1 Number the two equations.
On inspection, it can be seen that if the two equations are added, the variable y will be eliminated.

$x + y = 3 \quad (1)$
$2x - y = 9 \quad (2)$

2 Add equations (1) and (2).

(1) + (2): $3x = 12$

3 Solve for x. Divide both sides of the equation by 3.

$$\frac{3x}{3} = \frac{12}{3}$$
$\therefore x = 4$

4 Substitute $x = 4$ into equation (1) to find the corresponding y value.

Substitute $x = 4$ into (1).
$4 + y = 3$

5 Solve for y. Subtract 4 from both sides of the equation.

$4 + y - 4 = 3 - 4$
$\therefore y = -1$

6 Check by substituting $x = 4$ and $y = -1$ into equation (2).

LHS $= 2(4) - (-1)$
$= 8 + 1 = 9 =$ RHS

7 Write your answer as a pair of coordinates.

Solution is $(4, -1)$.

Example 30 Solving simultaneous equations by elimination

Solve the pair of simultaneous equations $3x + 5y = 8$ and $x - 2y = -1$.

Back to Menu >>>

Solution

1 Label the two equations (1) and (2).

$$3x + 5y = 8 \quad (1)$$
$$x - 2y = -1 \quad (2)$$

2 Multiply equation (2) by 3 to give a value of $3x$, as in equation (1). Label the new equation as (3).

Remember: Each term in equation (2) must be multiplied by 3.

$$(2) \times 3: \quad 3x - 6y = -3 \quad (3)$$

3 Subtract equation (3) from equation (1) to eliminate $3x$.

$$(1) - (3): \quad 11y = 11$$

4 Solve for y. Divide both sides of the equation by 11.

$$\frac{11y}{11} = \frac{11}{11}$$
$$\therefore y = 1$$

5 To find x, substitute $y = 1$ into equation (1).

Substitute $y = 1$ into (1).
$$3x + 5(1) = 8$$
$$3x + 5 = 8$$

6 Solve for x. Subtract 5 from both sides of the equation.

$$3x + 5 - 5 = 8 - 5$$
$$3x = 3$$

Divide both sides of the equation by 3.

$$\frac{3x}{3} = \frac{3}{3}$$
$$\therefore x = 1$$

7 Check by substituting $x = 1$ and $y = 1$ into equation (2).

$$\text{LHS} = 1 - 2(1)$$
$$= 1 - 2 = -1 = \text{RHS}$$

8 Write your answer as a pair of coordinates.

Solution is (1, 1).

Exercise 2J

1 Solve the following pairs of simultaneous equations by any algebraic method (elimination or substitution).

a $y = x - 1$
$3x + 2y = 8$

b $y = x + 3$
$6x + y = 17$

c $x + 3y = 15$
$y - x = 1$

d $x + y = 10$
$x - y = 8$

e $2x + 3y = 12$
$4x - 3y = 6$

f $3x + 5y = 8$
$x - 2y = -1$

g $2x + y = 11$
$3x - y = 9$

h $2x + 3y = 15$
$6x - y = 11$

i $3p + 5q = 17$
$4p + 5q = 16$

j $4x + 3y = 7$
$6x - 3y = -27$

k $3x + 5y = -11$
$-3x - 2y = 8$

l $4x - 3y = 6$
$-2x + 5y = 4$

Back to Menu >>>

2 Solve the following pairs of simultaneous equations by any suitable method.

a $y = 6 - x$
$2x + y = 8$

b $2x + 3y = 5$
$y = 7 - 2x$

c $3x + y = 4$
$y = 2 - 4x$

d $3x + 5y = 9$
$y = 3$

e $3x + 2y = 0$
$-3x - y = 3$

f $4x + 3y = -28$
$5x - 6y = -35$

2.11 Solving simultaneous linear equations using a graphics calculator

How to solve simultaneous equations using the TI-Nspire CAS

Solve the following pair of simultaneous equations:

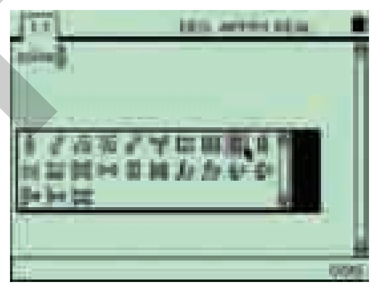
$$24x + 12y = 36$$
$$45x + 30y = 90$$

Steps

1 Start a new document and select **1:Add Calculator**.

a Press menu **/3:Algebra/1:Solve** to paste the **solve(** command onto the calculation screen.

b Press ctrl + and select the template shown opposite.



2 Enter the equations as shown. Use the tab key to move between entry boxes.

a Press tab to move to the end of the equations and type , X , Y.

b Press enter to display the solution, $x = 0$ and $y = 3$.

3 Check the solution $x = 0$ and $y = 3$ by substitution.

LHS $= 24(0) + 12(3) = 36 =$ RHS
LHS $= 45(0) + 30(3) = 90 =$ RHS

4 Write your answer as a pair of coordinates.

Solution is (0, 3).

Back to Menu >>>

How to solve simultaneous linear equations algebraically using the ClassPad

Solve the following pair of simultaneous equations:

$$24x + 12y = 36$$
$$45x + 30y = 90$$

Steps

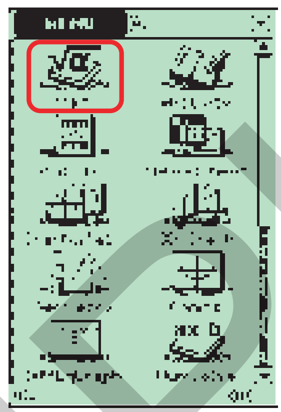
1 Open the built-in **Main** application.

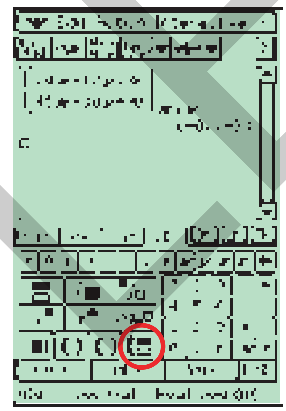
  a Press Keyboard on the front of the calculator to display the built-in keyboard.

  b Tap the 2D tab and locate simultaneous equations icon:

  c Enter the information

$$\begin{cases} 24x + 12y = 36 \\ 45x + 30y = 90 \end{cases}\Bigg|_{x,y}$$





2 Press EXE to display the solution, $x = 0$ and $y = 3$.

3 Check the solution $x = 0$ and $y = 3$ by substitution.

LHS $= 24(0) + 12(3) = 36 =$ RHS
LHS $= 45(0) + 30(3) = 90 =$ RHS

4 Write your answer as a pair of coordinates.

Solution is $(0, 3)$.

Exercise 2K

Solve the following simultaneous equations:

a $2x + 5y = 3$
  $x + y = 3$

b $3x + 2y = 5.5$
  $2x - y = -1$

c $3x - 8y = 13$
  $-2x - 3y = 8$

d $2h - d = 3$
  $8h - 7d = 18$

e $2p - 5k = 11$
  $5p + 3k = 12$

f $5t + 4s = 16$
  $2t + 5s = 12$

g $2m - n = 1$
  $2n + m = 8$

h $15x - 4y = 6$
  $-2y + 9x = 5$

i $2a - 4b = -12$
  $2b + 3a - 2 = -2$

2.12 Practical applications of simultaneous equations

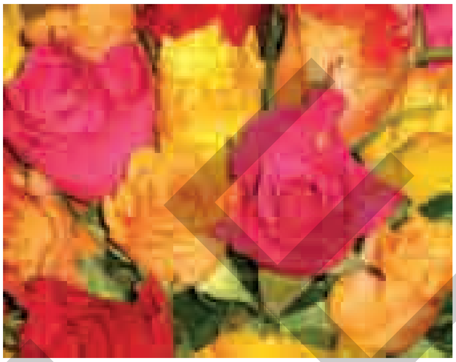
Simultaneous equations can be used to solve problems in real situations. It is important to define the unknown quantities with appropriate variables before setting up the equations.

Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4

Back to Menu >>>

Example 31 Using simultaneous equations to solve a practical problem

Mark buys 3 roses and 2 gardenias for \$15.50. Peter buys 5 roses and 3 gardenias for \$24.50. How much did each type of flower cost?



Solution

Strategy: Using the information given, set up a pair of simultaneous equations to solve.

1 Choose appropriate variables to represent the cost of roses and gardenias.

Let r be the cost of a rose and g be the cost of a gardenia.

2 Write two simultaneous equations using the information given in the question. Label the equations (1) and (2).

$$3r + 2g = 15.5 \quad (1)$$
$$5r + 3g = 24.5 \quad (2)$$

3 Eliminate one of the variables (g) by multiplying equation (1) by 3 and equation (2) by 2, to give $6g$ in both equations. Label the new equations (3) and (4), respectively.

$$(1) \times 3: \quad 9r + 6g = 46.5 \quad (3)$$
$$(2) \times 2: \quad 10r + 6g = 49 \quad (4)$$

4 Subtract equation (4) from equation (3) to eliminate the g term.

$$(3) - (4): \quad -r = -2.5$$

5 Divide both sides of the equation by –1 to find r.

$$\therefore r = 2.5$$

6 Substitute $r = 2.5$ into equation (1) to find g.

Substitute $r = 2.5$ into (1).

$$3(2.5) + 2g = 15.5$$
$$7.5 + 2g = 15.5$$

7 Solve for g. Subtract 7.5 from both sides of the equation.

$$7.5 + 2g - 7.5 = 15.5 - 7.5$$
$$2g = 8$$

Divide both sides of the equation by 2.

$$\frac{2g}{2} = \frac{8}{2}$$
$$\therefore g = 4$$

8 Check by substituting $r = 2.5$ and $g = 4$ into equation (2).

$$\text{LHS} = 5(2.5) + 3(4)$$
$$= 12.5 + 12 = 24.5 = \text{RHS}$$

9 Write your answer with the correct units.

Roses cost \$2.50 each and gardenias cost \$4 each.

Example 32 **Using simultaneous equations to solve a practical problem**

The perimeter of a rectangle is 48 cm. If the length of the rectangle is three times the width, determine its dimensions.

Solution

Strategy: Using the information given, set up a pair of simultaneous equations to solve.

1 Choose appropriate variables to represent the dimensions of width and length.

Let W = width
L = length.

2 Write two equations from the information given in the question. Label the equations as (1) and (2).

Remember: The perimeter of a rectangle is the distance around the outside and can be found using $2w + 2l$.

$$2W + 2L = 48 \quad (1)$$
$$L = 3W \quad (2)$$

3 Solve the simultaneous equations by substituting equation (2) in equation (1).

Substitute $L = 3W$ into (1).
$$2W + 2(3W) = 48$$

4 Expand the brackets.

$$2W + 6W = 48$$

5 Collect like terms.

$$8W = 48$$

6 Solve for w. Divide both sides by 8.

$$\frac{8W}{8} = \frac{48}{8}$$
$$\therefore W = 6$$

7 Find the corresponding value for l by substituting $w = 6$ into equation (2).

Substitute $W = 6$ into (2).
$$L = 3(6)$$
$$\therefore L = 18$$

8 Give your answer in the correct units.

The dimensions of the rectangle are width 6 cm and length 18 cm.

Exercise 2L

1 Jessica bought 5 textas and 6 pencils for \$12.75, and Tom bought 7 textas and 3 pencils for \$13.80.

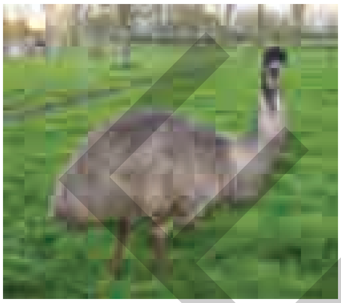
   a Using t for texta and p for pencil, find a pair of simultaneous equations to solve.
   b How much did one pencil and one texta cost?

2 Peter buys 50 L of petrol and 5 L of motor oil for \$93. His brother Anthony buys 75 L of petrol and 5 L of motor oil for \$122. How much do a litre of petrol and a litre of motor oil cost each?

3 Six oranges and ten bananas cost \$7.10. Three oranges and eight bananas cost \$4.60. Find the cost each of oranges and bananas.

4 The weight of a box of nails and a box of screws is 2.5 kg. Four boxes of nails and a box of screws weigh 7 kg. Determine the weight of each.

5 An enclosure at a wildlife sanctuary contains wombats and emus. If the number of heads totals 28 and the number of legs totals 88, determine the number of each species present.



6 The perimeter of a rectangle is 36 cm. If the length of the rectangle is twice its width, determine its dimensions.

7 Find a pair of numbers whose sum is 52 and whose difference is 8.

8 Bruce is 4 years older than Michelle. If their combined age is 70, determine their individual ages.

9 A chocolate thickshake costs \$2 more than a fruit smoothie. Jack pays \$27 for 3 chocolate thickshakes and 4 fruit smoothies. How much do a chocolate thickshake and a fruit smoothie cost each?

10 In 4 years time a mother will be three times as old as her son. Four years ago she was five times as old as her son. Find their present ages.

Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4

Key ideas and chapter summary

Formula

A **formula** is a mathematical relationship connecting two or more variables.

Examples:

$$C = 5t + 20$$
$$P = 2L + 2W$$
$$A = \pi r^2$$

Linear equation

A **linear equation** is an equation whose unknown values are always to the power of 1.

Examples:

$$y = 2x - 3$$
$$m + 3 = 7$$
$$3a - 2c = 8$$

Non-linear equation

A **non-linear equation** is one whose unknown values are *not* all to the power of 1.

Examples:

$$y = x^2 + 5$$
$$3y^2 = 6$$
$$b^3 = 27$$

Literal equation

A **literal equation** is an equation whose solution will be expressed in terms of other unknown values rather than numbers.
Example:
The solution for x of the equation $x + 3y = 7$ is $x = 7 - 3y$.

Linear recursion (iteration)

Linear recursion (iteration) is the process of using a repeated procedure.

Simultaneous equations

Two straight lines will always intersect, unless they are parallel. At the point of intersection the two lines will have the same coordinates. When we find the point of intersection, we are solving the equations simultaneously. Simultaneous equations can be solved graphically, algebraically or by using a graphics calculator program.
Example:
$3x + 2y = 6$
$4x - 5y = 12$
are a pair of simultaneous equations.

Skills check

Having completed the current chapter you should be able to:

- substitute values in linear relations and formulas
- construct tables of values from given formulas
- solve linear equations
- use linear equations to solve practical problems
- solve literal equations
- develop formulas from descriptions
- transpose formulas
- understand linear recursion relationships
- solve simultaneous equations graphically, algebraically and with a graphics calculator program.

Multiple-choice questions

1 If $a = 4$, then $3a + 5 =$

A 39 **B** 12 **C** 17 **D** 27

2 If $b = 1$, then $2b - 9 =$

A -11 **B** -7 **C** 12 **D** 21

3 If $C = 50t + 14$ and $t = 8$, then $C =$

A 522 **B** 1100 **C** 72 **D** 414

4 If $P = 2L + 2W$, then the value of P when $L = 6$ and $W = 2$ is:

A 48 **B** 16 **C** 12 **D** 30

5 If $x = -2$, $y = 3$ and $z = 7$, then $\dfrac{z - x}{y} =$

A 3 **B** $\dfrac{5}{3}$ **C** $\dfrac{-5}{3}$ **D** -3

6 If $a = 2$, $b = 5$, $c = 6$ and $d = 10$, then $bd - ac =$

A 38 **B** 24 **C** 7 **D** 484

7 The area of a circle is given by $A = \pi r^2$. If $r = 6$ cm, then the area of the circle is:

A 18.84 cm^2 **B** 37.70 cm^2 **C** 355 31 cm^2 **D** 113.10 cm^2

8 The solution to $4x = 24$ is:

A $x = 2$ **B** $x = 6$ **C** $x = 20$ **D** $x = 96$

9 The solution to $\dfrac{x}{3} = -8$ is:

A $\dfrac{8}{3}$ **B** 24 **C** $\dfrac{-8}{3}$ **D** -24

10 The solution to $2v + 5 = 11$ is:

A $v = 8$ **B** $v = 3$ **C** $v = 6$ **D** $v = 16$

Back to Menu >>>

Review

11 The solution to $4x^2 = 64$ is:

A $x = \pm 16$ **B** $x = \pm 2$ **C** $x = \pm 4$ **D** $x = \pm 8$

12 The solution to $3k - 5 = -14$ is:

A $k = 115.67$ **B** $k = 19$ **C** $k = 3$ **D** $k = -3$

13 The cost of hiring a car for a day is \$60 plus 0.25c per kilometre. Michelle travels 750 km. Her total cost is:

A \$810 **B** \$187.50 **C** \$247.50 **D** \$188.10

14 Given $v = u + at$ and $v = 11.6$ when $u = 6.5$ and $a = 3.7$, the value of t correct to 2 decimal places is:

A 1.378 **B** 9.84 **C** 1.37 **D** 1.38

15 The area of a trapezium is given by $A = \dfrac{(a+b)h}{2}$. An expression for h is:

A $\dfrac{2A}{a+b}$ **B** $\dfrac{A-2}{a+b}$ **C** $\dfrac{A-(a+b)}{2}$ **D** $2A - (a+b)$

16 A linear recursion relationship is given by $t_{n+1} = t_n + 26$. If the starting term t_1 is 14, then the sixth term is:

A 144 **B** 170 **C** 118 **D** 40

17 The solution to the pair of simultaneous equations

$y = 5x$

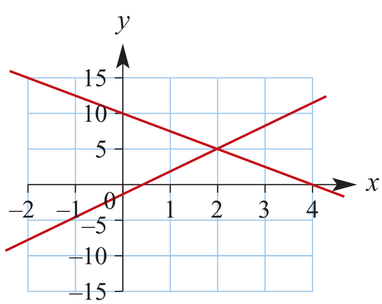
$y = 2x + 6$

is:

A $(-2, 0)$ **B** $(-1, -5)$ **C** $(3, 0)$ **D** $(2, 10)$

18 The point of intersection of the lines shown in the diagram is:

A $(5, 2)$
B $(0, 0)$
C $(0, 9)$
D $(2, 5)$



19 The solution to the pair of simultaneous equations

$$2x + 3y = -6$$
$$x + 3y = 0$$

is:

A $(-6, -2)$ **B** $(6, 2)$ **C** $(-2, 6)$ **D** $(-6, 2)$

Short-answer questions

1 Solve the following equations for x.
a $x + 5 = 15$ b $x - 7 = 4$ c $16 + x = 24$ d $9 - x = 3$
e $2x + 8 = 10$ f $3x - 4 = 17$ g $x + 4 = -2$ h $3 - x = -8$
i $6x + 8 = 26$ j $3x - 4 = 5$ k $\frac{x}{5} = 3$ l $\frac{x}{-2} = 12$

2 If $P = 2l + 2b$, find P if:
a $l = 12$ and $b = 8$ b $l = 40$ and $b = 25$.

3 If $A = \frac{1}{2}bh$, find A if:
a $b = 6$ and $h = 10$ b $b = 12$ and $h = 9$.

4 The formula for finding the circumference of a circle is given by $C = 2\pi r$, where r is the radius. Find the circumference of a circle with radius 15 cm correct to 2 decimal places.

5 For the equation $y = 33x - 56$, construct a table of values for values of x in intervals of 5 from -20 to 25.
a For what value of x is $y = 274$? b When $y = -221$, what value is x?

6 Solve the following non-linear equations.
a $a^2 = 49$ b $b^2 = 8836$ c $2c^2 = 128$ d $d^3 = 27$
e $e^3 = -64$ f $3f^3 = 81$ g $g^4 = 16$ h $h^5 = 3125$

7 I think of a number, double it and add 4. If the result is 6, what is the original number?

8 Four less than three times a number is 11. What is the number?

9 Find an expression for h in each of the following.
a $d = 3e - h$ b $A = 2\pi r^2 + 2\pi rh$

10 Rearrange the formula $m = 3s - 2t$ to make t the subject.

11 Find the point of intersection of the following pairs of lines.
a $y = x + 2$ and $y = 6 - 3x$ b $y = x - 3$ and $2x - y = 7$
c $x + y = 6$ and $2x - y = 9$

12 Solve the following pairs of simultaneous equations.
a $y = 5x - 2$ and $2x + y = 12$ b $x + 2y = 8$ and $3x - 2y = 4$
c $2p - q = 12$ and $p + q = 3$ d $3p + 5q = 25$ and $2p - q = 8$
e $3p + 2q = 8$ and $p - 2q = 0$

13 A linear recursion relationship is given by $t_{n+1} = t_n + 17$. Write down the first six terms if the starting value is 14.

14 A linear recursion relationship is given by $t_{n+1} = 2t_n - 4$. If the starting value is 75, write down the first eight terms.

Review

Extended-response questions

1 The cost, C, of hiring a boat is given by $C = 8h + 25$ where h represents hours.

a What is the cost if the boat is hired for 4 hours?

b For how many hours was the boat hired if the cost was \$81?

2 A phone bill is calculated using the formula $C = 25 + 0.50n$ where n is the number of calls made.

a Complete the table of values below for values of n from 60 to 160.

b What is the cost of making 160 phone calls?

n	60	70	80	90	100	110	120	130	140	150	160
C											

3 An electrician charges \$80 up front and \$45 for each hour, h, that he works.

a Write a linear equation for the total charge, C, of any job.

b How much would a 3-hour job cost?

4 Two families went to the theatre. The first family bought tickets for 3 adults and 5 children and paid \$73.50. The second family bought tickets for 2 adults and 3 children and paid \$46.50.

a Write down two simultaneous equations that could be used to solve the problem.

b What was the cost of an adult's ticket?

c What was the cost of a child's ticket?

5 A bank account has \$5000 in it. At the end of each month \$300 is withdrawn.

a How much is in the account at the end of 6 months?

b How much is in the account after 1 year?

c How long until there is no more money left?

6 Mark invests \$3000 at 3% per annum. The linear recursion relationship describing this investment is given by $t_{n+1} = 1.03t_n$. Show how \$3000 increases over 5 years.

Cambridge University Press • Uncorrected Sample Pages • 978-0-521-74049-4