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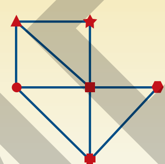
CHAPTER 25

MODULE 5

Revision: Networks and decision mathematics

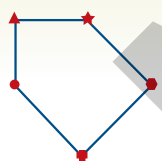
25.1 Multiple-choice questions

1 Consider this network graph.

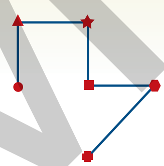


A subgraph of this graph is:

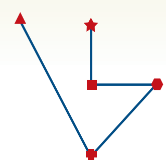
A



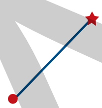
B



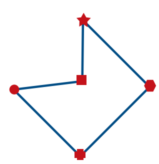
C



D

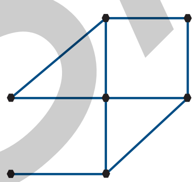


E

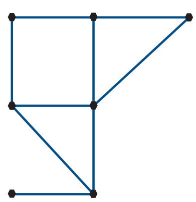


[VCAA 2004]

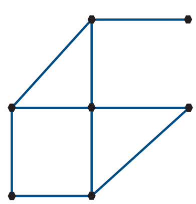
2 The diagrams show four connected planar graphs.



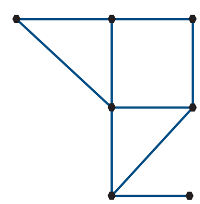
Graph *J*



Graph *K*



Graph *L*



Graph *M*

Equivalent graphs are:

A J and L only
B J and K and L only
C J and K and M only
D J and L and M only
E J and K and L and M

[VCAA 2004]

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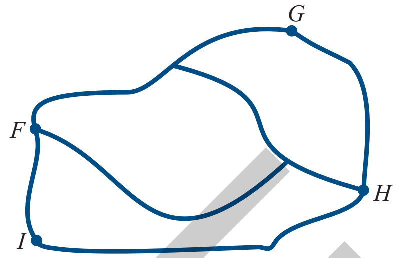
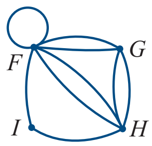
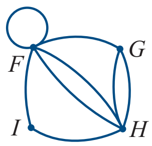
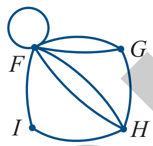
3 The diagram shows a map of the roads between four towns, F, G, H and I.

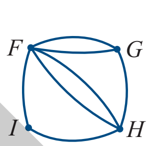


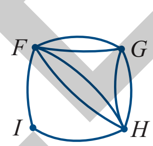
A network diagram that represents all the connections between the four towns on the map is:

A


B


C


D


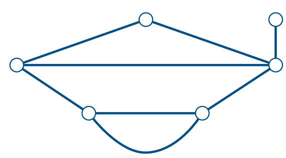
E


4 Five people are to be each allocated one of five tasks (A, B, C, D, E). The table shows the time, in hours, that each person takes to complete the tasks. The tasks must be completed in the least possible total amount of time. If no person can help another, Francis should be allocated task:

	A	*B*	*C*	*D*	*E*
Francis	12	15	99	10	14
David	10	9	10	7	12
Herman	99	10	11	6	12
Indira	8	8	12	9	99
Natalie	8	99	9	8	11

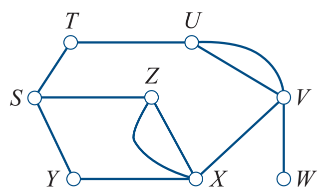
A *A* **B** *B* **C** *C* **D** *D* **E** *E*

5 The sum of the degrees of all the vertices in the network opposite is:



A 6 **B** 7 **C** 8 **D** 15 **E** 16

6 Adding which one of the following edges creates an Euler path?



A *ST* **B** *SU* **C** *SX* **D** *XW* **E** *ZY*

7 A connected planar graph has an equal number of vertices and faces. If there are 20 edges in this graph, the number of vertices must be:

A 9 **B** 10 **C** 11 **D** 20 **E** 22

8 Underground water pipes are needed to water a new golf course. Water will be pumped from the dam in the back corner of the course. To find the smallest total length of water pipe needed, we must find:

A a critical path **B** a minimal spanning tree **C** the shortest Euler circuit
D the shortest Hamiltonian circuit **E** the perimeter of the golf course

9 Which one of the following is a true statement about a critical path in a project?

A Knowledge of the critical path can be used to decide if any tasks in a project can be delayed without extending the length of time of the project.

B All tasks on the critical path must be completed before any other task in the same project can be started.

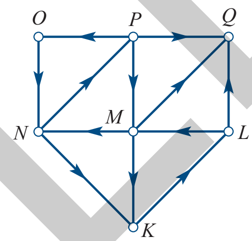
C Decreasing the times of tasks not on the critical path will decrease the length of time of the project.

D The critical path must always include at least two tasks in a project.

E There is only one critical path in any project.

10 For the directed graph shown, vertex O cannot be reached from vertex:

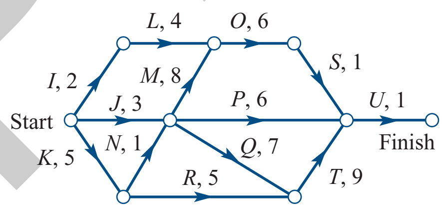
A L **B** M **C** N **D** P **E** Q



11 The network gives the times in hours to complete the 12 tasks required to finish a project. The critical path for this project is:

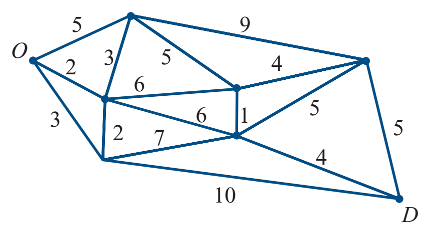
A J-P-U **B** K-R-T-U **C** J-M-O-S-U

D K-N-Q-T-U **E** K-N-M-O-S-U

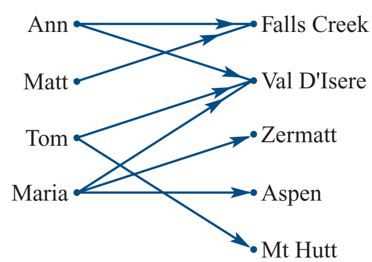


12 The shortest path between the origin, O, and destination, D, in the network shown here is:

A 11 **B** 12 **C** 13 **D** 14 **E** 15



13 Four students, talking about five ski resorts they have visited, represented their information on the bipartite graph shown here. Which one of the following statements is implied by this bipartite graph?



A Ann and Maria between them have visited fewer ski resorts than Matt and Tom between them.

B Matt and Tom have been to four ski resorts between them.

C Maria has visited fewer ski resorts than any of the others.

D Ann and Maria between them have visited all five ski resorts discussed.

E Ann and Tom between them have visited fewer resorts than Matt and Maria between them.

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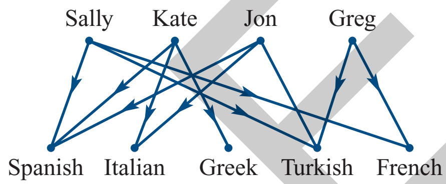
14 A gas pipeline is to be constructed to link several towns in the country. Assuming the pipeline construction costs are the same everywhere in the region, the cheapest network formed by the pipelines and the towns as vertices would form:

A a Hamilton circuit **B** an Euler circuit **C** a minimum length spanning tree
D a critical path **E** a complete graph

15 Which one of the following statements is not implied by this bipartite graph?

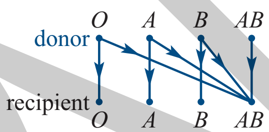


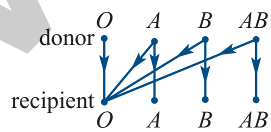
A There are more translators of French than Greek.
B Sally and Kate can translate five languages between them.
C Jon and Greg can translate four languages between them.
D Kate and Jon can translate more languages between them than can Sally and Greg.
E Sally and Jon can translate more languages between them than can Kate and Greg

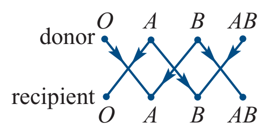
16 There are four different human blood types: O, A, B and AB. The relationships between donor and recipients for these blood types are as follows:

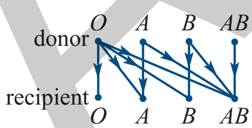
Type O can donate blood to any type.
Type AB can receive blood from any type.
Each type can donate blood to its own type.
Each type can receive blood from its own type.

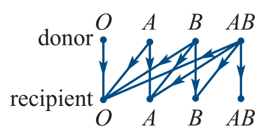
Which one of the following donor–recipient bipartite graphs correctly represents this information?

A


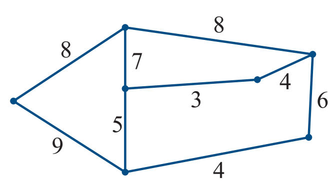
B


C


D


E


17 For the weighted graph shown, the length (total weight) of the minimum spanning tree is:



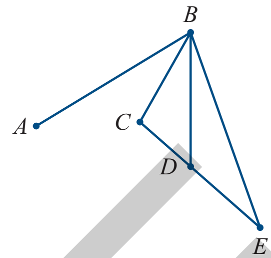
A 28 **B** 29 **C** 30 **D** 31 **E** 32

18 The number of edges for a complete graph with five vertices is:

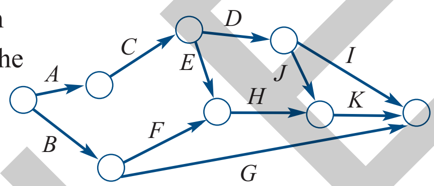
A 4 **B** 5 **C** 10 **D** 15 **E** 20

19 What additional arc could be added to the graph to ensure the resulting graph would contain an Euler circuit?

A *AB* **B** *AC* **C** *AD* **D** *AE* **E** *BC*



20 This network represents a project development with activities listed on the arcs of the graph. Which of the following statements must be true?

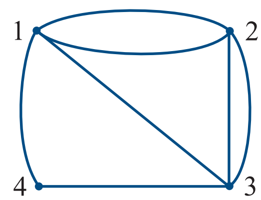


A *A* must be completed before *B* can start.
B *A* must be completed before *F* can start.
C *E* and *F* must start at the same time.
D *E* and *F* must finish at the same time.
E *E* cannot start until *A* is finished.

21 A connected graph with 12 edges divides a plane into 4 regions. The number of vertices in this graph will be:

A 6 **B** 10 **C** 12 **D** 13 **E** 14

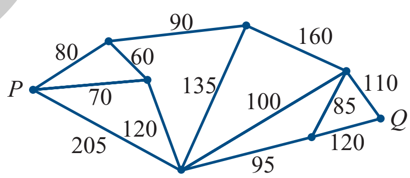
22 Which adjacency matrix (the matrix of vertex links) could represent this graph?



A $\begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 1 & 2 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ **B** $\begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

C $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 0 & 4 & 4 & 2 \\ 4 & 0 & 2 & 0 \\ 4 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{bmatrix}$ **E** $\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 0 & 2 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

23 A vehicle is travelling from town *P* to town *Q*. The journey requires the vehicle to travel along a network linking suitable fuel stops. The cost, in dollars, of travel between these is shown on the network where the nodes represent fuel stops.



What is the minimum cost, in dollars, for the trip?

A 400 **B** 405 **C** 410 **D** 420 **E** 440

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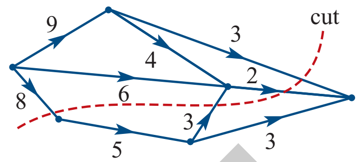
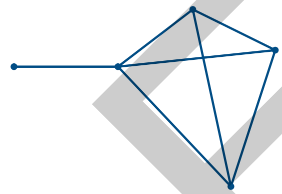
24 The capacity of the cut in the network flow diagram shown is:

A 0 **B** 2 **C** 10 **D** 13 **E** 16



25 The sum of the degrees of the vertices on the graph shown is:

A 12 **B** 13 **C** 14 **D** 15 **E** 16

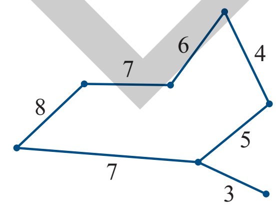


26 A connected planar graph divides the plane into a number of regions. If the graph has 9 vertices and these are linked by 20 edges, then the number of regions is:

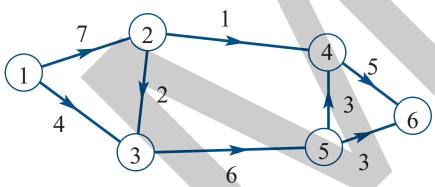
A 11 **B** 13 **C** 21 **D** 27 **E** 31

27 The sum of the weights of the minimum spanning tree of the weighted graph is:

A 2 **B** 30 **C** 32 **D** 33 **E** 35



The following graph relates to questions 28 and 29



28 The maximum flow in the network linking node 1 to node 6 is:

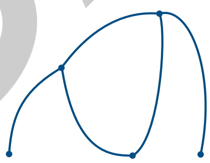
A 5 **B** 6 **C** 7 **D** 8 **E** 9

29 The number of ways that node 6 can be reached from node 1 is:

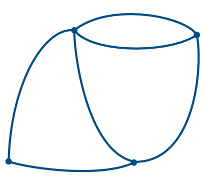
A 1 **B** 2 **C** 3 **D** 4 **E** 5

30 Which one of the following graphs has an Euler circuit?

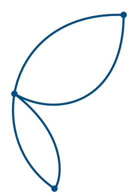
A



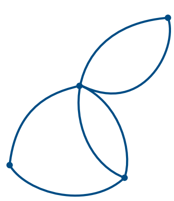
B



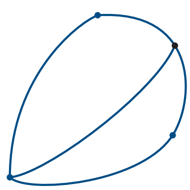
C



D



E



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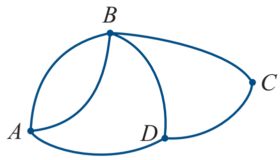
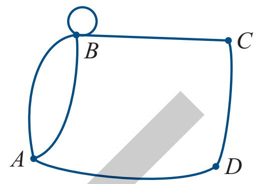
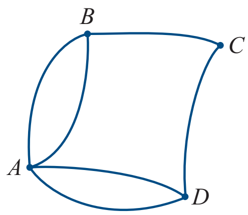
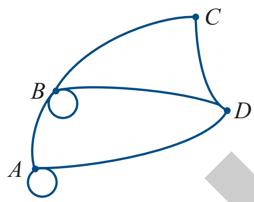
31 A graph corresponding to the matrix shown below is:

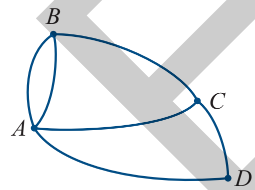
$$\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

A


B


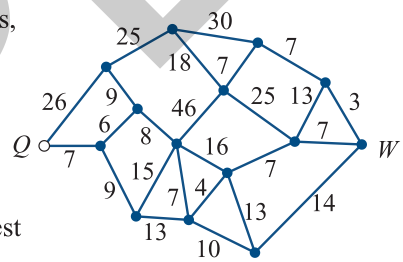
C


D


E


25.2 Extended-response questions

1 The network diagram shows the distances, in kilometres, along a series of roads that connect a quarry, Q, with worksites shown as nodes.



a One of these worksites is labelled as W.

i Indicate the shortest path from the quarry to W.

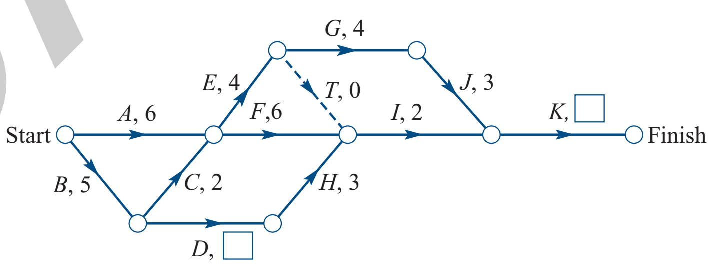
ii Determine the length, in kilometres, of the shortest path between the quarry Q and the worksite W.

b The engineer at the quarry wants to visit all worksites in the network. Beginning at Q, he wants to pass through each worksite only once before returning to the quarry.

i What mathematical term describes the route the engineer wants to take?

ii Show a complete route that the engineer could take to visit each worksite only once before returning to the quarry. [VCAA 2000]

2 All the activities and their durations (in hours) in a project at the quarry are shown in the network diagram below. The least time required for completing this entire project is 30 hours.



For each activity in this project, the table on the next page shows the completion time, the earliest starting time and the latest starting time.

(cont'd.)

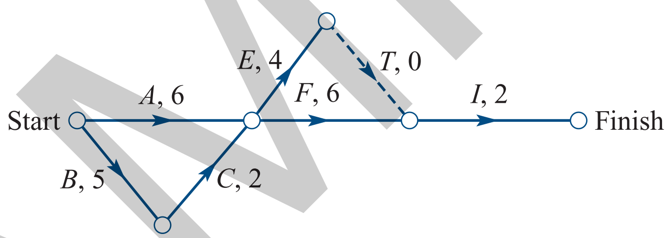
Revision

Activity	Completion time (hours)	Earliest starting time (hours)	Latest starting time (hours)
A	6	0	
B	5	0	0
C	2	5	5
D		5	9
E	4	7	7
F	6	7	
G	4	11	11
H	3	9	13
I	2	13	16
J	3	15	15
K		18	18

a Complete the missing times in the table.

b Write down the critical path for this project.

To speed up the project, several activities can be dropped. The diagram below shows the activities that must remain in this modified version of the project and their usual completion times.



c Determine the shortest time in which this modified project can be completed.

The completion time of some of the remaining activities in the modified project can be reduced at a cost.

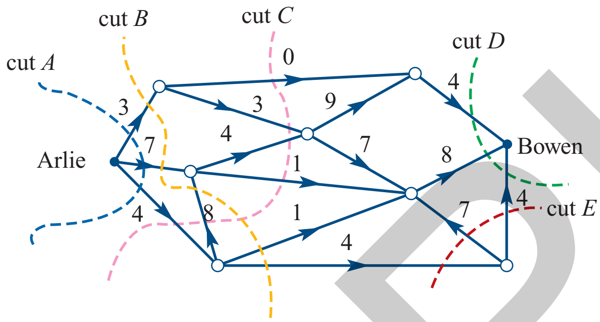
This table shows the reduced times (least possible time to complete an activity after maximum reduction of time).

The cost of this reduction, per hour, is also shown.

Activity	Usual completion time (hours)	Reduced time (hours)	Cost of reduction per hour ($)
A	6	3	50
B	5	4	100
C	2	2	–
E	4	2	20
F	6	4	50
I	2	2	–

d For this modified project, determine:
  i the activities that should be reduced in time to minimise the completion time of the project
  ii the maximum time, in hours, that can be saved by this reduction
  iii the minimum cost to achieve this time saving [VCAA pre 2000]

3 A train journey consists of a connected sequence of stages formed by edges on the directed network from Arlie to Bowen. The number of available seats for each stage is indicated beside the corresponding edge as shown in the diagram.

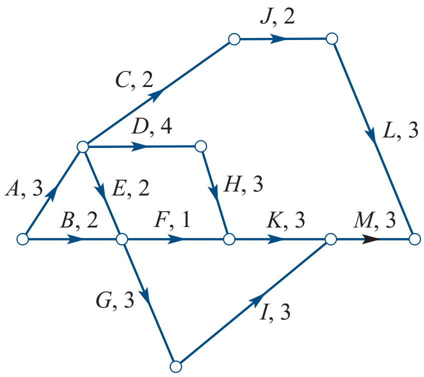


The five cuts, A, B, C, D and E, shown on the network, are attempts to find the maximum number of available seats that can be booked for a journey from Arlie to Bowen.

a Write down the capacity of cut A, cut B and cut C.
b Cut E is not a valid cut when trying to find the minimum cut between Arlie and Bowen. Why?
c Determine the maximum number of available seats for a train journey from Arlie to Bowen.

4 The Bowen Yard Buster team specialises in backyard improvement projects. The team has identified the activities required for a backyard improvement. The network diagram on the next page shows the activities identified and the actual times, in hours, needed to complete each activity, that is, the duration of each activity.
The table lists the activities, their immediate predecessor(s) and the earliest starting times (EST), in hours, of each of the activities. Activity X is not yet drawn on the network diagram.

(*cont'd.*)



	Immediate predecessor(s)	*EST*
A	–	0
B	–	0
C	*A*	3
D	*A*	3
E		3
F	*B*, *E*	5
G	*B*, *E*	5
H	*D*	7
I	*G*	
J	*C*, *X*	8
K	*F*, *H*	10
L	*J*	10
M	*I*, *K*	
X	*D*	7

a Use the information in the network diagram to complete the table.

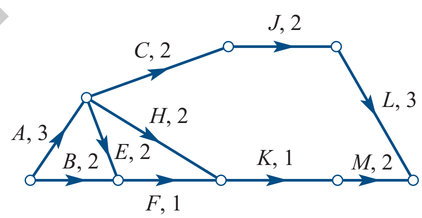
b Draw and label activity X on the network diagram above, including its direction and duration.

c The path A–D–H–K–M is the only critical path in this project.

  i Write down the duration of path A–D–H–K–M.

  ii Explain the importance of the critical path in completing the project.

5 To save money, Bowen Yard Busters decide to revise the project and leave out activities D, G, I and X. This results in a reduction in the time needed to complete activities H, K and M as shown.

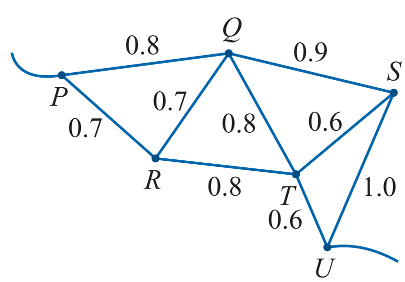


a For this revised project network, what is the earliest starting time for activity K?

b Write down the critical path for this revised project network.

c Without affecting the earliest completion time for this entire revised project, what is the latest starting time for activity M?

6 A rural town, built on hills, contains a set of roads represented by arcs in the network shown here. The numbers on the network refer to distances along the roads (in kilometres) and the letters refer to intersections of the roads. The arcs without endpoints refer to the two roads in and out of town.



a **i** What is the length of the shortest route through the town from P to U?

  ii A safety officer who enters the town at P needs to examine all intersections in the town before leaving from U to travel to the next town. To save time, she wants to pass through each intersection only once. State a path through the network of roads that would enable her to do this.

b A technician from the electricity company is checking the overhead cables along each street. The technician elects to follow an Euler path through the network streets (ignoring the roads in and out of town) starting at R and finishing at S.

i Complete the following Euler path: $R–Q–P–R–\square – \square – \square–T–U–S$

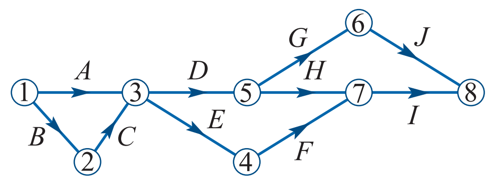
ii How would the technician benefit from choosing an Euler path?

c The local council plans to turn the main street of the town into a mall. The planning phase involves a number of activities whose normal completion times are supplied in Table 1. Also included in the table are the 'crash time' (possible time to which the activity time can be shortened) and the daily cost of this 'crashing'.

Table 1 Project completion times and costs

Activity	*Normal completion time (days)*	*Crash time (days)*	*Cost of crashing per day* ($)
A	10	8	400
B	5	5	–
C	3	2	400
D	5	4	600
E	4	4	–
F	6	5	500
G	6	4	200
H	7	5	300
I	5	5	–
J	4	3	400

The network for this project is as shown.



Using normal completion times as given in Table 1, determine the times missing from Table 2.

Table 2 Normal times for job starting

Activity	*Earliest start time (day)*	*Latest start time (day)*
A	0	0
B	0	2
C	5	7
D	10	10
E	10	12
F		16
G	15	
H	15	15
I	22	22
J	21	23

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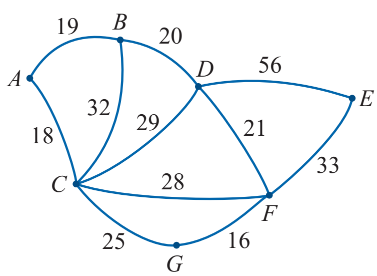
d **i** State the critical path in this network.
ii Determine the length of the critical path.

e **i** Complete Table 3, taking into account that some of the activities can be crashed, as shown in Table 1, to reduce the total completion time of the project.
ii Determine the shortest time in which the project can now be finished.
iii Apart from A, what three other activities must be shortened so the project is completed in minimum time?
iv What is the cost of achieving this time reduction for the whole project?

Table 3 Reduced times for job starting using crash data

Activity	*Earliest start time (days)*	*Latest start time (days)*
A	0	0
B	0	1
C	5	6
D	8	8
E	8	8
F	12	12
G	12	15
H	12	12
I	17	17
J		

7 A group of seven towns on an island have been surveyed for transport and communications needs. The towns (labelled A, B, C, D, E, F, G) form the network shown here. The road distances between the towns are marked in kilometres. (All towns may be treated as points being of no size compared to the network lengths.)



a Explain what is meant by the description of the graph as 'planar'.

b The roads between the towns define boundaries used by the local authority to establish rural planning subregions. (That is, the section bounded by roads AB, AC and BC would be one subregion. These subregions are non-overlapping.)
Treating the subregions as faces of the graph (with the exterior of the network as one subregion), the roads as edges and towns as vertices, show that Euler's formula linking the number of vertices, edges and faces in a planar graph, i.e.
number of vertices + number of faces = number of edges + 2, is satisfied.

An inspector of roads is stationed at B. Starting from B, she must travel the complete network of roads to examine them.

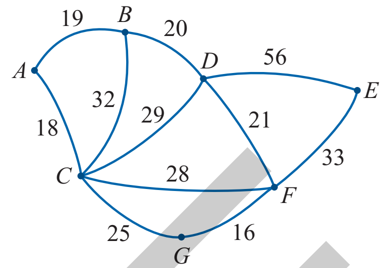
c If she wishes to travel the least distance where will she end up in the network?

d What will that distance be?

e Is the route unique? Briefly justify your answer.

f Determine the shortest distance that a fire truck stationed at E must travel to assist at an emergency at A.

g To establish a cable network for telecommunications on the island, it is proposed to put the cable underground beside the existing roads. What is the minimal length of cable required here if back-up links are not considered necessary; that is, there are no loops in the cable network?

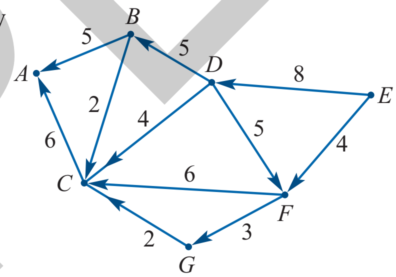


The Island Bank has outlets in each of the towns. The regional assistant manager stationed at C must visit each outlet every second Friday and then return to the office at C.

h Treating the towns as vertices and roads as edges in a graph, what is the distance of a journey that forms a Hamilton circuit in the graph?

i What is the length of the trip that gives the optimal (that is, shortest) route to the assistant manager?

j A reservoir at E pumps water through pipes along the network routes shown. The capacities of the flow are given in the digraph shown here in megalitres per day.
Occasionally, there are fire emergencies in the forest beside A and additional flow of water is used. What is the maximum flow that can reach A from E?



8 A development project involves completing a number of activities as shown in Table 1. With each activity, there is the optimistic assessment (i.e. the shortest time likely to occur) for how long it will take. Time is measured in days.

Table 1

Node–link	*Activity*	*Optimistic time (days)*	*Predecessor(s)*
1–2	A	4	–
1–3	B	2	–
2–4	C	1	A
3–4	D	6	B
3–5	E	5	B
3–6	F	7	B
4–7	G	5	C, D
5–7	H	1	E
6–8	I	2	F
7–9	J	10	G, H
8–9	K	6	I

The set of activities can be represented on a directed graph.

a Construct a graph for this project, labelling the activities on the arcs (edges) with their associated shortest durations.

b Determine the Earliest Start Time for each activity from your graph.

c How long is the estimated project time under this set of activity durations?

d Determine the latest start time for each activity from your graph.

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e State the critical path.

f If the final activity, K, had to be delayed, how many days could this delay take before the project schedule was disrupted?

The actual time to complete an activity is not known for sure. A pessimistic assessment of the activity duration (i.e. the longest time likely to occur) can also be used to assess the project as given in Table 2.

Table 2

Node–link	*Activity*	*Pessimistic time (days)*	*Predecessor(s)*
1–2	A	7	–
1–3	B	8	–
2–4	C	7	A
3–4	D	10	B
3–5	E	15	B
3–6	F	18	B
4–7	G	7	C, D
5–7	H	5	E
6–8	I	6	F
7–9	J	20	G, H
8–9	K	11	I

The earliest and latest start times for the activities under these conditions are shown in Table 3.

Table 3

Activity	*Earliest start time (days)*	*Latest start time (days)*
A	0	7
B	0	0
C	7	14
D	8	11
E	8	8
F	8	13
G	18	21
H	23	23
I	26	31
J	28	28
K	32	37

g What is the critical path in this case? How long is the estimated project time?

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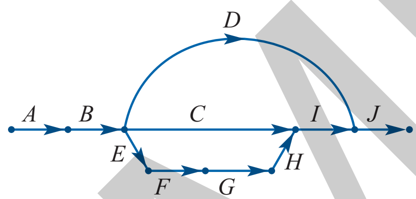
9 Jack's construction company builds a particular type of house using the project plan given in Table 1.

Activity	*Description*	*Predecessor*	*Duration (days)*
A	build foundation	–	5
B	build frame	*A*	8
C	build roof	*B*	12
D	do electrical wiring	*B*	5
E	put in windows	*B*	4
F	install insulation	*E*	1
G	install plumbing	*F*	1
H	put on siding	*G*	6
I	paint house	*C*, *H*	3
J	add fixtures/fittings	*D*, *I*	3

Table 1

A project network for this plan, with activities on arcs, is shown here.



a Using the information in Table 1 determine the times missing from Table 2.

b What is the earliest time in which the project can be completed using the information given in Table 1?

c What is/are the critical path/paths in this network?

d What is the float (slack time) for any activity not on a critical path?

Activity	*Earliest start time*	*Latest start time*
A	0	0
B	5	5
C	13	13
D	13	
E		13
F	17	
G	18	18
H	19	19
I	25	25
J	28	28

Table 2

(*cont'd.*)

Like many construction projects, this plan can have its time reduced by 'crashing' the project, that is, using more resources to finish parts of the job more quickly. Table 3 gives the cost of these reductions and the maximum extent to which each action can be taken.

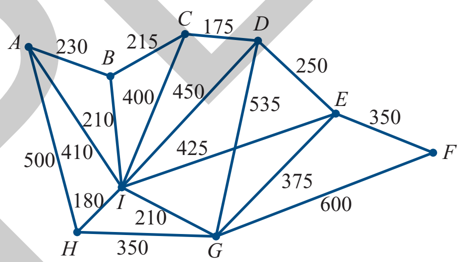
Activity	*Cost per day for reducing activity duration* ($)	*Maximum possible reduction* (*days*)
A	300	2
B	150	3
C	200	1
D	400	2
E	200	2
H	300	3
I	400	1
J	150	1

Table 3

e Using the information in Table 3, determine the shortest time in which the project can now be completed. Show all working.

f What is the minimum additional cost to achieve this?

g Jack's company is building the new Bigtown University. The company has constructed nine new faculty buildings in a layout as shown. The minimum distances in metres between adjacent buildings in the university are also shown.

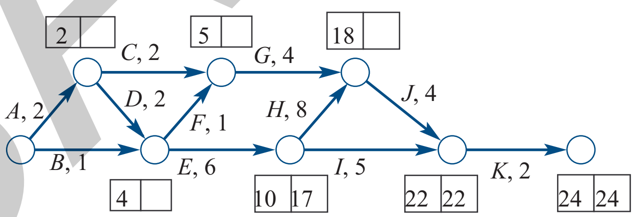


A computer network is to be built to serve the whole university.

i Draw a network that will ensure that all the buildings are connected to the network but that also minimises the amount of cable used. Label each node in the network.

ii What is the minimum length of cable required? [VCAA pre 2001]

10 The assembly of machined parts in a manufacturing process can be represented by the following network. The activities are represented by the letters on the arcs and the numbers represent the time taken (in hours) for the activities scheduled.



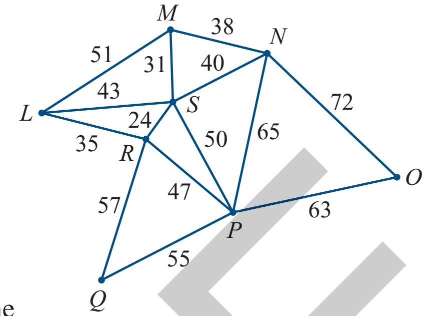
a The earliest start times (EST) for each activity except *G* are given in the table. Complete the table by finding the EST for *G*.

Activity	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
EST	0	0	2	2	4	4		10	10	18	22

b What is the shortest time required to assemble the product?

c What is the float (slack time) for Activity *I*?

11 A number of towns need to be linked by pipelines to a natural gas supply. In the network shown, the existing road links between towns L, M, N, O, P, Q and R and to the supply point, S, are shown as edges. The towns and the gas supply are shown as vertices. The distances along roads are given in kilometres.



a What is the shortest distance along roads from the gas supply point S to the town O?

b The gas company decides to run the gas lines along the existing roads. To ensure that all nodes on the network are linked, the company does not need to place pipes along all the roads in the network.

i What is the usual name given to the network within a graph (here, the road system) which links all nodes (towns and supply) and which gives the shortest total length?

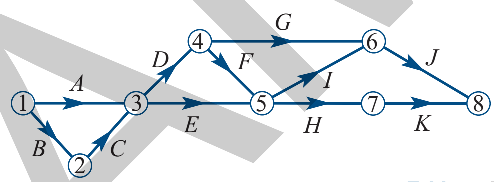
ii Sketch this network.

iii What is the minimum length of gas pipeline the company can use to supply all the towns by running the pipes along the existing roads?

c The gas company decides it wants to run the pipeline directly to any town which is linked by road to its supply at S. Towns not directly connected to S by road will be linked via other towns in the network.

What is the minimum length of pipeline that will enable all towns to be connected to the gas supply under these circumstances?

d In laying the pipeline, the various jobs involved have been grouped into a set of specific tasks A–K which are performed in the precedence described in the network below.



i List all the task(s) that must be completed before task E is started.

The durations of the tasks are given in Table 1.

Table 1 Task durations

Task	*Normal completion time (months)*
A	10
B	6
C	3
D	4
E	7
F	4
G	5
H	4
I	5
J	4
K	3

(cont'd.)

ii Use the information in Table 1 to complete Table 2.

e For this project:

i write down the critical path

ii determine the length of the critical path (that is, the earliest time the project can be completed)

f If the project managers are prepared to pay more for additional labour and machinery, the time taken to complete task A can be reduced to 8 months, task E can be reduced to 5 months and task I can be reduced to 4 months.

Under these circumstances:

i what would be the critical path(s)?

ii how long would it take to complete the project?

Table 2 Starting times for tasks

Task	*EST*	*LST*
A	0	0
B	0	
C	6	7
D	10	10
E		11
F	14	14
G	14	18
H	18	20
I	18	
J	23	23
K	22	24

g The pipeline construction team needs tractors at four different worksites. Four tractors are available but these are in four different locations. The cost (in dollars) of providing a tractor at each of the sites from each of the locations is given in Table 3.

Table 3 Cost of providing tractors (in dollars)

	Tractor based at			
Assigned to	*Location 1*	*Location 2*	*Location 3*	*Location 4*
Site 1	1130	830	2010	1140
Site 2	1020	1100	690	850
Site 3	2010	1320	1150	1410
Site 4	960	1210	2100	1530

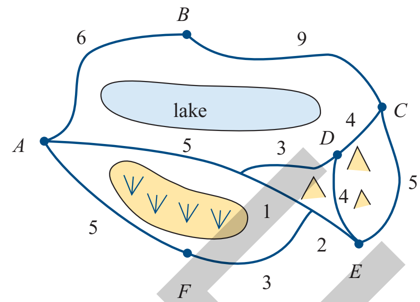
i Use the Hungarian algorithm, or otherwise, to complete the following table. From each location show where the tractors should be sent to minimise the total cost of providing tractors to the pipeline construction team.

Tractor at	*Assign to*
Location 1	Site 4
Location 2	
Location 3	
Location 4	

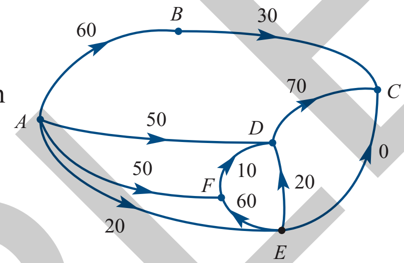
ii What is the minimum cost of providing tractors to the pipeline construction team?

[VCAA pre 2001]

12 The map shows six camp sites, A, B, C, D, E and F which are joined by paths. The numbers on the paths show lengths in kilometres of sections of the paths.



a The National Park Authority limits the number of people per day who can walk along each of the paths connecting the camp sites as shown in the graph. Note that, due to a landslide, path CE has been blocked and cannot be used.

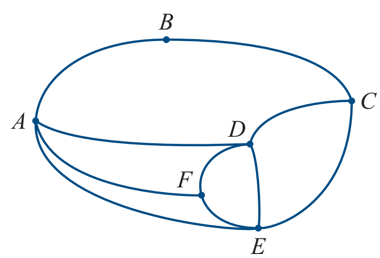


i What is the maximum number of people per day who can travel from A to C using the paths and directions as shown in the graph? Justify your answer.

ii The number of people allowed to use the paths each day in the reverse direction is given by the following table.

Path	*Number allowed*
B to A	60
C to B	30
C to D	50
C to E	0
D to E	20
F to E	0
D to F	5
E to A	0
D to A	50
F to A	20

*This diagram may be used to assist answering part **ii**.*



What is the maximum number of people per day who can travel from C to A?

b Camp sites A, B, C and D are to be supplied with food. Four local residents W, X, Y and Z offer to supply one campsite each. The cost in dollars of supplying one load of food from each resident to each campsite is tabulated.

	W	X	Y	Z
A	30	70	60	20
B	40	30	50	80
C	50	40	60	50
D	60	70	30	70

i Find the two possible matchings between campsites and residents so that the total cost is a minimum.

ii State this minimum cost.