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CHAPTER 7

CORE

Time series

- What is time series data?
- What are the features we look for in times series data?
- How do we identify and quantify trend?
- How do we measure seasonal variation?
- How do we use a time series for prediction?

7.1 Time series data

Time series data is a special kind of bivariate data, where the independent variable is time. An example of time series data is the birth rate (births per head of population), as given in the following table.

Year	*Birth rate*	*Year*	*Birth rate*	*Year*	*Birth rate*	*Year*	*Birth rate*
1931	1.039	1949	1.382	1967	1.342	1985	0.920
1932	0.967	1950	1.415	1968	1.360	1986	0.894
1933	0.959	1951	1.409	1969	1.360	1987	0.883
1934	0.939	1952	1.468	1970	1.349	1988	0.877
1935	0.941	1953	1.477	1971	1.400	1989	0.882
1936	0.967	1954	1.497	1972	1.296	1990	0.908
1937	0.981	1955	1.532	1973	1.179	1991	0.887
1938	0.976	1956	1.546	1974	1.123	1992	0.906
1939	0.986	1957	1.598	1975	1.049	1993	0.893
1940	1.042	1958	1.603	1976	0.980	1994	0.884
1941	1.094	1959	1.614	1977	0.951	1995	0.875
1942	1.096	1960	1.613	1978	0.930	1996	0.861
1943	1.148	1961	1.668	1979	0.908	1997	0.855
1944	1.179	1962	1.609	1980	0.901	1998	0.848
1945	1.267	1963	1.572	1981	0.924	1999	0.846
1946	1.379	1964	1.480	1982	0.921	2000	0.844
1947	1.416	1965	1.400	1983	0.920	2001	0.833
1948	1.376	1966	1.355	1984	0.883	2002	0.848

Birth rates in Australia by year, 1931–2002

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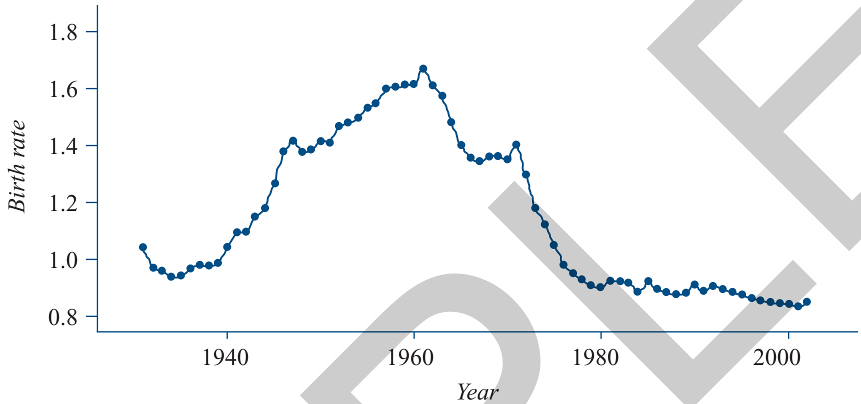
This data set is rather complex, and it is hard to see any patterns. However, as with other forms of bivariate data, we will start to get an idea about the relationship between the variables by drawing a scatterplot. When the scatterplot is a plot of time series data it is called a **time series plot**, with **time** always placed on the horizontal axis. A time series plot differs from a normal scatterplot in that the points will be joined by line segments in time order. A time series plot of the birth rate data is given below.



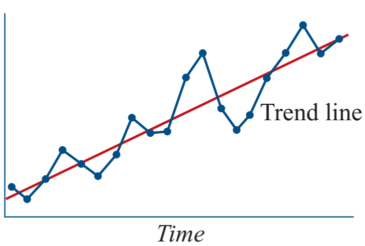
In general, a **time series plot** is a bivariate plot where the values of the dependent variable are plotted in time order. Points in a time series plot are joined by line segments.

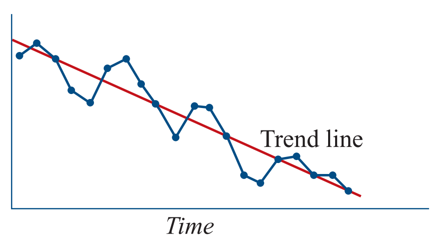
What are the features to look for in a time series plot? Time series data is often complex and shows seemingly wild fluctuations. The fluctuations are generally due to one or more of the following characteristics of the relationship:

- trend
- cyclical variation
- seasonal variation
- random variation

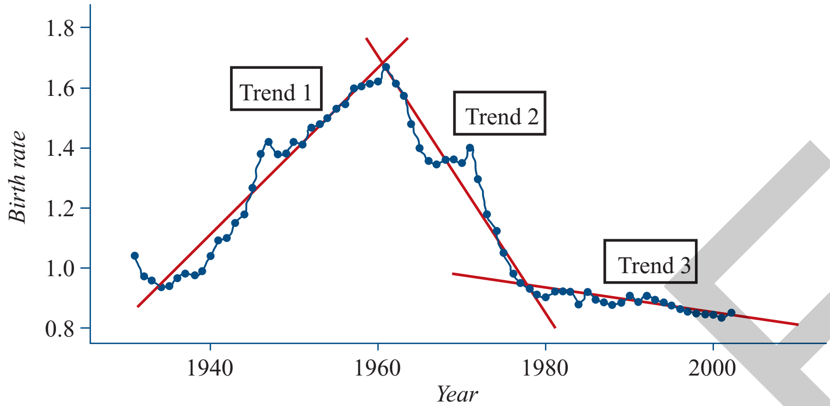
Trend

When we examine a time series plot we are often able to discern a general upward or downward movement over the long term, indicating a long-term change in the level of the variable. This overall pattern is called the **trend**. One way of identifying trends on a time series graph is to draw in a line that ignores the fluctuations but which reflects the overall increasing or decreasing nature of the plot. These are called **trend lines**. Trend lines have been drawn in on the time series plots below to indicate an *increasing* trend (line slopes upwards) and *decreasing* trend (line slopes downwards).





Sometimes, as in the birth rate time series plot, different trends are apparent for different parts of the plot. We can see this by drawing in trend lines on the plot.



Trend 1: From about 1940 to 1961 the birth rate grew quite dramatically. Those in the armed services came home from the war, and the economy grew quickly. This rapid increase in the birth rate is known as the Baby Boom.

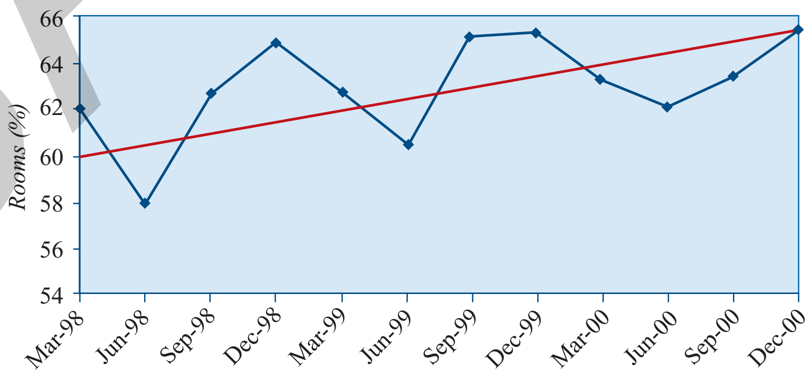
Trend 2: From about 1962 until 1980 the birth rate declined very rapidly. Birth control methods became more effective, and women started to think more about careers. This period has sometimes been referred to as the Baby Bust.

Trend 3: During the 1980s and up until this time, the birth rate continues to decline slowly for a complex range of social and economic reasons.

Seasonal variation

Seasonal variations are repetitive fluctuating movements which occur within a time period of one year or less. Seasonal movements tend to be more predictable than trends, and occur because of the variation in weather, such as sales of ice-cream for instance, or institutional factors, such as the increase in the number of unemployed at the end of the school year. The plot below shows the total percentage of rooms occupied in hotels, motels, etc., in Australia by quarter over the years 1998–2000.

The graph shows a general increasing trend, indicated by the upward sloping trend line. This indicates that the demand for accommodation is increasing over time.



However, over and above the general increasing trend, we can see that the demand for accommodation also appears to be seasonal. The demand for accommodation is at its lowest in the June quarter and peaks in the December quarter each year. Seasonality is identified by looking for peaks and troughs at the same time each year.

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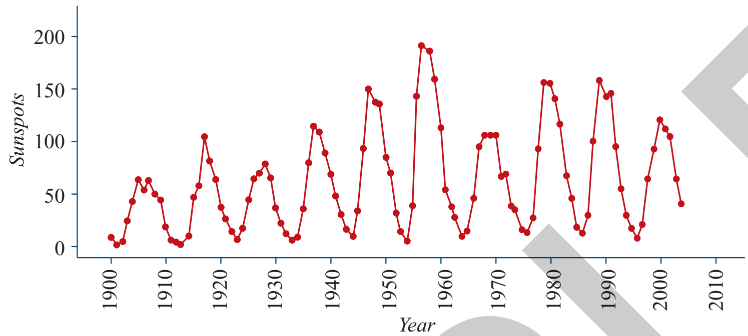
Cycles

The term **cycle** is used to describe longer term movements about the general trend line in the time series which are not seasonal. Some cycles repeat regularly, and some do not. The following plot shows the activity of sunspots, which are dark spots visible on the surface of the sun. This shows a fairly regular cycle of approximately 11 years.



Many business indicators, such as interest rates or unemployment figures, also vary in cycles, but their periods are usually less regular. In many contexts, including business, movements are only considered cyclical if they occur in time intervals of more than one year.

Random variation

Random or irregular variation is the part of the time series which cannot be classified into one of the above three categories. Generally, as with all variables, there can be many sources of random variation. Sometimes a specific cause such as a war or a strike can be isolated as the source of this variation. The aim of time series analysis is to develop techniques that can be used to measure things such as trend and seasonality in time series, given that there will always be random variation to cloud the picture.

Constructing time series plots

Most real-world time series data comes in the form of large data sets which are best plotted with the aid of a spreadsheet or statistical package. The availability of the data in electronic form via the web greatly helps the process. However, in this chapter, most of the time series data sets are relatively small and can be plotted using a graphics calculator.

CAS APPENDIX

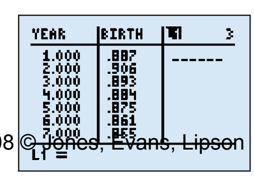
How to construct a time series plot using a graphics calculator

Construct a time series plot for the following data. The years have been recoded as 1, 2, . . . , 12, as is common practice.

1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
1	2	3	4	5	6	7	8	9	10	11	12
0.887	0.906	0.893	0.884	0.875	0.861	0.855	0.848	0.846	0.844	0.833	0.848

Steps

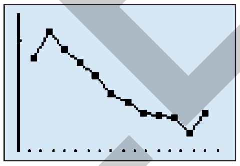
1 Enter the data into the calculator as shown (the calculator mode has been set to 3 decimal places here).



2 Select **STATPLOT**, turn the plot **On**. Move the cursor down to **Type:** and then across to the times series plot icon as shown. Press [ENTER] so that this plot is highlighted.

3 Move the cursor down to **Xlist:** and then use [2nd] [LIST] to access the **LIST** menu. Press [ENTER] to paste in the list **YEAR**. Repeat to paste the list **BIRTH** against **Ylist:** as shown.

4 Press [ZOOM] [9] to obtain a time series plot for birth rates.

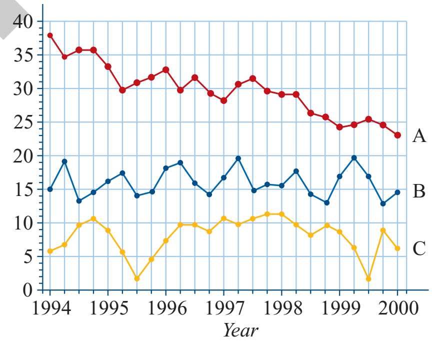


Exercise 7A

Throughout this chapter, use a graphics calculator whenever you wish

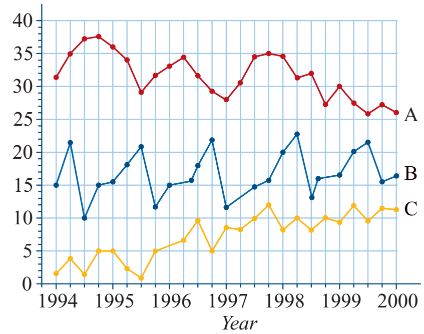
1 Complete a table like the one shown by indicating which of the listed characteristics are present in each of the time series plots shown below.

Characteristic	A	B	C
random variation			
increasing trend			
decreasing trend			
cyclical variation			
seasonal variation			



2 Complete the table by indicating which of the listed characteristics are present in each of the time series plots shown below.

Characteristic	A	B	C
random variation			
increasing trend			
decreasing trend			
cyclical variation			
seasonal variation			



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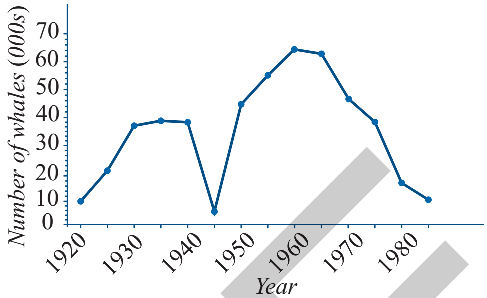
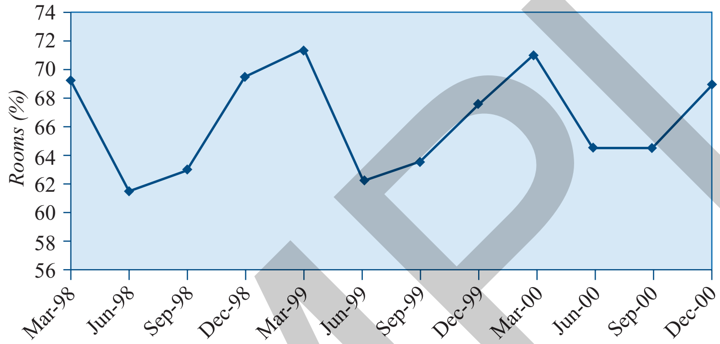
3 The time series plot opposite shows the number of whales caught during the period 1920–85. Describe the features of the plot.



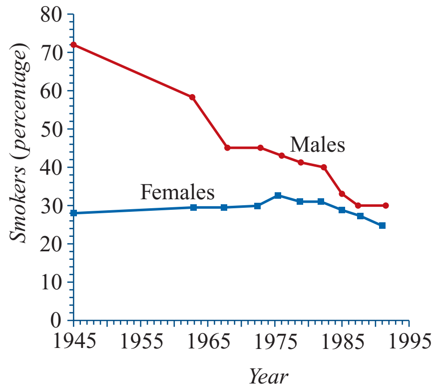
4 The hotel room occupancy rate (%) in Victoria over the period March 1998–December 2000 is depicted below. Describe the features of the plot.



5 The time series plot below shows the smoking rates (%) of Australian males and females who smoked over the period 1945–92.

a Describe any trends in the time series plot.

b Did the **difference** in smoking rates increase or decrease over the period 1945 to 1992?



6 Use the data below to construct a time series plot of the birth rate in Australia for 1960–70.

Year	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
Rate	1.613	1.668	1.609	1.572	1.480	1.400	1.355	1.342	1.360	1.360	1.349

7 Use the data below to construct a time series plot of the population (in millions) in Australia over the period 1993–2003.

Year	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Population	17.8	18.0	18.2	18.4	18.6	18.8	19.0	19.3	19.5	19.8	20.0

8 Use the data below to construct a time series plot for the number of school teachers (in thousands) in Australia over the period 1993–2001.

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Teachers	213	217	218	218	221	223	228	231	239	244	250

9 The table below gives the number of male and female teachers (in thousands) in Australia over the years 1993 to 2001.

Year	1993	1994	1995	1996	1997	1998	1999	2000	2001
Males '000	77.9	76.6	75.3	75.0	74.9	74.9	76.0	76.6	77.1
Females '000	139.9	141.2	145.5	148.5	152.5	156.0	163.4	167.4	172.5

a Construct a time series plot that shows both the male and female data on the same graph.
b Describe and comment on any trends you observe.

7.2 Smoothing a time series plot (moving means)

Excel

A time series plot can incorporate many of the sources of variation previously mentioned: trend, seasonality, cycles and random variation.

Because of the local variation, it is sometimes difficult to see the overall pattern. In order to clarify the situation, a technique known as **smoothing** may be helpful. Smoothing can be used to enhance and emphasise any trend in the data by eliminating the noisy jagged components, and allowing us to construct a line (possibly curved) that exhibits the trend of the times series. We shall discuss two of the most common smoothing methods, moving mean and moving median.

Moving mean smoothing

This method of smoothing involves the computation of **moving means**. The simplest method is to smooth over odd numbers of points, for example, 3, 5, 7.

The 3-moving mean

To use **3-moving mean smoothing**, replace each data value with the mean of that value and the values of its two neighbours, one on each side. That is, if y_1, y_2 and y_3 are sequential data values then:

$$\text{smoothed } y_2 = \frac{y_1 + y_2 + y_3}{3}$$

The first and last points do not have values on each side, so they are omitted.

For example, for the values shown in the table below:

Year	1	2	3
y	9	11	10
	(y_1)	(y_2)	(y_3)

$$\text{smoothed } y_2 = \frac{y_1 + y_2 + y_3}{3} = \frac{9 + 11 + 10}{3} = 10$$

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Similarly:

The 5-moving mean

To use **5-moving mean smoothing**, replace each data value with the mean of that value and the two values on each side. That is, if y_1, y_2, y_3, y_4, y_5, are sequential data values then:

$$\text{smoothed } y_3 = \frac{y_1 + y_2 + y_3 + y_4 + y_5}{5}$$

The first two and last two points do not have two values on each side, so they are omitted.

For example, for the values shown in the table below:

Year	1	2	3	4	5
y	9	11	10	12	13
	(y_1)	(y_2)	(y_3)	(y_4)	(y_5)

$$\text{smoothed } y_3 = \frac{y_1 + y_2 + y_3 + y_4 + y_5}{5} = \frac{9 + 11 + 10 + 12 + 13}{5} = 11$$

These definitions can readily be extended for smoothing over 7, 9, 11, etc, points. The larger the number of points we smooth over, the greater the smoothing effect.

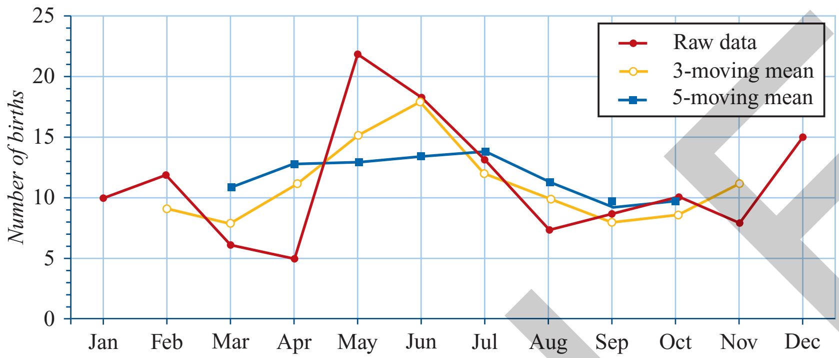
Example 1 — 3- and 5-moving mean smoothing

The following table gives the number of births per month over a calendar year in a small country hospital. Use the 3-moving mean and the 5-moving mean methods, correct to one decimal place, to complete the table.

Solution

Month	*Number of births*	*3-moving mean*	*5-moving mean*
January	10		
February	12	$\frac{10+12+6}{3} = 9.3$	
March	6	$\frac{12+6+5}{3} = 7.7$	$\frac{10+12+6+5+22}{5} = 11.0$
April	5	$\frac{6+5+22}{3} = 11.0$	$\frac{12+6+5+22+18}{5} = 12.6$
May	22	$\frac{5+22+18}{3} = 15.0$	$\frac{6+5+22+18+13}{5} = 12.8$
June	18	$\frac{22+18+13}{3} = 17.7$	$\frac{5+22+18+13+7}{5} = 13.0$
July	13	$\frac{18+13+7}{3} = 12.7$	$\frac{22+18+13+7+9}{5} = 13.8$
August	7	$\frac{13+7+9}{3} = 9.7$	$\frac{18+13+7+9+10}{5} = 11.4$
September	9	$\frac{7+9+10}{3} = 8.7$	$\frac{13+7+9+10+8}{5} = 9.4$
October	10	$\frac{9+10+8}{3} = 9.0$	$\frac{7+9+10+8+15}{5} = 9.8$
November	8	$\frac{10+8+15}{3} = 11.0$	
December	15		

The result of this smoothing can be seen in the graph below, which shows the raw data, the data smoothed with a 3-moving mean and the data smoothed with a 5-moving mean.



Note: In the process of smoothing, **data points are lost** at the beginning and end of the time series.

Choosing the number of points for the smoothing and centring

There are many ways of smoothing a time series. Moving means of group size other than three and five are common and often very useful. If data is considered to have some sort of seasonal component, then the moving means should be in groupings equivalent to the number of total periods. For example, daily data should be smoothed in groupings of five, or six, or seven, depending on the business. Quarterly data should be handled in groupings of four so that the seasonal component is being smoothed out and any trends are easier to perceive.

However, if we smooth over an even number of points, we run into a problem. The centre of the set of points is not at a time point belonging to the original series. Usually, we solve this problem by using a process called **centring**. Centring involves taking a 2-moving mean of the already smoothed values so that they line up with the original time values. This means that the process of smoothing takes place in two stages, as demonstrated in Example 2.

Example 2 **4-mean smoothing with centring (tabulated approach)**

Use 4-mean smoothing with centring to smooth the number of births per month over a calendar year in a small country hospital.

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Solution

Month	*Number of births*	*4-moving mean*	*4-moving mean with centring*
January	10		
February	12	$\frac{10+12+6+5}{4} = 8.25$	
March	6		$\frac{8.25+11.25}{2} = 9.75$
		$\frac{12+6+5+22}{4} = 11.25$	
April	5		$\frac{11.25+12.75}{2} = 12$
		$\frac{6+5+22+18}{4} = 12.75$	
May	22		$\frac{12.75+14.5}{2} = 13.625$
		$\frac{5+22+18+13}{4} = 14.5$	
June	18		$\frac{14.5+15}{2} = 14.75$
		$\frac{22+18+13+7}{4} = 15$	
July	13		$\frac{15+11.75}{2} = 13.375$
		$\frac{18+13+7+9}{4} = 11.75$	
August	7		$\frac{11.75+9.75}{2} = 10.75$
		$\frac{13+7+9+10}{4} = 9.75$	
September	9		$\frac{9.75+8.5}{2} = 9.125$
		$\frac{7+9+10+8}{4} = 8.5$	
October	10		$\frac{8.5+10.5}{2} = 9.5$
		$\frac{9+10+8+15}{4} = 10.5$	
November	8		
December	15		

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Exercise 7B

1 Find for the time series data in the table below:

t	1	2	3	4	5	6	7	8	9
y	5	2	5	3	1	0	2	3	0

a the 3-mean smoothed y value for $t = 4$
b the 3-mean smoothed y value for $t = 6$
c the 3-mean smoothed y value for $t = 2$
d the 5-mean smoothed y value for $t = 3$
e the 5-mean smoothed y value for $t = 7$
f the 5-mean smoothed y value for $t = 4$
g the 2-mean smoothed y value centred at $t = 3$
h the 2-mean smoothed y value centred at $t = 8$
i the 4-mean smoothed y value centred at $t = 3$
j the 4-mean smoothed y value centred at $t = 6$

2 Complete the following table.

t	1	2	3	4	5	6	7	8	9
y	10	12	8	4	12	8	10	18	2
3-mean smoothed y	–								–
5-mean smoothed y	–	–						–	–

3 The maximum daily temperature of a city over a period of 10 consecutive days is given below.

Day	1	2	3	4	5	6	7	8	9	10
Temperature (°*C*)	24	27	28	40	22	23	22	21	25	26
3-moving mean										
5-moving mean										

a Construct a time series plot of the temperature data. Label and scale axes.
b Use the 3-mean and 5-mean smoothing method to complete the table.
c Plot the smoothed temperature data on the same grid and comment on the plots.

4 The value of an Australian dollar in US dollars (exchange rate) over a 10-day period is given in the table.

Day	1	2	3	4	5	6	7	8	9	10
Exchange rate	0.743	0.754	0.737	0.751	0.724	0.724	0.712	0.735	0.716	0.711
3-mean smoothed										
5-mean smoothed										

a Construct a time series plot of the data. Label and scale the axes.
b Use the 3-mean and 5-mean smoothing method to complete the table.
c Plot the smoothed age data on the same grid and comment on the plots.

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5 Complete the following table by using 2-mean smoothing with centring.

Month	*Number of births*	*2-moving mean*	*2-moving mean with centring*
January	10		
February	12		
March	6		
April	5		
May	22		
June	18		
July	13		
August	7		
September	9		
October	10		
November	8		
December	15		

6 The data in the table shows internet usage (in gigabytes of information downloaded) at a university from April to December. Complete the table by using 2-mean smoothing with centring.

Month	*Internet usage*	*2-moving mean*	*2-moving mean with centring*
April	21		
May	40		
June	52		
July	42		
August	58		
September	79		
October	81		
November	54		
December	50		

7.3 Smoothing a time series plot (moving medians)

Median smoothing

Another simple and convenient way of smoothing the times series is to use **moving medians**.

The advantage of the moving median technique is that it is:

- primarily a graphical technique (although it can be done numerically) that enables the smoothed time series to be constructed directly from the original time series plot
- not influenced by a single outlier, thus any unusual values will be eliminated very quickly

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The 3-moving median

To use **3-moving median smoothing**, replace each data value with the median of that value and the values of its two neighbours, one on each side. That is, if y_1, y_2 and y_3 are sequential data values then:

$$\text{smoothed } y_2 = \text{median } (y_1, y_2, y_3)$$

The first and last points do not have values on each side, so they are omitted.

For example, for the values shown in the table below:

Year	1	2	3
(y)	9	11	10
	(y_1)	(y_2)	(y_3)

smoothed y_2 is the median of: 9, 11, 10

9, 10, 11

$\therefore$ smoothed $y_2 = 10$

Similarly:

The 5-moving median

To use **5-moving median smoothing**, replace each data value with the median of that value and the two values on each side. That is, if y_1, y_2, y_3, y_4, y_5, are sequential data values then:

$$\text{smoothed } y_3 = \text{median } (y_1, y_2, y_3, y_4, y_5)$$

The first two and last two points do not have two values on each side, so they are omitted.

For example, for the values shown in the table below:

Year	1	2	3	4	5
y	9	11	10	12	13
	(y_1)	(y_2)	(y_3)	(y_4)	(y_5)

smoothed y_3 is the median of: 9, 11, 10, 12, 13

9, 10, (11), 12, 13

$\therefore$ smoothed $y_3 = 11$

This definition can readily be extended for smoothing over 7, 9, 11, etc. points.

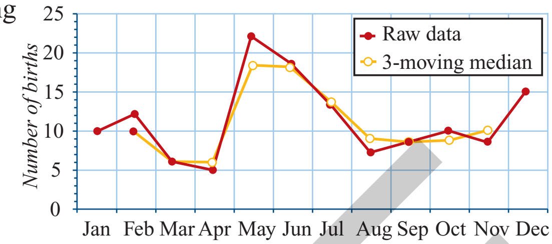
Example 3 — 3- and 5-median smoothing

The following table gives the number of births per month over a calendar year in a small country hospital. Use the 3-moving median and the 5-moving median methods, correct to one decimal place, to complete the table.

Solution

Month	*Number of births*	*3-moving median*	*5-moving median*
January	10		
February	12	Median = 10	
March	6	Median = 6	Median = 10
April	5	6	12
May	22	18	13
June	18	18	13
July	13	13	13
August	7	9	9
September	9	9	9
October	10	9	
November	8	10	
December	15		

The original time series plot and the 3-moving median plot are shown opposite.
While median smoothing can be done numerically, its real power is its potential to smooth time series graphs directly.

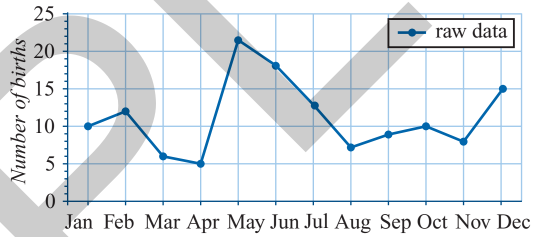


Example 4 3-median smoothing using a graphical approach

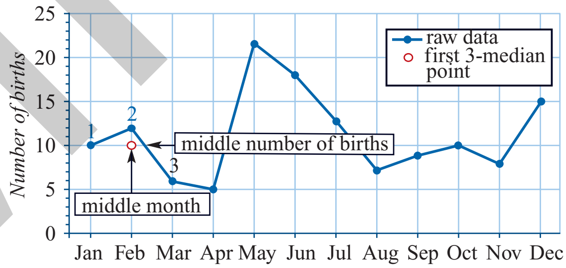
Construct a 3-median smoothed plot of the time series data shown.

Solution

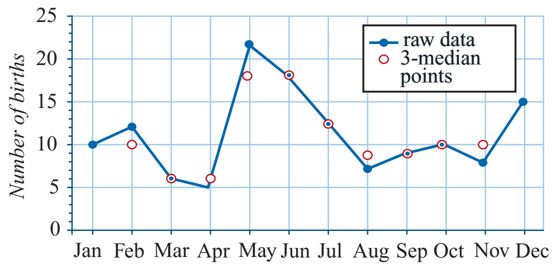
1 Construct a time series plot.



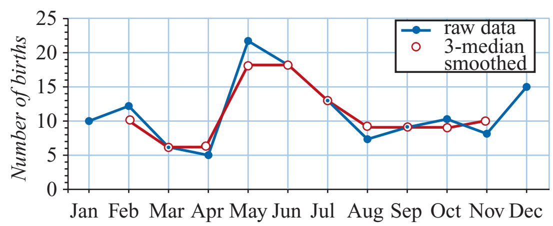
2 Locate on the time series plot the median of the *first* three points (Jan, Feb, Mar).



3 Continue this process by moving onto the next three points to be smoothed (Feb, Mar, Apr).
Mark in their median on the graph, and continue the process until you run out of groups of three.



4 Join up the median points with a line segment, see opposite.

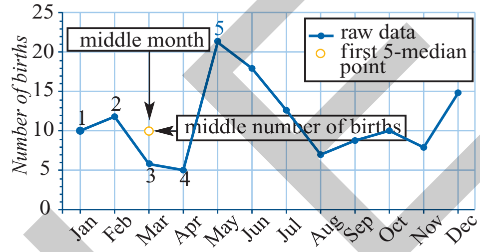


Example 5 5-median smoothing using a graphical approach

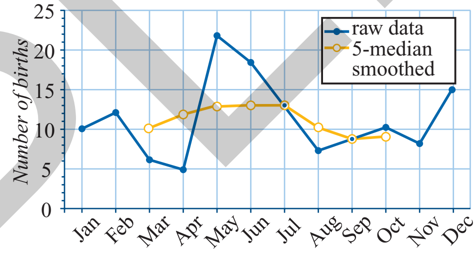
Construct a 5-median smoothed plot of the time series data shown.

Solution

1 Locate on the time series plot the median of the *first* five points (Jan, Feb, Mar, Apr, May) as shown opposite.



2 Then move onto the next five points to be smoothed (Feb, Mar, Apr, May, Jun) The process is then repeated until you run out of groups of five points. The 5-median points are then joined up with line segments to give the final smoothed plot as shown.



Note: The five-median smoothed plot is much smoother than the three-median smoothed plot.

When smoothing is carried out over an even number of data points, **centring** is again used to align the smoothed values with the original time periods.

Exercise 7C

1 Find for the time series data in the table:

t	1	2	3	4	5	6	7	8	9
y	5	2	5	3	1	0	2	3	0

a the 3-median smoothed y value for $t = 4$
b the 3-median smoothed y value for $t = 6$
c the 3-median smoothed y value for $t = 2$
d the 5-median smoothed y value for $t = 3$
e the 5-median smoothed y value for $t = 7$
f the 5-median smoothed y value for $t = 4$
g the 2-median smoothed y value centred at $t = 3$
h the 2-median smoothed y value centred at $t = 8$
i the 4-median smoothed y value centred at $t = 3$
j the 4-median smoothed y value centred at $t = 6$

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2 Complete the following table.

t	1	2	3	4	5	6	7	8	9
y	10	12	8	4	12	8	10	18	2
3-median smoothed y	–								–
5-median smoothed y	–	–						–	–

3 The maximum daily temperature of a city over a period of 10 consecutive days is given below.

Day	1	2	3	4	5	6	7	8	9	10
Temperature (°*C*)	24	27	28	40	22	23	22	21	25	26
3-moving median										
5-moving median										

a Use the 3-median and 5-median smoothing method to complete the table.

b Plot the 3-median smoothed temperature data and comment on the plot.

4 The value of an Australian dollar in US dollars (exchange rate) over a 10-day period is given in the following table.

Day	1	2	3	4	5	6	7	8	9	10
Exchange rate	0.743	0.754	0.737	0.751	0.724	0.724	0.712	0.735	0.716	0.711
3-moving median										
5-moving median										

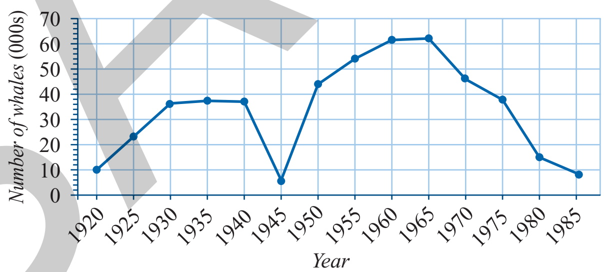
a Use the 3-median and 5-median smoothing method to complete the table.

b Plot the 3-median smoothed temperature data and comment on the plot.

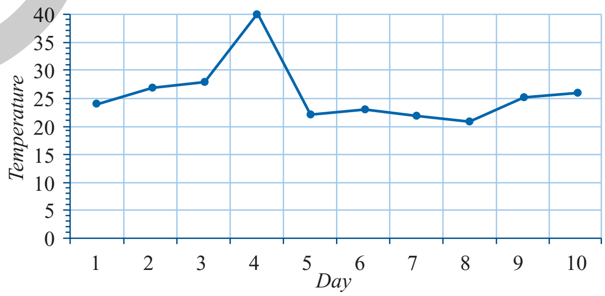
5 Use the graphical approach to smooth the time series plots below using:

i three-median smoothing **ii** five median smoothing

a



b



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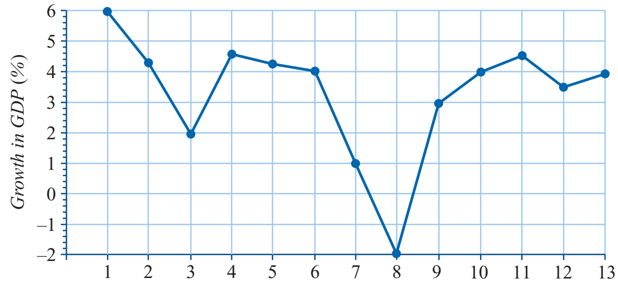
6 The time series plot below shows the percentage growth of GDP (gross domestic product) over a 13-year period.



a Smooth the times series graph:
  i using 3-median smoothing **ii** using 5-median smoothing

b What conclusions can be drawn about the variation in GDP growth from these time series plots?

7.4 Seasonal indices

When the data under consideration has a seasonal component, it is often necessary to remove this component by deseasonalising the data before further analysis. To do this we need to calculate **seasonal indices**. Seasonal indices tell us how a particular season (generally a day, month or quarter) compares to the average season.

Consider the (hypothetical) monthly seasonal indices for unemployment given in the table below:

Jan	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sept*	*Oct*	*Nov*	*Dec*	*Total*
1.1	1.2	1.1	1.0	0.95	0.95	0.9	0.9	0.85	0.85	1.1	1.1	12.0

Seasonal indices are calculated so that their **average** is **1**. This means that the **sum of the seasonal indices** equals the **number of seasons**. Thus, if the seasons are months, the seasonal indices add to 12. If the seasons are quarters, then the seasonal indices would add to 4, and so on.

Interpreting seasonal indices

- The seasonal index for unemployment for the month of February is 1.2.
  Seasonal indices are easier to interpret if we convert them to percentages. Remember, to convert a number to a percentage, just multiply by 100.
  A seasonal index of 1.2 for February, written in percentage terms, is 120%.
  A seasonal index of 1.2 (or 120%) tells us that February unemployment figures tend to be 20% **higher** than the monthly average. Remember, the average seasonal index is 1 or 100%.
- The seasonal index for August is 0.90 or 90%.
  A seasonal index of 0.9 (or 90%) tells us that the August unemployment figures tend to be only 90% of the monthly average. Alternatively, August unemployment figures are 10% **lower** than the monthly average.

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The seasonal index

A **season index** is defined by the formula:

$$\text{seasonal index} = \frac{\text{value for season}}{\text{seasonal average}}$$

In this formula, the season is a month, quarter or the like. The seasonal average is the monthly average, the quarterly average, and so on.

Seasonal indices have the property that the **sum** of the seasonal indices equals the **number of seasons**.

Example 6 Calculating seasonal indices (one year's data)

Mikki runs a shop and she wishes to determine quarterly seasonal indices based on her last year's sales, which are shown in the table opposite.

Summer	*Autumn*	*Winter*	*Spring*
920	1085	1241	446

Solution

1 The seasonal index is defined by:

$$\text{seasonal index} = \frac{\text{value for season}}{\text{seasonal average}}$$

The seasons are quarters. Write the formula in terms of quarters.

$$\text{seasonal index} = \frac{\text{value for quarter}}{\text{quarterly average}}$$

2 Find the quarterly average for the year.

$$\text{quarterly average} = \frac{920 + 1085 + 1241 + 446}{4} = 923$$

3 Work out the seasonal index (SI) for each time period.

$$SI_{Summer} = \frac{920}{923} = 0.997$$

$$SI_{Autumn} = \frac{1085}{923} = 1.176$$

$$SI_{Winter} = \frac{1241}{923} = 1.345$$

$$SI_{Spring} = \frac{446}{923} = 0.483$$

4 Check that the seasonal indices sum to 4 (the number of seasons). The slight difference here is due to rounding error.

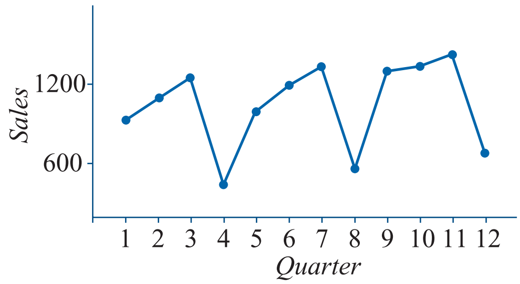
Check: $0.997 + 1.176 + 1.345 + 0.483 = 4.001$

5 Write out your answers as a table of the seasonal indices.

Seasonal indices

Summer	Autumn	Winter	Spring
0.997	1.176	1.345	0.483

Mikki actually has three previous year's data to work with and the time series plot confirms her belief that her sales are seasonal. The first quarter in the period has been labelled Quarter 1, and then each quarter labelled consecutively.



The next example illustrates how seasonal indices are calculated with several years' data. While the process looks more complicated, we just repeat what we did in Example 6 three times and average the results for each year at the end.

Example 7 Calculating seasonal indices (several years' data)

Suppose that Mikki has in fact three years of data, as shown. Use this data to calculate seasonal indices, correct to two decimal places.

Year	*Summer*	*Autumn*	*Winter*	*Spring*
1	920	1085	1241	446
2	1035	1180	1356	541
3	1299	1324	1450	659

Solution

The strategy is as follows:

- calculate the seasonal indices for Years 1, 2 and 3 separately as for Example 6 (as we already have the seasonal indices for Year 1 from Example 6 we will save ourselves some time by simply quoting the result)
- average the three sets of seasonal indices at the end to obtain a single set of seasonal indices

1 Write down the result for Year 1.

Year 1 seasonal indices:

Summer	Autumn	Winter	Spring
0.997	1.176	1.345	0.483

2 Now calculate the seasonal indices for Year 2.

Year 2 seasonal indices:

3 The seasonal index is defined by:

$$\text{seasonal index} = \frac{\text{value for season}}{\text{seasonal average}}$$

The seasons are quarters. Write the formula in terms of quarters.

$$\text{seasonal index} = \frac{\text{value for quarter}}{\text{quarterly average}}$$

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4 Find the quarterly average for the year.

$$\text{Quart. average} = \frac{1035 + 1180 + 1356 + 541}{4} = 1028$$

5 Work out the seasonal index (SI) for each time period.

$$SI_{Summer} = \frac{1035}{1028} = 1.007$$
$$SI_{Autumn} = \frac{1180}{1028} = 1.148$$
$$SI_{Winter} = \frac{1356}{1028} = 1.319$$
$$SI_{Spring} = \frac{541}{1028} = 0.526$$

6 Check that the seasonal indices sum to 4 (the number of seasons).

Check: $1.007 + 1.148 + 1.319 + 0.526 = 4.000$

7 Write out your answers as a table of the seasonal indices.

Summer	Autumn	Winter	Spring
1.007	1.148	1.319	0.526

8 Now calculate the seasonal indices for Year 3.

Year 3 seasonal indices:

9 Find the quarterly average for the year.

$$\text{Quart. average} = \frac{1299 + 1324 + 1450 + 659}{4} = 1183$$

10 Work out the seasonal index (SI) for each time period.

$$SI_{Summer} = \frac{1299}{1183} = 1.098$$
$$SI_{Autumn} = \frac{1324}{1183} = 1.119$$
$$SI_{Winter} = \frac{1450}{1183} = 1.226$$
$$SI_{Spring} = \frac{659}{1183} = 0.557$$

11 Check that the seasonal indices sum to 4 (the number of seasons).

Check: $1.098 + 1.119 + 1.226 + 0.557 = 4.000$

12 Write out your answers as a table of the seasonal indices.

Summer	Autumn	Winter	Spring
1.098	1.119	1.226	0.557

13 Find the 3-year averaged seasonal indices by averaging the seasonal indices for each season.

Final seasonal indices:

$$S_{Summer} = \frac{0.997 + 1.007 + 1.098}{3} = 1.03$$
$$S_{Autumn} = \frac{1.176 + 1.148 + 1.119}{3} = 1.15$$
$$S_{Winter} = \frac{1.345 + 1.319 + 1.226}{3} = 1.30$$
$$S_{Spring} = \frac{0.483 + 0.526 + 0.557}{3} = 0.52$$

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14 Check that the seasonal indices sum to 4 (the number of seasons).

Check: $1.03 + 1.15 + 1.30 + 0.52 = 4.00$

15 Write out your answers as a table of the seasonal indices.

Summer	Autumn	Winter	Spring
1.03	1.15	1.30	0.52

Having calculated these seasonal indices, what do they tell us?

A seasonal index of:

- 1.03 for summer tells us that sales in summer are typically 3% above average
- 1.15 for autumn tells us that sales in autumn are typically 15% above average
- 1.30 for winter tells us that sales in winter are typically 30% above average
- 0.52 for spring tells us that sales in spring are typically 48% below average

Using seasonal indices to deseasonalise the data

We can use seasonal indices to remove the seasonal component (deseasonalise) of a time series. To calculate deseasonalised figures, each entry is divided by its seasonal index as follows.

Deseasonalising data

Time series data is deseasonalised using the relationship:

$$\text{deseasonalised figure} = \frac{\text{actual figure}}{\text{seasonal index}}$$

The resulting data can then be examined for long-term trends.

Example 8 **Deseasonalising data**

The quarterly sales figures for Mikki's shop over a three-year period are given below.

Year	Summer	Autumn	Winter	Spring
1	920	1085	1241	446
2	1035	1180	1356	541
3	1299	1324	1450	659

Use the seasonal indices shown to deseasonalise these sales figures giving answers correct to the nearest whole number.

Summer	Autumn	Winter	Spring
1.03	1.15	1.30	0.52

Solution

1 $\text{deseasonalised figure} = \dfrac{\text{actual figure}}{\text{seasonal index}}$

2 To deseasonalise each sales figure in the table, divide by the appropriate seasonal index. For example, divide the figures in the summer column by 1.03. Round results to the nearest whole number.

Summer

$\dfrac{920}{1.03} = 893 \quad \dfrac{1035}{1.03} = 1005 \quad \dfrac{1299}{103} = 1261$

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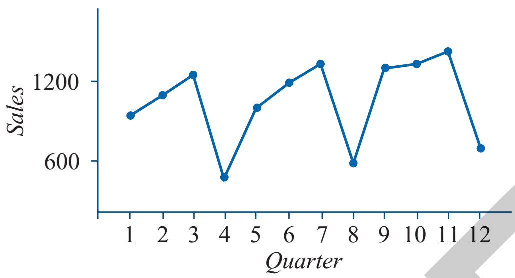
3 Repeat for the other seasons.
Write your answers out in a table.

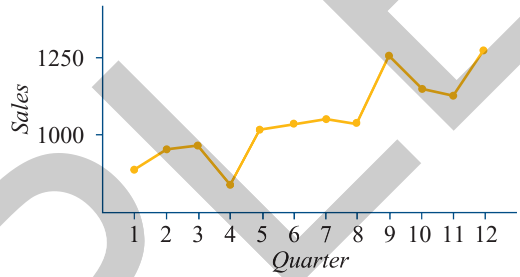
Deseasonalised sales figures

Year	Summer	Autumn	Winter	Spring
1	893	943	955	858
2	1005	1026	1043	1040
3	1261	1151	1115	1267

As can be seen from the graphs below, the time series plot of the deseasonalised sales figures shows that the regular seasonal pattern has indeed been removed from the data, and that any trend in sales is more apparent.



Raw data: original sales figures



Deseasonalised data: deseasonalised sales figures

Seasonal data should generally be treated in this way before further analyses are undertaken. (By the way, Mikki's shop sells ski gear.)

Exercise 7D

1 The table below shows the monthly sales figures and seasonal indices (for January to November) for a product produced by the U-beaut Company.

a Complete the table by calculating the missing seasonal index.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Seasonal index	1.2	1.3	1.1	1.0	1.0	0.9	0.8	0.7	0.9	1.0	1.1	

b Interpret the seasonal index for **i** February **ii** August.

2 Complete the following tables by calculating the missing seasonal indices.

a

Quarters	Q_1	Q_2	Q_3	Q_4
SI	1.04	1.18	1.29	

b

Terms	Term 1	Term 2	Term 3
SI	0.67		1.33

3 The table below shows the quarterly newspaper sales of a corner store for Year 1. Also shown are the seasonal indices for newspaper sales for the first, second and third quarters. Complete the table.

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Year 1	1256	1060	1868	1642
Year 1 deseasonalised				
Seasonal index	0.8	0.7	1.3	

4 The quarterly cream sales (in litres) made by the same corner store in Year 1, along with seasonal indices for cream sales for three of the four quarters, are shown in the table below. Complete the table.

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Year 1	68	102	115	84
Year 1 deseasonalised				
Seasonal index		1.10	1.15	0.90

5 Each of the following data sets records quarterly sales ($000s). Use the data to determine the seasonal indices for the four quarters. Give your results correct to two decimal places. Check that your seasonal indices add to 4.

a

Q1	Q2	Q3	Q4
48	41	60	65

b

Q1	Q2	Q3	Q4
60	56	75	78

6 Each of the following data sets records monthly sales ($000s). Use the data to determine the seasonal indices for the 12 months. Give your results correct to two decimal places. Check that your seasonal indices add to 12.

a

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
12	13	14	17	18	15	9	10	8	11	15	20

b

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
22	19	25	23	20	18	20	15	14	11	23	30

7 The number of waiters employed by a restaurant chain in each quarter of one year, along with some seasonal indices which have been calculated from the previous year's data, are given in the following table.

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Number of waiters	198	145	86	168
Seasonal index	1.30		0.58	1.10

a What is the seasonal index for the second quarter?

b The seasonal index for Quarter 1 is 1.30. Explain what this mean in terms of the average quarterly number of waiters.

c Deseasonalise the data.

8 The following table shows the number of students enrolled in a 3-month computer systems training course along with some seasonal indices which have been calculated from the previous year's enrolment figures. Complete the table by calculating the seasonal index for spring and the deseasonalised student numbers for each course.

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	Summer	*Autumn*	*Winter*	*Spring*
Number of students	56	125	126	96
Deseasonalised numbers				
Seasonal index	0.5	1.0	1.3	

9 The following table shows the monthly sales figures and seasonal indices (for January to December) for a product produced by the VMAX company.

a Complete the table by:

i calculating the missing seasonal index

ii evaluating the deseasonalised sales figures

b The seasonal index for July is 0.90. Explain what this means in terms of the average monthly sales.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Sales ($000s)	166	215	203	209	178	165	156	256	243	207	165	106
Sales (deseasonalised)												
Seasonal index	1.0		1.1	1.0	1.0	1.0	0.9	1.2	1.2	1.1	1.0	0.7

7.5 Fitting a trend line and forecasting

Fitting a trend line

If there appears to be a linear trend in the data, we can use regression techniques to fit a line to the data. Usually we use the least squares technique but, if there are outliers in the data, the 3-median line is more appropriate.

However, before we use either, we should always check our time series plot to see that the trend is linear. If it is not linear, data transformation techniques should be used to linearise the data first. The next example demonstrates using the least squares regression to fit a trend line to data which has no seasonal component.

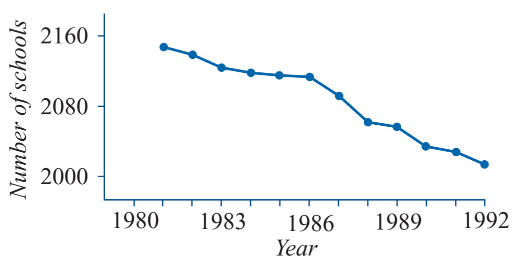
Example 9 **Fitting a trend line (no seasonality)**

Fit a trend line to the data in the following table, which shows the number of government schools in Victoria over the period 1981–92, and interpret the slope.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
Number	2149	2140	2124	2118	2118	2114	2091	2064	2059	2038	2029	2013

Solution

1 Construct a time series plot of the data to ensure linearity. If using a calculator, the first period of the time series is designated as '1', rather than as 1981.



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2 Use a calculator (with *Year* as the independent variable and *Number of schools* as the dependent variable) to find the equation of least squares regression line.

Number of schools = 2169 − 12.5 × year

Over the period 1981–92 the number of schools in Victoria was decreasing at an average rate of 12.5 per year.

Forecasting

Using a trend line fitted to a time series plot to make predictions about future values is known as **forecasting**.

Example 10 Forecasting (no seasonality)

How many government schools do we predict for Victoria in 2010 if the current decreasing trend continues?

Solution

Substitute the appropriate value for *year* in the equation determined using least squares regression. Since 1981 was designated as year '1', then 2010 is year '30'.

Number of schools = 2169 − 12.5 × year
= 2169 − 12.5 × 30
= 1794

Note: As with any relationship, extrapolation should be done with caution!

Taking seasonality into account

When data exhibits seasonality it is a good idea to deseasonalise the data first before fitting the trend line, as shown in the following example.

Example 11 Fitting a trend line (seasonality)

The deseasonalised quarterly sales data from Mikki's shop are shown below.

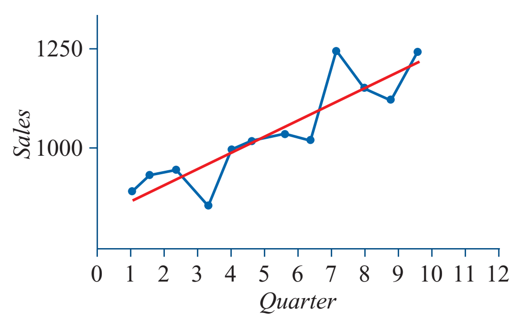
Quarter	1	2	3	4	5	6	7	8	9	10	11	12
Sales	893	943	955	858	1005	1026	1043	1040	1261	1151	1115	1267

Fit a trend line and interpret the slope.

Solution

1 Plot the time series.

2 Using the calculator (with *Quarter* as the IV and *Sales* as the DV) to find the equation of the least squares regression line. Plot it on the time series.



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3 Write down the equation of the least squares regression line.

sales = 838.0 + 32.1 × quarter

4 Interpret the slope in terms of the variables involved.

Over the 3-year period, sales at Mikki's shop increased at an average rate of 32 sales per quarter.

Making predictions with deseasonalised data

When using deseasonalised data to fit a trend line, you must remember that the result of any prediction is a deseasonalised value. To be meaningful, this result must then be 'seasonalised' by multiplying by the appropriate seasonal index.

Example 12 Forecasting (seasonality)

What sales do we predict for Mikki's shop in the winter of Year 4? (Because many items have to be ordered well in advance, retailers need to make such decisions.)

Solution

1 Substitute the appropriate value for time period in the equation determined using least squares regression. Since summer Year 1 was designated as quarter '1', then winter Year 4 is quarter '15'.

Sales = 838.0 + 32.1 × quarter
= 838.0 + 32.1 × 15
= 1319.5

Deseasonalised sales prediction for Winter Year 4 = 1319.5

2 This value is the **deseasonalised** sales figure for the quarter in question. This figure must be converted to the seasonalised (or predicted) sales figure. To do this, we multiply by the appropriate seasonal index for winter, which is 1.30

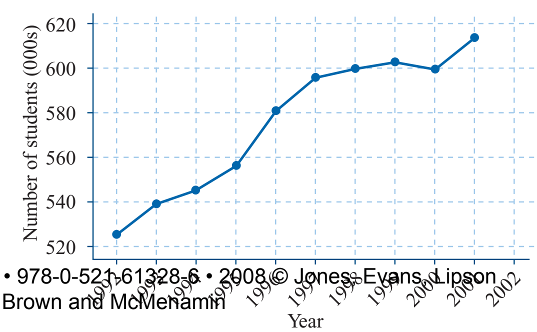
Seasonalised sales prediction for Winter Year 4
= 1319.5 × 1.30
= 1715

Exercise 7E

1 The data shows the number of students enrolled (in thousands) at university in Australia over the period 1992–2001.

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Number of students	525	539	545	556	581	596	600	603	600	614

The time series plot of this data is as shown opposite.



a Comment on the plot.
b Fit a least squares regression trend line to the data, using 1992 as Year 1, and interpret the slope.
c Use this equation to predict the number of students enrolled at university in Australia in 2008. Give your answer correct to the nearest 1000 students.

2 a The table below shows the *deseasonalised* washing-machine sales of a company over three years. Use least squares regression to fit a trend line to the data.

No. of purchases (deseasonalised)	1	2	3	4
Year 1	53	51	54	55
Year 2	64	64	61	63
Year 3	67	69	68	66

b Use this trend equation for washing-machine sales, together with the seasonal indices below, to forecast the sales of washing machines in the fourth quarter of Year 4.

Seasonal index	0.90	0.81	1.11	1.18

3 The table below shows the average number of questions asked by members of parliament during question time for the period 1976–1992.
a Construct a time series plot.
b Comment on the time series plot in terms of trend.
c Fit a trend line to the time series plot, find its equation (with 1976 as Year 1) and interpret the slope.
d Draw in the trend line on your time series plot.
e Use the trend line to forecast the average number of questions that will be asked in 2010.
f Does forecasting involve interpolating or extrapolating?

Year	*Average number (of questions)*	*Year*	*Average number (of questions)*
1976	19.8	1985	12.0
1977	16.5	1986	11.8
1978	16.1	1987	12.5
1979	16.4	1988	10.5
1980	15.2	1989	11.7
1981	16.3	1990	13.7
1982	15.4	1991	13.5
1983	12.7	1992	11.5
1984	12.1		

(*Source: The Age*, 1992)

4 The table below shows the percentage of total retail sales that were made in departmental stores over an 11-year period:

Sales (percentage)	12.3	12.0	11.7	11.5	11.0	10.5	10.6	10.7	10.4	10.0	9.4
Year	1	2	3	4	5	6	7	8	9	10	11

a Construct a time series plot.
b Comment on the time series plot in terms of trend.
c Fit a trend line to the time series plot, find its equation and interpret the slope.

d Draw in the trend line on your time series plot.

e Use the trend line to forecast the percentage of retails sales which will be made by departmental stores in Year 15.

5 The average ages of mothers having their first child in Australia over the years 1989–2002 are shown below.

Year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Age	27.3	27.6	27.8	28.0	28.3	28.5	28.6	28.8	29.0	29.1	29.3	29.5	29.8	30.1

a Fit a least squares regression trend line to the data, using 1989 as Year 1, and interpret the slope.

b Use this trend relationship to forecast the average ages of mothers having their first child in Australia in 2010.

6 The sale of boogie boards for a certain company over a two year period is given in the following table.

	Quarter 1	*Quarter 2*	*Quarter 3*	*Quarter 4*
Year 1	138	60	73	230
Year 2	283	115	163	417

The quarterly seasonal indices are given opposite.

Seasonal index	1.1297	0.4747	0.6248	1.7709

a Use the seasonal indices to calculate the deseasonalised sales figures for this period.

b Plot the actual sales figures and the deseasonalised sales figures for this period and comment on the plot.

c Fit a trend line to the deseasonalised sales data.

d Use the relationship calculated in **c**, together with the seasonal indices, to forecast the sales for the first quarter of Year 4.

7 The sales of motor vehicles for a large car dealer over a four year period and the quarterly seasonal indices are given in the tables opposite.

	Quarter 1	*Quarter 2*	*Quarter 3*	*Quarter 4*
Year 1	202	396	274	238
Year 2	212	350	246	238
Year 3	241	453	362	355
Year 4	253	471	389	325

Seasonal index	0.7314	1.3400	1.0091	0.9196

a Use the seasonal indices to calculate the deseasonalised sales figures for this period.

b Plot the actual sales figures and the deseasonalised sales figures for this period and comment on the plots.

c Fit a trend line to the deseasonalised sales data.

d Use the relationship calculated in **c**, together with the seasonal indices, to forecast the sales for the fourth quarter of Year 5.

8 The median duration of marriage to divorce (years) in Australia over the years 1992–2002 is given in the following table.

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Duration	10.5	10.7	10.9	11.0	11.0	11.1	11.2	11.3	11.6	11.8	12.0

a Fit a least squares regression trend line to the data, using 1992 as Year 1, and interpret the slope.

b Use this trend relationship to forecast the median duration of marriage to divorce in Australia in 2010.

MC TEST D&D TEST

Key ideas and chapter summary

Time series data — **Time series data** is a collection of data values along with the times (in order) at which they were recorded.

Time series plot — A **time series plot** is a bivariate plot where the values of the dependent variable are plotted in time order. Points in a time series plot are joined by line segments.

Features to look for in a time series plot
- trend
- seasonal variation
- cyclic variation
- random variation

Trend — The tendency for values in the time series to generally increase or decrease over a significant period of time.

Seasonal variation — The tendency for values in the time series to follow a seasonal pattern, increasing or decreasing predictably according to time periods such as time of day, day of the week, month, or quarter.

Cyclic variation — The tendency for values in the time series to go up or go down on a regular basis, but over a period *greater* than a year.

Random variation — The component of variation in a time series that is irregular or has no pattern. Random variation is present in most time series.

Smoothing — A technique used to eliminate some of the variation in a time series plot so that features such as seasonality or trend are more easily identified.

Moving mean smoothing
- In 3-moving mean smoothing, each original data value is replaced by the mean of itself and the value on either side.
- In 5-moving mean smoothing, each original data value is replaced by the mean of itself and the two values on either side.

Moving median smoothing
- In 3-moving median smoothing, each original data value is replaced by the median of itself and the value on either side of it.
- In 5-moving median smoothing, each original data value is replaced by the median of itself and the two values on either side.

Centring — If smoothing takes place over an even number of data values, then the smoothed values do not align with an original data value. A second stage of smoothing (either 2-moving mean or 2-moving median) is carried out to centre the smoothed values at an original data value.

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Seasonal indices

These are calculated when the data shows seasonal variation. Seasonal indices quantify the seasonal variation. For seasonal indices, the average is 1 (or 100%).

Calculating seasonal indices

A seasonal index is defined by the formula:

$$\text{seasonal index} = \frac{\text{value for season}}{\text{seasonal average}}$$

In this formula, the season is a month, quarter, etc. The seasonal average is the monthly average, the quarterly average, etc. If the seasons are months, the sum of the seasonal indices is 12; if quarters, the sum is 4, etc.

Deseasonalisation

The process of accounting for the effects of seasonality in a time series is called deseasonalisation. Time series data is deseasonalised using the relationship:

$$\text{deseasonalised figure} = \frac{\text{actual figure}}{\text{seasonal index}}$$

Trend

When there appears to be a linear trend in the time series, regression techniques can be used to fit a trend line. If a time series shows seasonal variation, it is usual to deseasonalise the data before fitting the trend line.

Forecasting

Once the equation for the trend line has been calculated it can be used to make predictions about what values the time series might take in the future. When a trend line is fitted to deseasonalised data, forecasted values need to be reseasonalised using the rule:

$$\text{seasonal forecast} = \text{deseasonalised forecast} \times \text{seasonal index}$$

Skills check

Having completed this chapter you should be able to:

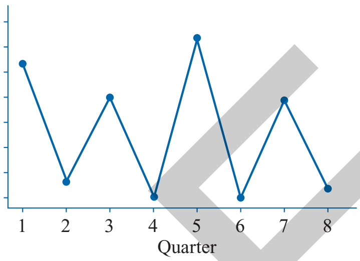
- recognise time series data
- construct a times series plot
- identify the presence of trend, seasonality, cycles and random variation in a time series plot
- smooth the time series plot to help clarify any trend, using moving means or medians and centring if necessary
- calculate and interpret seasonal indices
- calculate and interpret the linear trend, using least squares regression
- use the linear trend relationship, with or without seasonal indices, for forecasting

Review

Multiple-choice questions

1 The pattern in the time series in the graph shown is best described as:

A trend **B** cyclical but not seasonal
C seasonal **D** random
E average



2 For the time series given in the table, the 3-moving mean centred at time period 4 is closest to:

Time period	1	2	3	4	5	6
Data value	2.3	3.4	4.4	2.7	5.1	3.7

A 2.7 **B** 4.1 **C** 4.4 **D** 3.9 **E** 3.7

3 For the time series given in the table, the 5-moving median centred at time period 3 is (to the nearest whole number):

Time period	1	2	3	4	5	6	7	8
Data value	99	74	103	92	88	110	109	118

A 88 **B** 91 **C** 103 **D** 92 **E** 90

4 The seasonal indices for the number of customers at a restaurant are as follows.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1.0	p	1.1	0.9	1.0	1.0	1.2	1.1	1.1	1.1	1.0	0.7

The value of p is:

A 0.5 **B** 0.7 **C** 1.0 **D** 12 **E** 0.8

5 The seasonal indices for the number of bathing suits sold at a Surf Shop are given in the table.

Quarter	Summer	Autumn	Winter	Spring
Seasonal index	1.8	0.4	0.3	1.5

If the number of bathing suits sold one summer is 432, then the deseasonalised figure (to the nearest whole number) is:

A 432 **B** 240 **C** 778 **D** 540 **E** 346

6 The number of visitors recorded at a tourist centre each quarter one year is as shown.

Quarter	Summer	Autumn	Winter	Spring
Visitors	1048	677	593	998

Assuming that there is a seasonal component to the number of visitors to the centre, the seasonal index for autumn is closest to:

A 0.25 **B** 1.0 **C** 1.23 **D** 0.82 **E** 0.21

Questions 7 and 8 refer to the following information

The average ages at marriage for males over the period 1995–2002 are given in the following table.

Year	1995	1996	1997	1998	1999	2000	2001	2002
Age at marriage (males)	27.3	27.6	27.8	27.9	28.2	28.5	28.7	29.0

A least squares regression trend line fitted to the data (with 1995 as Year 1) was found to have the following rule:

$Age = 27.06 + 0.236 \times Year$

7 Using this trend line we predict that the average age of marriage of males in 2010 would be:

A 30.0 **B** 30.2 **C** 30.4 **D** 30.6 **E** 30.8

8 From the slope of the trend line it can be said that:

A on average the age of marriage for males is increasing by about 3 months per year
B on average the age of marriage for males is decreasing by about 3 months per year
C older males are more likely to marry than younger males
D no males married at an age younger than 27 years
E on average the age of marriage for males is increasing by 0.236 months per year

Questions 9 and 10 refer to the following information

Suppose that the seasonal indices for the price of petrol are:

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Index	1.2	1.0	0.9	0.8	0.7	1.2	1.2

Deseasonalised prices for a petrol outlet for Week 1 (in cents/litre) are given in the following table:

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Price	88.3	85.4	86.7	88.5	90.1	91.7	94.6

9 The equation of the least squares regression line which could enable us to predict the deseasonalised price is:

A $Price = 84.34 + 1.246 \times Day$ **B** $Price = -49.66 + 0.601 \times Day$
C $Price = 1.246 + 84.34 \times Day$ **D** $Price = 0.601 - 49.66 \times Day$
E $Price = 84.34 - 1.246 \times Day$

10 Based on this equation the forecast price of petrol (in cents) for Friday of Week 2 is:

A 100.5 **B** 83.8 **C** 120.6 **D** 91.8 **E** 110.2

Extended-response questions

1 The infant mortality rate (number of deaths under one year per 100 000 live births) in Victoria over the period 1990–2002 is given in the following table.

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Mortality rate	523	428	366	347	327	308	308	300	283	331	268	284	305
3-moving mean													
3-moving median													

a Use 3-moving mean and 3-moving median smoothing to complete the table (give your answers to the nearest whole number).

b Plot the original data, together with the mean and median smoothed data, and comment on the plots.

2 The table below shows the average mortgage interest rate for the period 1987–97.

Year	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
Interest rate	15.50	13.50	17.00	16.50	13.00	10.50	9.50	8.75	10.50	8.75	7.55
3-moving mean											

a Construct a time series plot for average mortgage interest rate during the period 1987–97.

b Use the 3-moving mean method to complete the table.

c Plot the smoothed interest rate data and comment on any trend revealed.

d Fit a trend line to the data and find its equation (with 1987 as Year 1). Interpret the slope.

e Use the trend line to forecast interest rates in 1998. In making this forecast, are you interpolating or extrapolating?

f When does the trend predict that the interest rates will fall to zero? Do you think that this will ever happen? Why? What assumption are we making in our prediction that will probably not hold true in the future?

3 **a** Complete the table for the sales of meat pies.

Month	*Sales*	*3-mean smoothed*	*3-median smoothed*
Feb	5700		
March	7400		
April	6400		

(*cont'd.*)

Review

b The sales of pies are known to be seasonal. The pie manufacturer has produced the following quarterly seasonal indices for the pie sales.

Q1	Q2	Q3	Q4
0.6	1.2	1.4	0.8

The trend equation for deseasonalised data is:

$Sales = 12000 + 100 \times Quarter\ number$

i Estimate the (deseasonalised) quarterly sales for the second quarter of Year 2, if the first quarter of Year 1 is Quarter number 1.

ii Use the appropriate seasonal index to obtain a forecast for the second quarter of Year 2.

4 **a** Complete the table for the sales of ice-cream.

Month	*Sales*	*3-mean smoothed sales*	*3 median smoothed sales*
Feb	9700		
March	9900		
April	7400		

b The sales of ice cream are known to be seasonal. The ice cream manufacturer has produced the following quarterly seasonal indices for the sales of ice-cream.

Q1	Q2	Q3	Q4
1.5	0.7	0.6	1.2

The trend equation for deseasonalised data is:

$Sales = 10\,000 + 80 \times Quarter\ number$

i Estimate the (deseasonalised) quarterly sales for the third quarter of Year 2, if the first quarter of Year 1 is Quarter number 1.

ii Use the appropriate seasonal index to obtain a forecast for the third quarter of Year 2.

5 The seasonal indexes for the four quarters for a particular product have been calculated from sales data over many years. This data gives quarterly sales for Year 1.

Season	*Summer*	*Autumn*	*Winter*	*Spring*
Year 1	1976	2940	3195	4900
Seasonal index	0.80	1.05	0.90	1.25

a Calculate the deseasonalised sales figure for summer.

b A least squares regression trend line has been fitted to the deseasonalised sales figures. The equation of the trend line is:

$Sales = 1910 + 510 \times Time\ period$

where summer, Year 1, is time period 1.

i Estimate the (deseasonalised) quarterly sales for the spring of Year 3.

ii Use the seasonal index to obtain a better forecast for the spring of Year 3.

c The seasonal index for spring is 1.25. Explain what this means in terms of the quarterly sales.