

Question 1

(a) $3.7278\dots \div 3.728$ (4 sig. fig.)

(b)
$$\frac{n^2 - 25}{n-5} = \frac{(n-5)(n+5)}{n-5} = n+5$$

$$\begin{aligned} (c) \quad 2^{2x+1} &= 32 \\ 2^{2x+1} &= 2^5 \\ \therefore 2x+1 &= 5 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

(d) $\frac{d}{dx} [\ln(5x+2)] = \frac{5}{5x+2}$

$$\begin{aligned} (e) \quad 2 - 3x &\leq 8 \\ -3x &\leq 6 \\ \therefore x &\geq -2 \end{aligned}$$

$$\begin{aligned} (f) \quad \frac{4}{\sqrt{5} + \sqrt{3}} &= \frac{4}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} \\ &= \frac{4(\sqrt{5} - \sqrt{3})}{5 - 3} \\ &= \frac{4(\sqrt{5} - \sqrt{3})}{2} \\ &= 2(\sqrt{5} - \sqrt{3}) \\ &= 2\sqrt{5} - 2\sqrt{3} \end{aligned}$$

$$(g) \quad \text{Number defective} = 0.02 \times 800 = 16$$

Question 2

(a) $x^2 - 6x + 2 = 0$

(i) $\alpha + \beta = \frac{-(-6)}{1} = 6$

(ii) $\alpha\beta = \frac{2}{1} = 2$

$$\begin{aligned} (\text{iii}) \quad \frac{1}{\alpha} + \frac{1}{\beta} &= \frac{\beta + \alpha}{\alpha\beta} \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

$$\begin{aligned} (\text{b}) \quad 2 \sin x &= -\sqrt{3} \quad 0 \leq x \leq 2\pi \\ \sin x &= -\frac{\sqrt{3}}{2} \\ x &= \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3} \\ &= \frac{4\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

$$\begin{aligned} (\text{c}) \quad y &= (2x+1)^4 \\ \frac{dy}{dx} &= 4(2x+1)^3(2) \\ &= 8(2x+1)^3 \end{aligned}$$

$$\begin{aligned} \text{At } x = -1: \quad y &= (-2+1)^4 \\ &= 1 \quad \Rightarrow (-1, 1) \\ \frac{dy}{dx} &= 8(-2+1)^3 \\ &= -8 \quad \Rightarrow m = -8 \end{aligned}$$

$$\begin{aligned} \therefore \text{ tangent is } \quad y - 1 &= -8(x+1) \\ y - 1 &= -8x - 8 \\ y &= -8x - 7 \end{aligned}$$

$$\begin{aligned} (\text{d}) \quad y &= x^2 e^x \\ \frac{dy}{dx} &= x^2 (e^x) + e^x (2x) \text{ by the product rule} \\ &= x e^x (x+2) \end{aligned}$$

$$\begin{aligned} (\text{e}) \quad \int \frac{1}{3x^2} dx &= \int \frac{1}{3} x^{-2} dx \\ &= \frac{1}{3} \left(\frac{x^{-1}}{-1} \right) + C \\ &= -\frac{1}{3x} + C \end{aligned}$$

Question 3

- (a) \$3, \\$3.5, \\$4, \dots\$ is an arithmetic sequence with $a = 3$ and $d = 0.5$

$$(i) T_n = a + (n-1)d \text{ where } n = 25$$

$$T_{25} = 3 + (25-1) \times 0.5 \\ = 15$$

\therefore the 25th floor will cost \$15 million

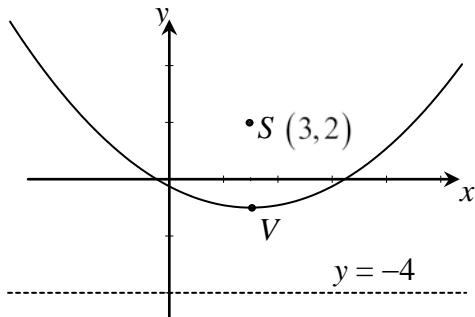
$$(ii) S_n = \frac{n}{2} [2a + (n-1)d] \text{ where } n = 119$$

$$S_{110} = \frac{110}{2} [2 \times 3 + (110-1) \times 0.5] \\ = 3327.5$$

\therefore the 110 floors will cost \$3327.5 million

- (b) $S = (3, 2)$ and the directrix is $y = -4$

\therefore the parabola is concave up



$$2a = 2 - (-4)$$

$$= 6$$

$$a = 3$$

$$\therefore V = (3, 2 - 3)$$

$$= (3, -1)$$

- (c)(i) y-intercept of $3x + 4y - 12 = 0$ occurs when

$$x = 0:$$

$$\therefore 4y - 12 = 0$$

$$y = 3$$

$$\therefore B = (0, 3)$$

$$(ii) m_{l_1} = -\frac{3}{4} \quad \text{using } m = -\frac{A}{B}$$

$$m_{l_2} = \frac{-4}{-3} = \frac{4}{3}$$

$$\begin{aligned} \text{Now } m_{l_1} \times m_{l_2} &= -\frac{3}{4} \times \frac{4}{3} \\ &= -1 \end{aligned}$$

$$\therefore l_1 \perp l_2$$

$$(iii) d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

where $A = 3, B = 4, C = -12$ and $(x_1, y_1) = (0, 0)$

$$\begin{aligned} \therefore d &= \frac{|0 + 0 - 12|}{\sqrt{3^2 + 4^2}} \\ &= \frac{12}{5} \end{aligned}$$

$$(iv) OE = \frac{12}{5} \quad (OE \perp AB)$$

$OB = 3$ from (i)

$\therefore OB^2 = OE^2 + BE^2$ by Pythagoras

$$9 = \frac{144}{25} + BE^2$$

$$\begin{aligned} BE^2 &= 9 - \frac{144}{25} \\ &= \frac{81}{25} \end{aligned}$$

$$\therefore BE = \frac{9}{5} \quad (BE > 0)$$

$$\begin{aligned} (v) \text{ Area } \Delta BOE &= \frac{1}{2} \times BE \times OE \\ &= \frac{1}{2} \times \frac{9}{5} \times \frac{12}{5} \\ &= 2.16 \end{aligned}$$

\therefore the area is 2.16 unit²

Question 4

$$(a) \frac{d}{dx} \left(\frac{x}{\sin x} \right) = \frac{(\sin x)(1) - (x)(\cos x)}{\sin^2 x}$$

by the quotient rule

$$= \frac{\sin x - x \cos x}{\sin^2 x}$$

$$\begin{aligned}
 \text{(b)} \quad & \int_e^{e^3} \frac{5}{x} dx = 5[\ln x]_e^{e^3} \\
 &= 5\ln e^3 - 5\ln e \\
 &= 15\ln e - 5\ln e \\
 &= 10\ln e \quad \text{but } \ln e = 1 \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & \frac{dy}{dx} = 6x - 2 \quad \text{through } (-1, 4) \\
 & y = \frac{6x^2}{2} - 2x + C \\
 & y = 3x^2 - 2x + C \\
 & \text{sub } (-1, 4): 4 = 3(-1)^2 - 2(-1) + C \\
 & \quad 4 = 5 + C \\
 & \quad C = -1 \\
 & \therefore y = 3x^2 - 2x - 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)(i)} \quad & y = \sqrt{9 - x^2} \\
 & y = (9 - x^2)^{\frac{1}{2}} \\
 & \frac{dy}{dx} = \frac{1}{2}(9 - x^2)^{-\frac{1}{2}}(-2x) \quad \text{by the chain rule} \\
 & \quad = -\frac{x}{\sqrt{9 - x^2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \int \frac{6x}{\sqrt{9 - x^2}} dx = -6 \int -\frac{x}{\sqrt{9 - x^2}} dx \\
 & = -6\sqrt{9 - x^2} + C
 \end{aligned}$$

- (e) The region below $y = 4 - x^2$ is $y < 4 - x^2$
The region below $y = |x| - 2$ is $y > |x| - 2$
The shaded region is where
 $y < 4 - x^2$ and $y > |x| - 2$

Question 5

- (a) 27, 54, 108, ... is a geometric series with $a = 12$ and $r = 2$

$$\begin{aligned}
 \text{(i)} \quad & n = 12 \text{ and } T_n = ar^{n-1} \\
 & \therefore T_{12} = 27(2)^{11} = 55296 \\
 & \therefore \text{there are 55 296 members on Day 12}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \text{Now } T_n > 10000000 \\
 & \therefore ar^{n-1} > 10000000 \\
 & 27(2)^{n-1} > 10000000 \\
 & (2)^{n-1} > \frac{10000000}{27} \\
 & (n-1)\ln 2 > \ln\left(\frac{10000000}{27}\right) \\
 & n-1 > \frac{\ln\left(\frac{10000000}{27}\right)}{\ln 2} \quad \text{as } \ln 2 > 1 \\
 & n > \frac{\ln\left(\frac{10000000}{27}\right)}{\ln 2} + 1 \\
 & n > 19.49.... \\
 & \therefore \text{it will occur on Day 20}
 \end{aligned}$$

$$\text{(b)(i)} \quad P(\text{red on Monday}) = \frac{3}{5}$$

- (ii) As there are 3 red shirts and only 2 yellow, the colour worn on 3 days must be red.

$$\begin{aligned}
 P(\text{same colour on 3 days}) &= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \\
 &= \frac{1}{10}
 \end{aligned}$$

- (iii) There are 2 possible scenarios:
RYR or YRY

$$\begin{aligned}
 & P(\text{not same colour on consecutive days}) \\
 &= P(\text{RYR}) + P(\text{YRY}) \\
 &= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} \\
 &= \frac{1}{5} + \frac{1}{10} \\
 &= \frac{3}{10}
 \end{aligned}$$

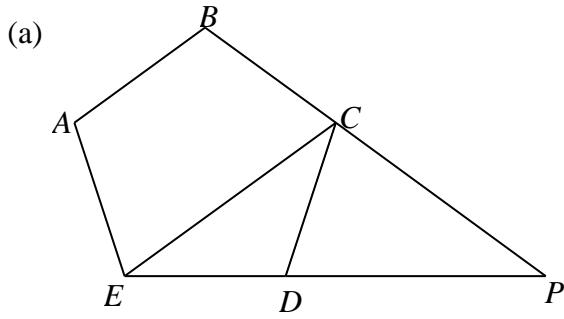
(c)

t	0	5	10	15	20
v	173	81	127	195	168
weight	1	4	2	4	1

$$\begin{aligned}
 \int_0^{20} v dt &= \frac{10-0}{6} [173 + 4(8) + 2(127) + 4(195) + 168] \\
 &= 2831\frac{2}{3}
 \end{aligned}$$

\therefore the jogger travels approximately 2832 m
i.e. about 2.8 km (2 sig. fig)

Question 6



$$(i) \angle CDP = \frac{360^\circ}{5}$$

(exterior \angle regular pentagon)
 $\therefore \angle CDP = 72^\circ$

$$(ii) \angle PCD = 72^\circ \text{ (exterior } \angle \text{ regular pentagon)} \\ \angle CPD = (180 - 2 \times 72)^\circ \quad (\angle \text{ sum of } \triangle PCD) \\ = 36^\circ$$

Now $CD = ED$ ($ABCDE$ regular)

$\therefore \angle CED = \angle ECD$
 (base angles of isosceles triangle EOD)

$$2\angle CED = 72^\circ \quad (\text{exterior } \angle \text{ of } \triangle ECD) \\ \angle CED = 36^\circ \\ = \angle CPD \quad (\text{from above})$$

$\therefore \triangle EPC$ is isosceles (2 equal angles)

$$(b) \text{ Now } PA^2 + PB^2 = 40 \\ \therefore (x+1)^2 + y^2 + (x-3)^2 + y^2 = 40 \\ x^2 + 2x + 1 + y^2 + x^2 - 6x + 9 + y^2 = 40 \\ \therefore 2x^2 - 4x + 2y^2 = 30 \\ x^2 - 2x + y^2 = 15 \\ x^2 - 2x + 1 + y^2 = 15 + 1 \\ (x-1)^2 + y^2 = 16$$

Which is a circle with centre $(1, 0)$ and radius 4.

$$(c)(i) \text{ At } P, x = 0 \quad \therefore y = 2 \cos 0 = 2 \\ \therefore P = (0, 2)$$

$$(ii) \int_0^{\frac{\pi}{2}} 2 \cos x dx = \left[2 \sin x \right]_0^{\frac{\pi}{2}} \\ = 2 \sin \frac{\pi}{2} - 2 \sin 0 \\ = 2$$

(iii) C

$$(iv) \text{ Area} = A + B + C \quad (\text{note Area } B > 0) \\ = 4A \\ = 4(2) \\ = 8$$

$$(v) \int_{\frac{\pi}{2}}^{2\pi} 2 \cos x dx = -\text{Area } B + \text{Area } A \\ = -4 + 2 \\ = -2$$

Question 7

$$(a) f(x) = x^3 - 3x + 2$$

$$(i) f'(x) = 3x^2 - 3 \\ f''(x) = 6x$$

Stationary points occur when $f'(x) = 0$

$$\therefore 3x^2 - 3 = 0 \\ x^2 - 1 = 0 \\ x^2 = 1 \\ x = \pm 1$$

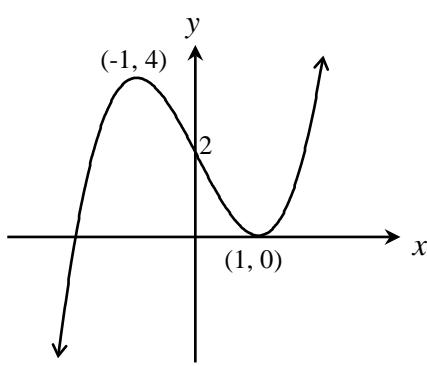
$$\text{At } x = 1: f(1) = 1^3 - 3(1) + 2 \\ = 0 \Rightarrow (1, 0)$$

$$f''(1) = 6(1) > 0 \Rightarrow \text{a min}$$

$$\text{At } x = -1: f(-1) = (-1)^3 - 3(-1) + 2 \\ = 4 \Rightarrow (-1, 4) \\ f''(-1) = 6(-1) < 0 \Rightarrow \text{a max}$$

$\therefore (1, 0)$ is a local minimum and
 $(-1, 4)$ is a local maximum

(ii)



(b) $\dot{x} = 8 - 8e^{-2t}$

(i) When $t = 0$: $\dot{x} = 8 - 8e^0 = 0$
 \therefore it is initially stationary

(ii) $\ddot{x} = -2(-8e^{-2t})$

$\therefore \ddot{x} = 16e^{-2t}$

Now $e^{-2t} > 0$ for all $t > 0$

$\therefore 16e^{-2t} > 0$ for all $t > 0$

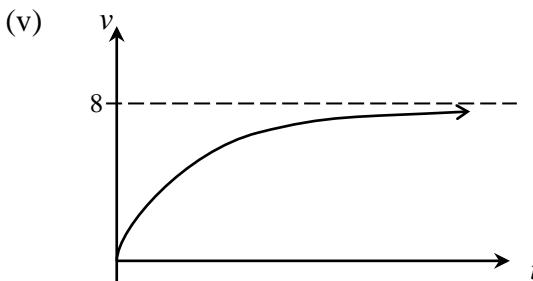
\therefore the acceleration is always positive
 for all $t > 0$.

(iii) The particle starts at rest and is acted upon by an acceleration of 16 ms^{-2} . This moves it in the positive direction. As the subsequent acceleration is always positive, the particle will continue to move in this direction with ever increasing velocity.

(iv) As $t \rightarrow \infty$, $e^{-2t} \rightarrow 0$

$\therefore 8 - 8e^{-2t} \rightarrow 8$

i.e. the velocity approaches 8



Question 8

(a)(i) $22^2 = 20^2 + x^2 - 2x(20)\cos 60^\circ$ (cos rule)

$$22^2 = 20^2 + x^2 - 40x\left(\frac{1}{2}\right)$$

$$484 = 400 + x^2 - 20x$$

$$\therefore x^2 - 20x - 84 = 0$$

(ii) Using the quadratic formula:

$$x = \frac{20 \pm \sqrt{(-20)^2 - 4(1)(-84)}}{2}$$

$$= \frac{20 \pm \sqrt{736}}{2} \quad \text{but } x > 0 \text{ as it is a distance}$$

$$= \frac{20 + \sqrt{736}}{2}$$

$$= 23.564\dots$$

\therefore the distance is 24 km to the nearest kilometre

(b)(i) $V = \pi \int_0^h x^2 dy$

$$= \pi \int_0^h y dy \quad \text{as } x^2 = y$$

$$= \pi \left[\frac{y^2}{2} \right]_0^h$$

$$= \pi \left[\frac{h^2}{2} - 0 \right]$$

$$= \frac{\pi h^2}{2}$$

(ii) $V_{\text{cylinder}} = \pi r^2 h$ where $r = HC; h = OH$

At C : $y = h \Rightarrow x^2 = h$

$$x = \sqrt{h} \text{ as } x > 0$$

$$\therefore HC = \sqrt{h}$$

$$\therefore V_{\text{cylinder}} = \pi (\sqrt{h})^2 h$$

$$= \pi h^2$$

$$\therefore \frac{V_{\text{paraboloid}}}{V_{\text{cylinder}}} = \frac{\frac{\pi h^2}{2}}{\pi h^2}$$

$$= \frac{1}{2}$$

(b)(i) $6\% \text{ pa} = 0.5\% \text{ per month} = 0.005$

The first \$100 grows to \$100(1.005)⁴²⁰

The second \$100 grows to \$100(1.005)⁴¹⁹

The third \$100 grows to \$100(1.005)⁴¹⁸

.....
 The last \$100 grows to \$100(1.005)

$$P = 100(1.005) + 100(1.005)^2 + \dots + 100(1.005)^{420}$$

$$= \frac{a(r^n - 1)}{r - 1} \quad \text{where } a = 100(1.005)$$

$$r = 1.005$$

$$n = 420$$

$$= \frac{100(1.005)((1.005)^{420} - 1)}{1.005 - 1}$$

$$= 143183.83\dots$$

$\therefore \$P = \143183 to the nearest dollar

(ii) (1)

$$A_1 = (29227 + M)(1.005)$$

$$A_2 = (A_1 + M)(1.005)$$

$$= \left[\underbrace{(29227 + M)(1.005)}_{A_1} + M \right] (1.005)$$

$$= 29227 \times 1.005^2 + M(1.005)^2 + M(1.005)$$

$$\therefore A_2 = 29227 \times 1.005^2 + M(1.005 + 1.005^2)$$

(2)

$$A_{240} = 29227 \times 1.005^{240} + M(1.005 + 1.005^2 + \dots + 1.005^{240})$$

$$800000 = 29227 \times 1.005^{240} + M \left(\frac{1.005(1.005^{240} - 1)}{1.005 - 1} \right)$$

$$M \left(\frac{1.005(1.005^{240} - 1)}{1.005 - 1} \right) = 800000 - 29227 \times 1.005^{240}$$

$$M(201(1.005^{240} - 1)) = 800000 - 29227 \times 1.005^{240}$$

$$\therefore M = \frac{800000 - 29227 \times 1.005^{240}}{201(1.005^{240} - 1)}$$

$$= 1514.48 \text{ (2 dp)}$$

\therefore she needs to invest \$1514.48 each month

Question 9

(a)(i) In ΔABC and ΔADE

$$1. \frac{AB}{AD} = \frac{1}{2} \quad (\text{B is the midpoint})$$

2. $\angle A$ is common

$$3. \frac{AC}{AE} = \frac{1}{2} \quad (\text{C is the midpoint})$$

$\therefore \Delta ABC \parallel\!\!\!|| \Delta ADE$

(2 sides in proportion and the included angle equal)

$$(ii) \frac{AB}{BD} = \frac{AC}{CE} \quad (\text{given})$$

$\therefore BC \parallel DE$ (ratios are preserved)

In ΔBCF and ΔDEF

1. $\angle BCF = \angle FDE$ (alternate $BC \parallel DE$)

2. $\angle BFC = \angle DFE$ (vertically opposite)

$\therefore \Delta BCF \parallel\!\!\!|| \Delta EDF$ (equiangular)

$\therefore \frac{BC}{ED} = \frac{BF}{EF}$ (matching sides of similar triangles)

$$\therefore \frac{1}{2} = \frac{BF}{EF}$$

$$\therefore BF : FE = 1 : 2$$

$$\begin{aligned} (\text{b})(\text{i}) \quad \text{Flow } A - \text{Flow } B &= 2 + \frac{t^2}{t+1} - \left(1 + \frac{1}{t+1} \right) \\ &= 1 + \frac{t^2}{t+1} - \frac{1}{t+1} \\ &= \frac{t+1+t^2-1}{t+1} \\ &= \frac{t+t^2}{t+1} \\ &= \frac{t(1+t)}{t+1} \\ &= t \end{aligned}$$

\therefore the flow rate differs by t litres per minute

(ii) The change in volume is $\frac{dV}{dt} = t$

$$\therefore V = \int_0^4 t dt$$

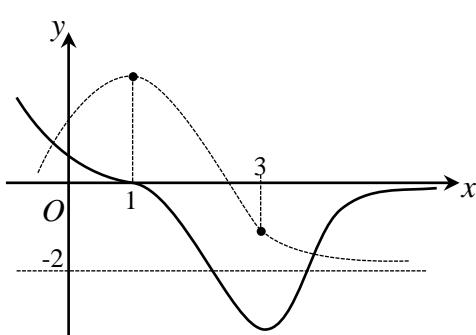
$$= \left[\frac{t^2}{2} \right]_0^4$$

$$= \frac{16}{2} - 0$$

$$= 8$$

\therefore there is 8L more liquid A than liquid B

(c)



$$\begin{aligned}
 \text{(d)(i)} \quad \frac{1}{\sqrt{n} + \sqrt{n+1}} &= \frac{1}{\sqrt{n} + \sqrt{n+1}} \times \frac{\sqrt{n} - \sqrt{n+1}}{\sqrt{n} - \sqrt{n+1}} \\
 &= \frac{\sqrt{n} - \sqrt{n+1}}{n - (n+1)} \\
 &= \frac{\sqrt{n} - \sqrt{n+1}}{n-n-1} \\
 &= \frac{-1}{\sqrt{n} - \sqrt{n+1}} \\
 &= \sqrt{n+1} - \sqrt{n}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{99} + \sqrt{100}} \\
 &= (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + \dots \\
 &\quad \dots + (\sqrt{99} - \sqrt{98}) + (\sqrt{100} - \sqrt{99}) \\
 &= \sqrt{100} - \sqrt{1} \\
 &= 10 - 1 \\
 &= 9
 \end{aligned}$$

Question 10

$$\text{(a)} \quad I = 10^{-12} \times e^{0.1L}$$

$$\text{(i)} \quad \text{Now } L = 110$$

$$\begin{aligned}
 \therefore I &= 10^{-12} \times e^{0.1(110)} \\
 &= 5.987 \dots \times 10^{-8} \\
 &\doteq 6.0 \times 10^{-8} \quad \text{(2 sig. fig.)}
 \end{aligned}$$

\therefore the intensity is about 6.0×10^{-8} watt/m²

$$\text{(ii)} \quad I > 8.1 \times 10^{-9}$$

$$\therefore 10^{-12} \times e^{0.1L} > 8.1 \times 10^{-9}$$

$$e^{0.1L} > 8.1 \times 10^3$$

$$e^{0.1L} > 8100$$

$$\ln e^{0.1L} > \ln 8100$$

$$0.1L \ln e > \ln 8100 \quad \text{but } \ln e = 1$$

$$0.1L > \ln 8100$$

$$L > \frac{\ln 8100}{0.1}$$

$$L > 89.996\dots$$

\therefore about 90 decibels

(iii) Let the original loudness be K .

Then the original intensity is

$$I = 10^{-12} \times e^{0.1K}$$

When the intensity doubles,

$$I = 2(10^{-12} \times e^{0.1K})$$

We need the value of L at this intensity

$$\therefore 2(10^{-12} \times e^{0.1K}) = 10^{-12} \times e^{0.1L}$$

$$2(e^{0.1K}) = e^{0.1L}$$

$$2 = \frac{e^{0.1L}}{e^{0.1K}}$$

$$2 = e^{0.1L - 0.1K}$$

$$\ln 2 = \ln e^{0.1L - 0.1K}$$

$$\ln 2 = (0.1L - 0.1K) \ln e$$

$$\ln 2 = 0.1(L - K)$$

$$\frac{\ln 2}{0.1} = L - K$$

$$L - K = 10 \ln 2$$

$$\doteq 7$$

\therefore the increase is about 7 decibels

(b)(i) Let the length of the arc be l .

$$\therefore l = r\theta$$

$$\therefore P = l + 2r$$

$$P = r\theta + 2r$$

$$P = r(\theta + 2) \quad \text{as required}$$

$$\text{(ii)} \quad A = \frac{1}{2} r^2 \theta$$

$$\text{But } P = r(\theta + 2)$$

$$\therefore \theta + 2 = \frac{P}{r}$$

$$\theta = \frac{P}{r} - 2$$

$$\therefore A = \frac{1}{2} r^2 \left(\frac{P}{r} - 2 \right)$$

$$A = \frac{1}{2} Pr - r^2 \quad \text{as required}$$

$$(iii) \quad \frac{dA}{dr} = \frac{1}{2}P - 2r$$

$$\frac{d^2A}{dr^2} = -2$$

Max/min occurs when $\frac{dA}{dr} = 0$

$$\therefore \frac{1}{2}P - 2r = 0$$

$$\frac{1}{2}P = 2r$$

$$r = \frac{1}{4}P$$

Then $\frac{d^2A}{dr^2} = -2 < 0 \Rightarrow$ a max occurs

\therefore the area is maximised when $r = \frac{1}{4}P$

$$(iv) \quad \text{Now } \theta = \frac{P}{r} - 2$$

$$\therefore \text{when } r = \frac{1}{4}P: \quad \theta = \frac{P}{\frac{1}{4}P} - 2$$

$$= 4 - 2$$

$$= 2$$

\therefore the angle is 2 radians

(v) For the area to exist, $\theta > 0$

$$\therefore \frac{P}{r} - 2 > 0$$

$$\frac{P}{r} > 2$$

$$P > 2r$$

$$r < \frac{P}{2}$$

But in a circle, the greatest possible angle is 2π

$$\therefore \frac{P}{r} - 2 < 2\pi$$

$$\frac{P}{r} < 2\pi + 2$$

$$\frac{P}{r} < 2(\pi + 1)$$

$$P < 2r(\pi + 1)$$

$$r > \frac{P}{2(\pi + 1)}$$

Combining these two restrictions gives

$$\frac{P}{2(\pi + 1)} < r < \frac{P}{2}$$

as required

End of solutions